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Problems with variation in teaching/learning Geometry: an example of Chinese Cultural Transposition

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This paper discusses some theoretical/methodological observation and some qualitative results coming from a Cultural Transposition experience, implemented in the Italian school context (grade 8), according to the methodology of variation, as one of the most significant problem solving approach in Chinese schools. The framework of the Cultural Transposition and the methodology of variation are presented as an important condition for “decentralizing” the didactic practice from a specific social and cultural context. We argue that looking at different teaching/learning mathematics strategies coming from East-Asia cultures can favor some cultural contaminations at school and allow students to a significant and unusual thought about “inclusion” and “diversity” in mathematics. Our variation problems are designed on 3D Geometry and are aimed to guide students in discovering the relationship between pyramid and cone areas/volumes.

Keywords: Cultural Transposition, Chinese problems with variation, Teaching Geometry.

Introduction

In a society dominated by cognitive zapping (Veen & van Staalduinen, 2009), which is constantly changing due to socio-cultural transformations, school is also changing. Learning and teaching trajectories are in fact, in many cases, varying in all countries. Looking at the new scenarios that the school is experiencing in recent years, the classroom realities that teachers and students observe are in fact nowadays changing, enriched by new approaches, new stimuli, new routines, new didactical processes that come from inside and outside social and cultural classroom contexts. These scenarios are in many cases complex and difficult to study for the mathematics education Communities. One pioneering work (Bishop, 1988) highlighted the importance of recognizing mathematics practices as social phenomena that are embedded in those cultures and those societies that generated them. D’Ambrosio (2010) pointed out that taking care about cultural and social issues into mathematical practices contributes to the understanding of cultures and the mathematics itself. Nowadays the awareness of taking into account cultural and historical contextualization in Math classroom teaching practices and the crucial assumption that culture permeates mathematics education practices is well known by all the mathematics education Communities. As Radford underlined, it has to be clear for teachers that the “configuration and the content of mathematical knowledge is properly and intimately defined by the culture in which it develops and in which it is subsumed”. (Radford, 1997). Barton’s research (Barton, 2008) reinforced the importance of a cultural perspective to study mathematics education phenomena, looking for example at different languages, and cultural manners. Agreeing to this perspective, we argue that culture gives students an opportunity to engage in mathematics focusing on concepts of inclusion, integration ... to discover diversity as a resource! In some cases, the participation of many countries in international student competence tests (such as PISA) gives also the opportunity not only to compare different countries

obtained results (Schleicher, 2019), but also to reflect on the educational practices, social, political and cultural environments that determined these performances. The last good results (as the previous ones) obtained by students from East Asia, especially those from Hong Kong and Shanghai, have prompted comparisons to reflect on the “reasons” for this educational success. It is well known (Spagnolo & Di Paola, 2010) that in these countries some learning approaches that compared to those of their Western peers produce different mathematical skills, in many cases more useful. According to Di Paola (2016) and Mellone et al. (2019; 2021) in these learning/teaching approaches there are many cultural factors linked to different cultural assumptions, educational practices and related cognitive styles (Bartolini Bussi & Martignone, 2013; Mellone et al., 2019). The same feeling is nowadays claimed by many teachers and researchers around the world: they in fact confirm how, in many cases, East Asian students, already educated in their own country, show more developed knowledge and skills in many mathematics subjects.

Theoretical framework

Bartolini Bussi & Martignone (2013), studying didactical phenomena discussed in the previous paragraph, emphasized the possible correlation between mathematical knowledge and the cultural context, cultural beliefs, in which they are and have been inserted and in which, therefore, mathematical knowledge is elaborated, assimilated and transmitted. In recent years, some other research works, through qualitative and/or quantitative approaches (Bartolini Bussi et al. 2017; Di Paola 2016; Mellone et al., 2019) pointed out how the *cultural diversity* could become, in this sense, an opportunity in mathematics Education (Kaiser, 2018). Researchers and teachers, coming into contact with educational practices adopted in other cultural contexts, are able to *deconstruct* (Derrida, 1967) them, reconsidering the themes of educational intentionality defined as background of their educational practices. Mellone et al. (2019), inspired by the Skovsmose’s (1994) approach, defined the framework *Cultural Transposition* as “a condition for decentralizing the didactic practice of a specific cultural context through contact with the didactic practices of different cultural contexts”. *Cultural Transposition* (CT) is a perspective that can allow the meeting between different mathematics education school practices/approaches, coming out from different cultural contexts, and define a potential space for reflection and awareness, and also development for researchers, educators, teachers, students (Mellone et al., 2021). CT involves those who implement/observe math teaching practices, coming from other cultures, to a “*deconstruction*” process useful to a re-interpretation of their own thought and consequently a possibility to change/improve personal (cultural) beliefs, values, and didactical principles. As Derrida stated defining the deconstruction, “*an analysis of the different levels in which a culture is stratified*” (Derrida, 1967). According to this perspective it is possible to recognize, valorize and include possible differences coming from “other” cultural Communities, linked to different values, principles, beliefs systems, happens, for example, in schools Communities (Mellone et al. 2021). To “get in touch” with different educational practices, coming out from different social and cultural contexts, can help researcher and teachers not only to become more aware of their social and cultural paradigm in regards to the classroom teaching practices but also to deconstruct their thought decentralizing their cultural expectation and assumptions and rethinking mathematics educational practices (Bartolini Bussi et al, 2013) in terms of “inclusion” and significant us of

possible social, cultural “diversities”. Of course this “changing process” is complex and needs more and more opportunities for reflection and *contamination* (Bartolini Bussi et al., 2017; Jullien, 2006). From the school student’s perspective, *diversity and difference* in learning mathematics strategies appears, in this sense, a great opportunity, a good chance for something that maybe should not be favored without this “revolution” of prospective and without this change in the cultural system of reference. This could permit us to look at diversity and difference in mathematics and in mathematics teaching within the realms of the cultural, the social and the political. Spagnolo & Di Paola (2010), a pioneer of this kind of subject, presented this approach as a continuous open dialogue between cultures, societies, histories ... useful to *cross* the didactics of mathematics. In the last years several researches (e.g. Bartolini Bussi et al., 2013; Di Paola, 2016; Mellone et al., 2021; Spagnolo & Di Paola 2010) discussed different CT experiences in Western school contexts. Several of them look to Chinese culture and in particular the use of Chinese practices of “*problem with variation*” for an early approach to Algebra in Primary school. What is a rather new (Leung, 2003), research subject discussed in this paper, is the use of this methodology in teaching/learning mathematics in Middle school and, in particular, in teaching/learning Geometry.

“*Problems with variation*” is considered one of the most significant mathematics education tools used in Chinese Primary school (e.g. Bartolini Bussi et al. 2017; Fan et al., 2004; Mellone et al., 2019; Sun, 2011). In the last twenty years, many researchers underlined the importance of the variation approach as a necessary condition for deep learning (e.g. Marton & Booth, 1997; Sun, 2011) and in particular for the learning of mathematics. variation is typically expressed by Rowland (2008) as a practice in structured exercises which varies considerably from country to country and from text to text. Sun (2011) underlines the use of variation problems as a tool “to discern and compare the invariant features of the relationship among concepts, solutions and contexts, and provide opportunities for making connections, since comparison is considered the pre-condition to perceive the structures, dependencies, and relationships that may lead to mathematical abstraction.” (p.107). In Chinese language it is *Bianshi* (變式,) where *bian* stands for “*changing*” and *shi* means “form.”. Yakes and Star (2009) looked to variation as a critical means for comparing and developing flexibility for learning mathematics already from the first school years. In China, the variation approach to problem solving is linked to many disciplines and used in all school levels from the first grades. The issue of variations in problems perfectly reproduces one of a Chinese proverb: “*no clarification, without comparison*”, and it is “*in contrast*” to the assumption used in many textbook in different countries: “to consolidate one topic, or skill, before moving on to another” (Rowland, 2008). According to Rowland, comparing this problem solving approach with the one used, in many cases, in the Italian school (textbooks and/or practice), it is in fact possible to point out a strong difference, in some cases, common in several Western countries and related to the use of isolated problems/exercises, organized in progressive steps, strongly partitioned between them and so not very useful to define a possible abstract thinking looking relationships between concepts, strategies, algorithms (Cai & Nie 2008; Spagnolo & Di Paola, 2010). These considerations can be useful to underline the significant opportunity to proceed with *contamination experiments* (Jullien, 2006) aimed to get in touch with different “good practices”, coming from different cultural contexts and linked to different language; different historical tradition; ideologies; school systems; governance structures... In this sense, the *contamination* appears as an important condition for teachers to “decentralize” their own didactic practice of a specific cultural context to

something more wild, different, and in many cases more useful for their own mathematics students and could be included. Bartolini Bussi et al. (2013), Di Paola et al. (2016), Mellone et al. (2021) discussed this approach in Italian schools for an early approach to Algebra in Primary school. As we already discussed, very few research papers refer instead to the inclusion and the use of “*problem with variation*” in teaching/learning mathematics in Western Middle school (grade 8) and, in particular, in Geometry. The CT discussed in this paper is aimed to look at a different teaching/learning mathematics practice, coming from East-Asia cultures, as an important opportunity for a significant and unusual (for the Western culture) 3D Geometry problem solving. In particular the proposed variation problems are aimed to guide Italian students in discovering, almost autonomously, the relationship between pyramids and cones area/volume. According to the declared aim, our lens was focused on students' reaction to this approach and, in particular, the research question to which we tried to answer was: *What kind of process do grade 8 students of Italian (Western) culture show in variation geometric problem solving?*

Methodology

In this section we discuss our CT in teaching/learning Geometry, implemented in the Italian school context (four classrooms of grade 8), according to the methodology of variation. It is important to underline that our intention was not an attempt to *translate*, or even worse *import* a Chinese practice/teaching strategy into the Italian culture and more in general in the Western one. On the contrary, the educational path we analyze, developed by the author of this paper and the group of teachers and researchers with whom we have collaborated for some years, is aimed to create a real CT, with an interesting inclusion of the different Chinese model into the Italian didactical practice in grade 8. With the aim to reply to the research question expressed before, our CT was particularly focused to help students to transfer their mathematics knowledge from a well-known context to another apparently different context, using the variation approach. The two contexts refer to 3D Geometry and in particular to the concepts of pyramid, cone and their area and volume. Authors of this paper were involved in all phases of the education path, the design, the implementation of it with school teachers and the analysis of the collected data. The grade 8 students (around 100) were engaged for almost 20 hours. All of them were conscious about experimenting with a pedagogical method from another culture; during the CT path they explored it, initially by themselves, after being guided by the Authors of this paper.

According to the declared aim and what the literature discusses about educational practices based on the same research subject (e.g. Bartolini Bussi e al., 2017, Mellone et. al., 2019), the sections of the CT path referred to the proposed mathematical subject were designed following this frame:

1. Pre/Post-questionnaire about 3D solids knowledge (pyramid and cone and their area/volumes),
2. Single students resolution of different variation problems about pyramid and cone solids.
3. Students interview about their own variation problem solutions (difficulties and mistakes personally implemented CT).

Before to start the first phase, with the aim to propose useful tasks in all research phases of the implemented educational path, we provided a structured and very fine *a-priori* analysis of Chinese and Italian textbooks (schemes, images, writing, task ...) and of methodological implicit assumptions that variation approach could have had on the involved students about the inclusion of this didactic methodologies in the one commonly adopted by Italian teachers. All research phases were video and audio recorded. These collected data were qualitatively and quantitatively analyzed (with cluster and implicative analysis). Researchers and teachers studied these data step by step during all CT path phases in order to eventually redefine and redesign the path frames. According to the contamination theory, protocols and interviews were examined with a specific focus on possible interesting used of different signs (words, pictures, arrows, colors, ...) useful to describe students approach to variation and, in particular, to underline possible difficulties or readiness to define relationship and among concepts, solutions and contexts. Cluster and implicative analysis gave us the possibility to put in evidence possible stable behaviors in the analyzed students sample. In this paper we don't present these data; we are referring only to some qualitative findings.

An example of an implemented variation problem task (the first triplet) is shown in Figure 1. Starting from a problem concerning only the pyramid solid and its area (first problem on the left), we asked the students to solve two more problems on the cone solid (regarding its area and volume). According the variation problems theory and the contamination one, discussed before, the defined triplet was structured in order to favor the possibility to find, independently, the relationships (similarities and differences) between the two proposed texts and the images of geometric solids, without an explicit “presentation” by the teacher of the second one (about the cone).

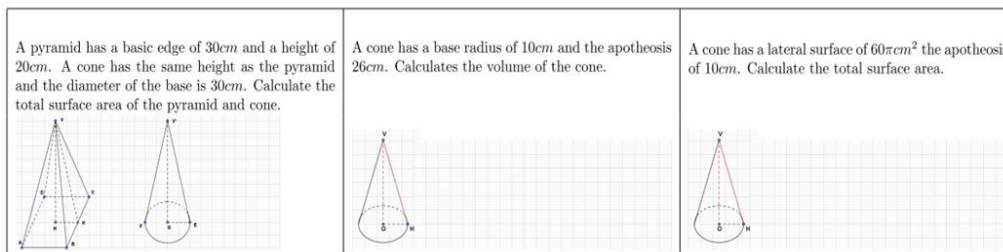


Figure 1. Our CT: an example of a first triplet of variation problem

As we also mentioned before, all students knew the concept of pyramid and its area and volume calculation and properties; none of them knew (we formally tested it) the cone solid. Students, to solve this task needed to autonomously “transfer” (using variation approach) knowledge and skills acquired on the pyramid solid, to the cone. In the three problems the graphical representation has been also inserted in order to help students to better focus on the relationships and possible links in area and volume calculation. This choice retraces what Chinese textbooks and Chinese teachers commonly propose in classrooms.

Empirical findings and first conclusion

In order to give an overview of the obtained results and related discussion, in this section, we briefly present some finding, come from the analysis of students protocols referred to the phase 2 of the implemented CT path and some data coming from a qualitative analysis of the same two students interview (phase 3 of the CT) about their own variation problem solutions.

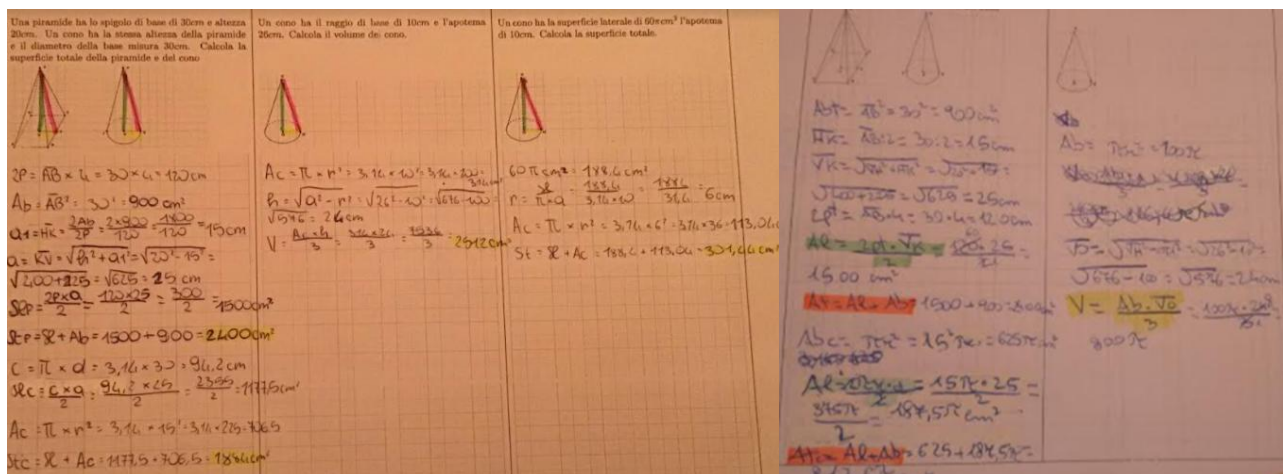


Figure 2. Maria’s and Valentina’s variation problem protocols

Maria’s and Valentina’s protocols appear interesting for replying to our research question. In both students sheets it is, in fact, possible to observe an noteworthy and autonomous use of the colors in the proposed variation problems. Maria used colors to highlight analogies and relationships between part of solid. A solid that she knows and a solid that she doesn’t know. Valentina did the same but taking into account the solving strategies and the formulas, instead of the graphic solid representation. Thanks to the Chinese variation approach, she autonomously discovers a relationship between two used formulas. In both cases (we found similar approaches in many student protocols) the opportunity to be *contaminated* by the Chinese variation methodology (Mellone et al., 2019, 2021) guided them in discovering in the problem solving activities something new as the relationship between pyramids and cones area/volume. Their approach to variation problems and their “surprise” to the possibility to autonomously learn “new mathematics' ' (Maria used these terms during her interview), clearly emerged also during their subsequent interview. The following part of the dialogue between Maria and her teacher, appears interesting:

- Teacher: Could you better explain to me what you mean?
- Maria: I highlighted with the same color what behaves in the same way.
- ...
- Maria: Here I have something that I know and something unknown into the same problem. I tried to look, to search for analogies and relationships. It's a interesting problem. I like it.
- Teacher: What did you find?
- Maria: All are right triangles, the first two are just the same ... same numbers! So I highlighted them the same way.
- ...
- Maria: Prof., could we do the same argumentation between Prisms and Cylinders? We studied them... We could find a general “rule”, we could use less memory and more variation. Problem solving can become more “simple”.
- Teacher: You can try, ...
- Maria: It is great. I didn't know anything before today about it. We have to look also to other countries (smiling) ...

As Maria, almost all students benefited from the Chinese contamination of variation problems. they declared that working with this methodology, they became more aware of their knowledge and they were able to construct step by step a general solving approach to this kind of problems (the same finding in Cai & Nie, 2008). Of course a few of them (Giovanni, Marco and Francesca speeches are some examples) didn't immediately reach up to this expertise.

Giovanni: We haven't studied the cone solid, the teacher was wrong ... that's why I only solved the pyramid problem.

Marco: I don't know the cone and I don't remember the pyramid, sorry.

Francesca: We have not studied the cone, but the teacher gave us a task with the pyramid and the cone; we don't know anything about the cone ... I tried to use what I knew about the pyramid but I couldn't find the way.. why did the teacher put the pyramid next to the cone? I will try to think about it. We didn't do this same kind of problems before

As we said before, this “changing process” is complex and long (Bartolini Bussi et al., 2014); Giovanni, Marco and Francesca need more and more opportunities for reflection and *contamination*.

According to Spagnolo & Di Paola (2010), Sun (2011) and Mellone et al. (2021) findings, our students approach underlines how the implemented CT path gave them a good chance to stimulate and improve a possible abstract thinking, finding relationships between concepts, strategies, algorithms in variation problems. The defined variation problems gave them an important opportunity to deepen abstract geometrical thinking - structure, strategies, relationship, concepts ... - (Leung, 2003) than does typical isolated problems on the two investigated solids as, in many cases, happens in Italian (but also in other Western countries) school context. They discovered a “new”, “diverse” culture and different related problems solving approaches that enrich their teaching practices and, according to our finding, their mathematics learning (Barton, 2008). Thanks to the CT path they had the opportunity to engage in mathematics through a cultural lens, helpful to discover and use diversities as resources for their future teaching. This gave us, as researchers, the chance to rethink our own cultural expectations and assumptions about possible important cultural, social and political issues in Mathematics Education research.

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