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Social (in)equality through mathematical modelling? – Results of a case study

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Solving realistic mathematical tasks with multiple possible solutions can require many competencies. At the same time, they can allow students to engage with a situation mathematically according to their own preferences. Previous studies seem to indicate that – through socialisation – socio-economically privileged students are more likely to acquire skills dealing with such tasks. This paper approaches the described issue qualitatively by comparing modelling processes of privileged and unprivileged student pairs. It turns out that privileged pairs, on average, spend more time on making real-world assumptions and they show a broader spectrum of assumptions compared to unprivileged pairs. Thus, it is discussed to what extent differences and similarities can be traced back to students' habitus and how modelling tasks may thus increase both social inequality and social equality.

Keywords: Social (in)equality, mathematical modelling, qualitative content analysis, Bourdieu.

Introduction

In a variety of studies social background is associated with school success (e. g., OECD, 2019). This can be explained, among other things, by parents who are more or less likely to support their children's learning financially, culturally, socially and psychologically (OECD, 2019). Moreover, certain characteristics tend to be taught in socio-economically privileged households, such as argumentation and communication, discussing learning strategies or perseverance in problem solving (e. g., Weininger & Lareau, 2009). In the German educational standards for mathematics (KMK, 2004) such characteristics are described as competencies students are meant to develop. It seems that privileged students internalise a habitus through socialisation (Bourdieu, 1994/1998), which rather finds acceptance in school. Furthermore, in mathematics class students should apply their knowledge in real-world situations in order to be able to view natural, social and cultural phenomena from a mathematical perspective (KMK, 2004). The role of the social background when dealing with real-world tasks is, thereby, controversially discussed. While, for example, Piel and Schuchart (2014) show social class differences to be more likely to occur for reality-based tasks than for purely mathematical tasks, Ay et al. (2021) find socio-economic status to be less strongly related to the solving of modelling tasks compared to other tasks. This paper focuses on this field of research from a modelling perspective, by analysing and comparing modelling processes of privileged and unprivileged student pairs qualitatively (Schreier, 2012). Using Bourdieu's habitus, it thereby shall be discussed to what extent differences can be explained by students' socialisation and in what way results from other fields of research can be confirmed.

Theoretical framework

This raises the question of why socio-economic status can be related to certain school behaviours. For instance, the number of books on the shelf cannot yet explain how social inequality occurs.

According to the sociologist Pierre Bourdieu, there is a connection between one's socio-cultural position and the individual lifestyles. A mediator, Bourdieu calls *habitus*, stands between social position and individual patterns of thought and action. Hereby, one's habitus represents the disposition towards the world, such as habits, ways of life, attitudes, values, aesthetic standards, etc. According to the habitus-theory, most actions of individuals do not pursue an intention, but are an expression of their acquired dispositions (Bourdieu, 1994/1998). The habitus creates a scheme that individuals use to classify and judge other situations, objects, persons, actions etc. – and ultimately themselves. However, individuals usually do not consciously activate these schemes. Clearly, people from the same social class do not completely match in their habitus. Nevertheless, they are most likely to have had more similar experiences and, thus, match more in their behaviour than people from other classes (Bourdieu, 1972/1977). Thereby, a person's habitus can often be detected in little things.¹

Social (in)equality and the processing of tasks

When addressing social (in)equality, a variety of factors such as migration background, language, gender and school systems, can be relevant. In this study, socio-economic status is considered as a central distinguishing characteristic since it measures family resources and their social position and thus aligns with Bourdieu's concepts. For some time now, empirical studies based on sociological theories regard how children process tasks. In one study, children are given 24 pictures showing food. They are asked to group the pictures so that they fit together well. It turns out that the unprivileged children sort the pictures more often according to their own experience, such as 'tastes good', while the privileged children are more likely to sort the pictures according to more abstract criteria such as 'vegetables' (Holland, 1981). Similar results can be found in other studies. Students are shown sketches of two tables showing pizzas and seats. On one of the tables, there is one more pizza, but also two more people can be seated here. Students are asked which table they would join and why (Lubienski, 2000). In her study, the unprivileged students are more likely to focus on real-world concerns (e. g., arriving late) instead of using the task to learn generalizable methods (using relations to make comparisons). The author states that this experience-based orientation could hinder unprivileged students in understanding the mathematical ideas behind the situation. On the other hand, the author finds several instances of unprivileged students being concerned with getting the algorithm that solves the task, getting frustrated and giving up more quickly when facing barriers.

However, looking at social diversity in a fruitful way also entails considering the needs and strengths of unprivileged students. Previous research suggests that it might just be topics that are problematic, communicative and relevant to students' real world that enables them, regardless of social background, to participate in the classroom according to their own abilities and experiences (Nasir & Cobb, 2007).

¹ Bourdieu's habitus-theory is often accused of determinism. Bourdieu argues that individuals can be free and creative within a certain frame. According to him, certain life courses are not predetermined, but more or less likely.

Mathematical modelling

Mathematical modelling can be defined “as the solving of a realistic problem” (Maaß, 2010, p. 288). Modelling problems are often accompanied by authentic situations, missing relevant information and multiple possible approaches. During a modelling process students need to identify and collect relevant information, translate a respective situation into mathematical terms, structures and relations, work within the mathematical model, interpret and check results with respect to the corresponding situation (KMK, 2004). This distinguishes modelling from embedded tasks, as embedded tasks use contexts from the real-world, but “have no real relation to reality. The factual context is of no importance regarding the solution” (Greefrath et al., 2017, p. 933). Following the rules of the game of embedded tasks, students can usually be successful in the mathematics classroom if they ignore the context of a situation, use recently learned formulas and don’t question the motivation as well as the action of involved persons (Verschaffel et al., 2000). When approaching modelling tasks, these rules hardly apply. Instead, real-worlds assumptions and everyday knowledge are crucial for modelling.

Synthesis and research questions

Now, why should one draw on Bourdieu’s concepts of habitus when discussing social (in)equality in mathematical modelling? The metaphor of a game might clarify this. When playing a game, players need a practical sense or a *feel for the game*. That is the “mastery of the logic or of the immanent necessity of a game – a mastery acquired by experience of the game, and one which works outside conscious control and discourse” (Bourdieu, 1987/1990, p. 61). This construct coincides to some extent with the game in mathematics classroom (Verschaffel et al., 2000). Both have rules to be followed in order to be successful, have an inherent logic that is not revealed and are acquired through experience. Looking at previous empirical findings, it seems that many unprivileged students lack a feel for the game when they draw tasks with (seemingly) multiple possible solutions on everyday experiences rather than more abstract constructs such as the mathematics beyond (Cooper, 2007). According to the “habitus as the feel for the game” (Bourdieu, 1987/1990, p. 63), different immanent rules apply in different fields so that habitual behaviours are manifested in social classes. Bourdieu (1984) describes the habitus of lower social classes as a *taste of necessity*, focusing in everyday life on practical, functional and technically necessary things, on conformity and on immediate satisfaction of needs. Such taste may thus become apparent when observing students organising food according to their desires (Holland, 1981) or discuss planning to meet their friends (Lubienski, 2000).

It remains unclear, to what extent these concepts can be applied to mathematical modelling. While the inclusion of reality-based experiences is highlighted in other studies as being disadvantageous for task processing, it might just be seen as an advantage in modelling. For example, assumptions are essential and, thus, a real-world model is still being developed. Therefore, characteristics attributed to unprivileged students might be advantageous here. Still, a wide set of competencies which can be seen as useful for modelling has been attributed to privileged households in various studies (e. g., Weininger & Lareau, 2009). This leads to the following research questions:

To what extent do socio-economically privileged and unprivileged pairs differ in making assumptions in the modelling process? To what extent can conclusions be drawn about students' habitual behaviour?

Methodology

A qualitative approach is chosen with the aim of understanding, reconstructing and interpreting contexts and processes. 24 tenth-grade students (around age 15) from two urban secondary schools in western Germany belonging to four different classes partake in this study. The student body of both schools is considered culturally diverse and heterogeneous in performance due to schools' locations and concepts. Pairs of students are shown a picture of a giant pizza (www.bit.ly/2WRXOSU) and get the information that they are planning a party for 80 people and the task to figure out how many pizzas to order. The context comprises a realistic situation and the giant pizza can be ordered in the students' home region and, thus, may have a connection to their everyday life. To enable drawing conclusions, the pairs are put together according to their socio-economic status. Therefore, student and parent questionnaires are carried out for measuring students' HISEI (*Highest International Socio-Economic Index of Occupational Status of both parents*). The HISEI is determined by the professions of the parents and takes income and educational level into account (Ganzeboom et al., 1992). Students who fall into the upper quartile of the nationwide comparison of the HISEI are considered socio-economically privileged. Reversely, students from the lower quartile are considered unprivileged. The survey is divided into the three phases of observation, stimulated recall and interview. A qualitative content analysis (Schreier, 2012) serves as foundation for the data evaluation. Therefore, all phases are being recorded, transcribed, and coded. The evaluation follows a quantitative-qualitative approach by comparing categories quantitatively, identifying qualitative differences and supporting concise differences by means of transcript excerpts. To ensure coding quality, codings were carried out and compared by two independent persons. In addition, inductive subcategories were developed based on the research interest. For example, processes that serve to develop a real-world model are divided into subcategories like *simplifying*, *organising* and *assuming*. Simplifying contains e. g. repeatedly reading text segments or identifying missing information. Organising is often reflected by students drawing or measuring, and assuming entails that assumptions and premises are set or estimated using everyday knowledge. Whereas simplifying represents a surface-level processing strategy, organising and assuming represent deep-level processing strategies (Schukajlow et al., 2021). This paper will present some results on these subcategories. For readability and recognition, privileged students are assigned a three-syllable name and unprivileged students a two-syllable name.

Results

The modelling task Giant Pizza allows multiple possible approaches and solutions. The students can decide individually, which meaning they attach to the photo, to what extent they estimate using everyday knowledge and objects of comparison and to what extent they use mathematics to solve the task. Accordingly, a wide variety of approaches are found. Firstly, there are five pairs (Dominik & Krystian, Vivien & Oliver, Tobias & Benedikt, Samuel & Nathalie, Dawid & Leon) who choose a mathematical approach by estimating the diameter of both the giant and an ordinary pizza,

determining their areas, comparing them and generating a real result. Secondly, some pairs (Julia & Florian, Samuel & Nathalie, Michael & Paulina, Kaia & Mila, Ronja & Hürrem) choose an extra-mathematical procedure by dividing the giant pizza visually and concluding the number of giant pizzas to be ordered. Thirdly, there are pairs (Amba & Bahar, Lena & Pia, partly Sofi & Aram) where the result is rather guessed. Regardless of their approach, all couples come to a final solution.

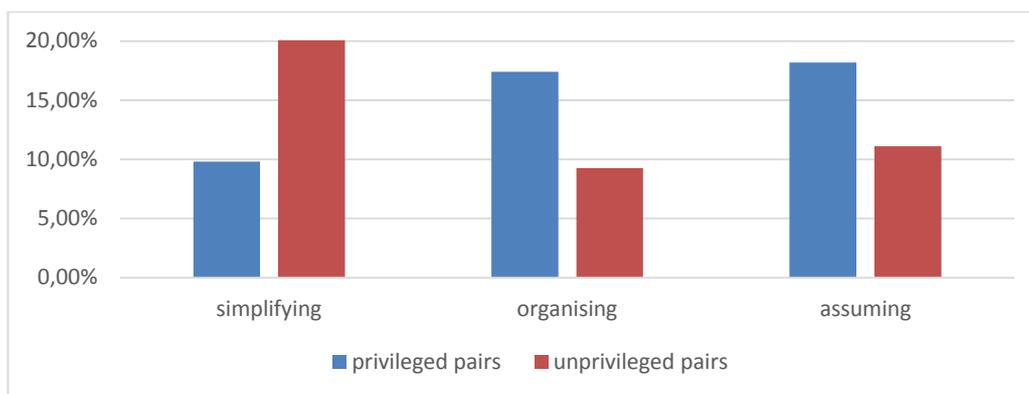


Figure 1: Contribution of the categories

Figure 1 shows that privileged and unprivileged pairs differ regarding their processes. On average, the unprivileged pairs deal more extensive with simplifying processes, whereas privileged pairs put a stronger focus on organising and assuming. In many processes, estimations play an important role, although not all students use them as part of a solution approach. Only the five initially mentioned pairs use everyday knowledge to estimate the diameter of the giant pizza. As objects of comparison, they use parameters such as the height of people or the dimensions of trailers. Additionally, there is a variety of other estimates and premises that pairs consider relevant (see Table 1).

Table 1: pairs’ everyday knowledge and objects of comparison

... observed only in unprivileged pairs	... observed in both groups	... observed only in privileged pairs
size of the angle of a pizza slice	diameter of an ordinary pizza; pizza per guest; width of a person; gender / age of the guests; comparison with the school class	height of a person; size of a family pizza; size of a salami slice; length and span of a hand; length of a forearm; size of a pizza plate; dimensions of a trailer; size of a bun; thickness of the giant pizza; duration of a party; other foods at the party

On average, the privileged pairs do not only spend more time on assuming, they also show a broader spectrum of assumptions. Some are used to compare sizes, others for visualisation and others for validating the results. Five pairs use visualisations to generate a real result. Two of them (Julia & Florian, Michael & Paulina) thereby express objects of comparison, such as the hand of the women and the size of a pizza plate, to develop their model. Taking a look at the process of Kaia & Mila shows that they also use visualisation as basis for their approach. “How many pizzas do you think you can fit in here, normal ones? [...] 1, 2, 3, ((tracing circles)) 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, shall we just say 20?” (Kaia, 04:42) As other students, Kaia carries out the visualisation without sketching. The pair does not estimate the diameter of the giant pizza and does not express objects of comparison to find out the number of ordinary pizzas that match the giant pizza. In the interview Kaia explains: “And there’s the picture, then I have a rough idea of it and

can put a guess in there accordingly.” (01:18) Even though she does not include any objects of comparison explicitly, she seems to have a sense of the size. Instead, she expresses other assumptions:

- 02:40 Kaia: [...] if you're planning a party [...] you have to assume that people are our age.
02:54 Mila: Okay.
02:55 Kaia: [...] if I we're planning a party (.) I would do about half boys, half girls, right?
[...]
03:07 Let's say then that women usually eat a little less than men [...]

Kaia thus relates the task to a situation that is close to her life: “It says YOU and a party with 80 people. [...] people at a young age, they certainly invite people from their grade or from some hobby area, football or something else.” (03:07, stimulated recall) Other pairs are rather guessing a result. Amba & Bahar, for example, keep discussing what the intention of the task might be. Bahar explains what to do instead of calculating. “I don't think we're supposed to calculate anything there [...] we're just supposed to say how many pizzas, whether that's enough.” (03:22) Asking Amba about this during the stimulated recall she explains: “it's a task where you just have to estimate and not really calculate. You can tell by feeling how much you need or don't need. [...] You don't have to think much.” (01:16) Although they make a few assumptions and visualise to some extent, they mostly go back to repeatedly reading the task to find some hidden information. In the end, they guess a result.

Discussion

The aim of this paper is to uncover differences and similarities in the modelling processes of socio-economically privileged and unprivileged students as well as to draw conclusions about habitual behaviour. For this, 12 pairs of students work on the modelling task Giant Pizza. All pairs are organising or assuming in some way, for example sketching or using everyday knowledge. Nevertheless, the pairs differ noticeably in how deeply they engage with these processes and whether those lead to a further development of their models. The work assignment of the modelling task does not indicate that estimation is to be done using everyday knowledge or that mathematisation should be carried out. It stays hidden, to what extent reality is to be considered. Five pairs choose to calculate and compare the areas of the giant pizza to an ordinary one using estimation and everyday knowledge. Four of those pairs are assigned to the socio-economically privileged group. Five pairs divide the giant pizza visually and conclude the number of pizzas to be ordered. Here, all of those who expressed objects of comparison for their visualisation are socio-economically privileged. This is also reflected in the quantitative comparison. The unprivileged pairs engage more in reading repeatedly and talking about the relevance of the text elements, whereas the privileged pairs spend more time organising and assuming. Further, the processes of privileged pairs are characterised by a wider range of estimates and objects of comparison, which they use to develop their models and verify their results. It seems like surface-level processing strategies (Schukajlow et al., 2021) are rather used by unprivileged pairs while the privileged pairs tend to focus more on deep-level strategies. Relating these results to Bourdieu's game (1990), there are inherent necessities or logics in the modelling task. It seems that the privileged pairs are more likely to recognise them (as stated by Cooper, 2007). These necessities are contrary to the rules that usually apply for embedded mathematical problems (Verschaffel et al., 2000), where the situation

has no relevance for processing. Those school socialised routines do not apply here, and thus habitual differences may be meaningful in explaining the observed differences.

With regard to the taste of necessity of lower social classes (Bourdieu, 1979/1984), the process of Kaia & Mila in particular stands out as one of the few unprivileged pairs who intensively deal with assumptions and everyday knowledge. While most privileged pairs estimate measurements and use objects of comparison, Kaia & Mila make assumptions regarding the organisation of their own party, for example the gender balance, the age of the guests and the leisure activities people are invited from. In accordance with the findings of Bourdieu (1984), Holland (1981) and Lubienski (2000) one could say, that the approach of Kaia & Mila conforms to everyday useful purposes. A notable difference to Lubienski's study however is that she considers it less appropriate when students refer to everyday experiences. Here, using everyday knowledge is not inadequate, but conversely, of central importance. Yet, differences are apparent in the use of everyday experiences.

Nevertheless, socio-economically unprivileged pairs appear across all described approaches. Besides, even though they are less likely to bring in estimates, a few seem to have a sense of dimensions of objects of comparison. Also, all pairs still achieve a final result, although the approaches vary in depth and adequacy. Unprivileged students who are more likely to give up (Lubienski, 2000), cannot be observed – despite frustration. Further, three unprivileged pairs use adequate mathematical or visual approaches to find a result. In addition, also pairs who follow a guessing approach show organising and assuming to some extent. Due to mathematical errors or misconceptions, such processes remain partly infertile or get stuck.

At this point, some aspects should be discussed critically. Bourdieu does not operationalise his constructs essentially, and his empirical findings refer to France in the 1960s. Thus, his constructs run the risk of being overinterpreted and his work can hardly consider more recent phenomena such as educational expansion. Nonetheless, current empirical studies can confirm Bourdieu's findings to some extent and, thereby, make his constructs more tangible. Bourdieu's theories and constructs are primarily sociological in nature and not directly designed for application in didactic research. Other factors that may be relevant here but are not (or can't be) controlled include other characteristics of social background and the individual school context in which the students find themselves. Moreover, it could also be fruitful to analyse this data through the lens of Bernstein's work on realisation rules and language codes. For methodological reasons, a dichotomization of socio-economic status is carried out into privileged and unprivileged pairs. This represents a simplification of reality, as socio-economic background is a complex construct that entails individual pathways in specific cases (Weininger & Lareau, 2009). In addition, more cases need to be studied to confirm the results.

In this paper, systematic differences and similarities between socio-economically privileged and unprivileged pairs of students in mathematical modelling can be identified. Thereby, behavioural patterns become apparent which may be attributed to students' socio-economic background, be it in relation to Bourdieu's habitus or to more recent empirical studies. At the same time, many fertile approaches are evident regardless of socio-economic status. Overall, mathematical modelling tasks seem to contain aspects that are rather difficult for unprivileged students. At the same time, the case

analysis provides evidence that all students in some way can engage in the context and achieve a result (for a quantitative comparison see Ay et al., 2021). This paper, thus, provides indications that mathematical modelling contains aspects that may increase social inequality as well as social equality. In addition to investigating these findings further, it needs to be discussed what must be done so that all students can benefit from mathematical modelling regardless of their background.

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