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## Article Qualitative analysis of a 3D multiphysics model for nonlinear ultrasonics and vibration induced heating at closed defects

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- Abstract: Upon exciting a material using elastic waves, the locally induced deformation at the
- interfaces of internally closed defects may cause nonlinear wave mechanics and dynamics in
- <sup>3</sup> the form of clapping and friction. As a result, both phenomena instigate spectral broadening of
- 4 the excitation spectrum as well as the production of heat, directly originating from the defect.
- 5 To better understand and account for the physics behind the dissipation of energy by internally
- 6 closed defects as a result of the wave-interface interaction, dedicated models can be developed. In
- r this work, we propose a 3D finite element multiphysics model that is capable of simultaneously
- describing the generation of nonlinearities and heating at a defect's interface experiencing clapping
- and friction induced by elastic wave propagation. The model consists of three different modules.
- <sup>10</sup> The first module describes the elastic wave propagation in a virgin/bulk material, whereas the
- second module captures the contact physics at the defect level. The third module is implemented
   to calculate the diffusion of thermal energy in the specimen. The contact physics module accounts
- <sup>13</sup> for anharmonic and hysteretic effects, describing the nonlinear behavior of the defect's interfaces,
- which is echoed in both the ultrasound spectrum and in the vibration induced heating. A qualita-
- tive analysis of the computational model, integrating the three modules, is performed to validate
- the approach. Examples show that nonlinear spectral components are indeed observed as a result
- <sup>17</sup> of the friction and the clapping experienced by the faces of the defect. At the same time, a localized
- 18 temperature increase due to the induced friction is noted, and its response at the outer surface of
- <sup>19</sup> the sample is examined. The qualitative validation approves that the model is ready to be tested
- <sup>20</sup> further quantitively, and to compare its predictions to experiments.

21 Keywords: Ultrasound; Modelling; Friction

#### 22 1. Introduction

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Ultrasonic vibrothermography is an auspicious Non-Destructive Testing (NDT) technique for the detection of closed defects, either fully internal or surface breaking. The principle of ultrasonic vibrothermography is simple: through excitation by ultrasonic waves, frictional behavior at the defect's interfaces takes place. As a result, the vibrational energy of the elastic wave will locally be converted into thermal energy, generating heat around the defect, which subsequently dissipates through the medium by way of thermal diffusion. In practice, the distributed heat can be recorded by an infrared camera, possibly detecting the heat signature and its origin. Next to defect detection, defect characterization has become one of the major research interests in NDT, in particular to anticipate the severity of the defect. However, to obtain reliable and high quality defect classification by ultrasonic thermography, theoretical models capable of interpreting the obtained thermograms are necessary.

Contact models come in all shapes and sizes. Some models only take into account clapping (the normal behavior) [1], but more realistic models also include friction (the tangential behavior). The contact profile of the two bodies in contact also greatly influences the response of the applied loading procedure. Some studies assume the bodies in

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contact to have flat faces [2] whereas other models introduce non-flat surface, allowing 39 to account for roughness [3-7]. When a nontrivial topography at the contact area is assumed, a contact situation of partial slip needs to be introduced, next to the two trivial 41 contact states of contact loss and sliding. In the partial slip state, both stick and slip areas 42 can be simultaneously present at the contact interface, as a result of Coulomb's frictional 43 law. For the implementation into the model, the difficulty of meshing all asperities at the contact faces on a microscopic level is avoided by considering a multiscale approach in 45 which mesoscopic cells are introduced in the mesh. Using this approach in a 1D (normal) 46 loading, the complexity of the stress and displacement fields, along with the contact 47 profile and possible roughness can be fitted into a single relationship between normal displacement and normal load. 49 The authors of this work, together with others, previously developed a 2D contact 50 model. The description of the contact model follows the Method of Memory Diagrams 51 (MMD) to update the contact loads in a semi-analytical manner, not needing to approach 52 the state of the load through variational means [4,5]. Apart from updating the stress at 53 the level of the contact, the 2D contact model is also capable of calculating the instan-54 taneously generated heat induced by friction at the contact interface [8,9]. Traditional implementations on the calculation of dissipated energy generally start from the as-56 sumption of cyclic contact behavior, hence a simple closed hysteretic loop, and calculate the area of the hysteric loop to find the dissipated energy per cycle [10,11]. However, 58 despite the fact that the loading procedure may be monofrequent, internal reflections and 59 nonlinear effects will result in complex contact behavior, creating complicated and/or 60 non-closed, hysteretic areas, instead of a single closed loops. Apart from this, having non-static conditions, such as an evolving crack, fatigue of the material, a changing 62 excitation amplitude or frequency as an influence of the bond between actuator and 63 sample ... will also have an effect on the hysteresis area. Therefore, the instantaneous 64 energy dissipation method has been extended to accommodate the MMD contact model as illustrated in previous work for realistic excitation procedures in 2D case studies [8]. 66 Also a passive excitation will result in a nonperiodic contact behaviour as well, making 67 this method of calculating the dissipated energy also valuable for modelling structural health monitoring scenarios through passive excitations. The 2D simulation model clearly reveals the simultaneous presence of nonlinear elastic wave generation, i.e. the 70 generation of higher harmonics of the excitation frequency at the defect location, along 71 with the thermographic response at the defect and its radiation towards the boundaries. 72 In the present contribution, we develop and discuss a three dimensional model, 73 based on the aforementioned two dimensional model. We perform a qualitative analysis

order to validate the capability of the three dimensional model to simulate realisticsituations.

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#### 78 2. Materials and Methods

As the current three dimensional model is based on the previous two dimensional model described by the authors of this work, the most important ingredients of the 80 computational model will be briefly recapped [4,5,8]. The model consist of three different 81 modules. The first module describes the elastic wave propagation in a virgin material. 82 The second module captures the contact physics at the defect level. This module accounts 83 for anharmonic and hysteretic effects, describing the nonlinear behavior of the defect's 84 interfaces, which is echoed in both the ultrasound spectrum and in the vibration induced heating. The third module is implemented to calculate the diffusion of thermal energy 86 within the specimen. The full model is built in COMSOL Multiphysics<sup>®</sup> and uses its 87 internal descriptions for the first and third module. The contact response is described 88 through the Method of Memory Diagrams (MMD) in three dimensions and is linked to 89 the computational model through a MATLAB<sup>®</sup> interface. 90

by comparing the obtained numerical results to results described in the literature in

#### 91 2.1. Contact description

By combining the above contact description with the two modules to solve the wave equation in bulk materials and the heat transfer equation in solids, a consistent model 93 can be built to simulate the nonlinear elastic behavior and the vibrothermal response at the defect's contact faces. These are reflected in the nonlinear spectral components in the 95 frequency spectrum and in the thermograms. The extension of the MMD description in 3D can be retrieved in analogy with the two dimensional case. The MMD contact 97 model represents a semi-analytical method that allows to study the dynamic response of a hysteretic tangential contact in the simultaneous presence of a normal contact loading, respectively describing friction and clapping. The normal ( $\sigma$ ) and tangential ( $\vec{\tau}$ ) loads 100 are calculated in each point, given the normal indentation a and tangential displacement 101  $\dot{b}$  at that location. The method accounts for roughness at the internal boundaries of the 102 internal defect, assuming the asperities are convex and axisymmetric, following the 103 Mindlin description on contact mechanics [12] and the reduced elastic principle [13,14]. 104 These assumptions allow to avoid an expensive computational procedure of updating 105 the stress profiles to the current position by introducing analytical functions that need 106 to be iteratively updated, as the contact progresses. As mentioned in the introduction, 107 roughness at the contact interfaces introduces a contact state called partial slip when 108 the tangential load is smaller than the product of the frictional coefficient ( $\mu$ ) and the 109 normal load, i.e. when the Coulomb condition for slipping is not yet met  $(|\tau| \le \mu \sigma)$ . 110 These microscopic aspereties, which are assumed to be convex and axisymmetric, are 111 locally unified in a general mesoscopic cell (which will be the size of the mesh element 112 in the simulations) and generalized into one single convex axisymmetric profile, which 113 exerts an equivalent load to the opposing face. These equivalent loads result in an identical contact behaviour compared to the original family of asperities. Sub-figure (a) 115 of Figure 1 shows an axisymmetric contact in a state when no tangential displacement is 116 applied, after the center of the contact has been indented over a distance q. Sub-figure (b) 117 represents the status of the contact area obtained by increasing the normal indentation 118 to distance *a* at the center after which a non-zero tangential displacement is applied. As 119 the contact profile is convex, the outer regions will experience a smaller normal load 120 compared to the interior region. Subsequently, the Coulomb friction criterion will be met 121 in the outer regions ( $|\tau| = \mu \sigma$ ), as the tangential load is the same in the entire contact 122 region. As of this, regions with a radius between s and c will generate friction, whereas 123 the inner contact region (with a radius smaller than *s*) will remain in place and does not 124 slide, as  $|\tau| \leq \mu \sigma$  in this region. 125

To implement the presence of stick, slip and partial slip into the contact model, the normal and tangential loads acting in each point of a contact need to be determined. This can be done by relating them to the displacements. The load-displacement relationship which accounts for the normal dimension is based on both an empirically [15] and a numerically [16] derived quadratic relationship:

$$= C \cdot a^2, \tag{1}$$

where C is a material dependent constant and is  $3.6 \cdot 10^{21} Pa \cdot m^{-2}$  for aluminium. The tangential interaction occurs through the MMD, where the piece-wise memory vector function  $\vec{D}(\alpha)$  plays the central role, containing the memory of the hysteretic contact. This vector function is updated through the current tangential displacement vector  $\vec{b}$ and used to calculate the tangential load vector  $\vec{\tau}$ :

 $\sigma$ 



**Figure 1.** Representation of the 3D axisymmetric contact description, as a generalization of a set of arbitrary contacts, experiencing no tangential displacement (figure a) versus experiencing a tangential displacement (figure b). As a tangential loading is applied to the system, at some contact points, the Coulomb threshold will be met, introducing slip inside the regions with a radius between *s* and *c*, where *c* is the radius of the contact and *s* is the radius of the stick area. The more central parts of the contact, due to the convex assumption of the contact, will not experience slip, as the Coulomb threshold is not met.

$$\begin{cases} b_x = \theta \mu \int_0^a D_x(\alpha) \, d\alpha \\ b_y = \theta \mu \int_0^a D_y(\alpha) \, d\alpha \\ \tau_x = \mu \int_0^a D_x(\alpha) \frac{d\sigma(\alpha)}{d\alpha} \, d\alpha \\ \tau_y = \mu \int_0^a D_y(\alpha) \frac{d\sigma(\alpha)}{d\alpha} \, d\alpha \end{cases}$$
(2)

where  $\theta$  is a material parameter which depends on Poisson's ratio  $\nu$  [4]. The rules follow the exact same steps as in the 2D case, as explained and illustrated in [4], except that the orthogonal parts of the memory vector function  $\vec{D}(\alpha)$ , now comprise an area of maximum  $\cos \gamma$ , respectively  $\sin \gamma$ , where  $\gamma$  is the angle of the tangential displacement, as they represent orthogonal projections of  $\vec{\tau}$ . Note that we have assumed isotropy at the moment for the material parameters  $\theta$  and  $\mu$ , however, an extension to anisotropic media can be obtained by including the tensor notation of  $\theta$  and  $\nu$ .

The energy dissipation in 3D, which determines the amount of energy lost due to friction, can be retrieved in a similar way as in the 2D case, as has been described in Truyaert et al. [9]. This energy is calculated as the inner product of both the tangential load  $(\vec{\tau}(\rho))$  and the tangential slip  $(\Delta \vec{\Sigma}(\rho))$ , inside the slip region,

$$\Delta W = \int_{s}^{c} \left( \vec{\tau}(\rho) \cdot \Delta \vec{\Sigma}(\rho) \right) 2\pi \rho \, \mathrm{d}\rho.$$
(3)

In this expression, the lower and upper boundaries of the integral, *s* and *c*, denote the

radii of the related circles describing the contact, as depicted in Figure 1. Equation 3 can

be conveniently rewritten in terms of the displacement vectors and normal load, which

are all inputs of the model, in order to obtain,

$$\Delta W = 2\mu \left[ \left| \left| \vec{\Delta b} \right| \right| - \theta \mu \Delta a \right] \cdot \left( \sigma(a) - \sigma(q) + \left. \frac{\mathrm{d}\sigma}{\mathrm{d}a} \right|_{a=q} (q-a) \right), \tag{4}$$

where *q* corresponds to the normal indentation which would be needed to obtain the non-slipping contact region, as in the right part of Figure 1, and  $||\vec{\Delta b}||$  is the Euclidean norm of the tangential slip displacement vector.

#### 154 2.2. Computational model

By combining the above described nonlinear and history dependent contact mechanics with the two modules to both solve the wave equation in bulk materials and the heat transfer equation in solids, a consistent model can be built to simulate the nonlinear dynamics of the defect's contact faces. These microscopic and mesoscopic features will be reflected in the macroscopically observable nonlinear spectral components and thermal properties. Simulations performed with the developed model need to be time-dependent in order to correctly capture the generation of the nonlinear effects.

The time stepping procedure, of which a flowchart is present in Figure 2, is as follows:

(a) Computation of displacements and temperature distribution

Starting from the local normal and tangential stresses ( $\sigma$  and  $\vec{\tau}$ ), the Structural 165 Mechanics module of COMSOL<sup>®</sup> calculates the displacements within the entire 166 domain at each time instance t of the procedure. The local normal and tangen-167 tial displacements ( $u_n$  and  $\vec{u_t}$ ) on the contact interface are stored as output of the 168 COMSOL<sup>®</sup> module, and used as an input for the displacement driven MMD 169 module which is implemented in MATLAB<sup>®</sup> in the next calculation step (see (b)). 170 Simultaneously, the heat diffusion problem in response to an internal boundary 171 heat source at the contacting surfaces is solved through the Heat Transfer module 172 of COMSOL<sup>®</sup>, updating the temperature distribution in the entire computational 173 region. In contrast to the contact module which is only addressed for the defect specific nodes, the PDE's for structural mechanics and thermal diffusion are constantly 175 updating all properties in the whole computational region. 176

177 (b) Computation of contact stresses and energy losses

During this intermediate step, the parameters at the contact interface are updated. The displacement-driven MMD module for the nonlinear and history dependent 179 contact dynamics, implemented in MATLAB<sup>®</sup>, calculates the new stress parameters 180 at the integration points on the contact interface. Simultaneously, the friction-181 induced energy loss is calculated for each of these points. The calculated stress 18: values and energy losses are then returned to COMSOL® in order to update 183 the mechanical (i.e. internal thin elastic layer) and thermal (i.e. internal heat 184 source) boundary conditions at the contact interface in respectively the Structural 185 Mechanics and Heat Transfer modules. In this step, only the boundary conditions 186 at the defect interface are updated. 18

188 (c) Repeated calculations

Steps (a) and (b) are repeated for the next time instance  $t + \Delta t$ , continuously updating the entire computational region first and then the boundary conditions on the crack. This goes on until the desired calculation time is reached ( $t = t_{end}$ ).



**Figure 2.** Small flowchart of the computational procedure where the locations of the updates of parameters related to the contact description are highlighted.



Figure 3. Geometry of the more complex cut.

Both the structural mechanics and thermal module are solved using an appropriate 192 mesh, with its corresponding discretizations and shape functions to the order of the 193 chosen mesh, depending on the given problem. The PDE's for these two modules are 194 solved based on their corresponding mesh, but can communicate a common parameter 195 to each other. In this model, the common parameters (the stresses) are calculated in the description of the contact mechanics through an external MATLAB<sup>®</sup> script and are 197 fed into both COMSOL<sup>®</sup> modules, adapted to the appropriate mesh of the module. 198 To qualitatively illustrate the simulation procedure and results, a model consisting of 199 a rectangular block of aluminum of dimension 18 cm-by-10 cm-by-1 cm in x, y and z200 direction will be considered. On the largest surface of the block, at z = 0 m and centered 201 about x = 0.05 m and y = 0.05 m, a circular zone with a diameter of 1 cm acts as a 202 transducer, exerting a normal displacement in the form of a continuous sinusoidal wave 203 of frequency 70 kHz, with an amplitude of 100 nm. Two different defects are studied. 204 A first synthetic 2D defect in the form of a zero thickness cut is considered located at 205 = 15 cm. The cut is oriented perpendicular to the surface, 3.3 cm deep and runs 206 through the full 1 cm depth of the sample. A second zero thickness defect has a more 207 complex shape, where the defect is open at the front and top side of the sample, while 208 the geometry of the defect is closed through the material. The defect is curved in all 209 three dimensions, runs to the same depth of 3.3 cm depth and the opening at the front 210 side is located at the same place of the first defect. A normal prestress is applied at the 211 defect interface for both studied defects, to increase the time spent in contact, allowing 212 for a larger amount of time where the defect experiences friction instead of clapping. The geometry of both these defects are taken as such to test either a planar cut through 214 the whole sample and a more complex defect which does not run through the whole 215 sample and is curved. The areas around both defects is meshed denser than the other 216 region of the sample. The meshdensity in the healty region of the sample is meshed 217 at 12 elements for each wavelength, whereas the denser region consists of 84 elements 218 for each wavelength. Not only is this area denser to propagate the higher harmonics 219 generated by the clapping and friction of the defect, it is also needed to obtain a stable 220 contact description. All meshelements are quadratic. 221

#### 222 3. Results

First, the planar defect will be discussed, after which the more complex geometry is treated. These two different defects are studied in order to have a benchmark for the model (planar defect), which is used to study the numerical stability and performance of the model. The second defect is introduced in order to observe the results of a more realistic defect.

228 3.1. Planar defect

Figure 4 displays the frequency spectrum of the normal displacement,  $u_z$ , at a point on the x-y plane near the planar defect region. It can be clearly seen that higher

harmonics of the excitation frequency are present in the response signal. To ensure that 231 these harmonics directly originate from the defect, the amplitudes at the fundamental 232 frequency and its higher harmonics were examined on the surface of the sample and their 233 distribution is visualized in Figure 5 for the displacement tangential to the defect and in 234 Figure 6 for the displacement in the normal direction to the defect. The results reveal a 235 concentration of the magnitudes of the second and third harmonic in the neighbourhood 23 of the defect and confirm that the origin of these generated frequencies is to be attributed 237 to the interaction of its interfaces: friction (Figure 5) and clapping (Figure 6). From 238 Figures 5 and 6a, b and c it can be concluded that the harmonics are predominantly 239 generated near the tip of the defect, the reason being that the region of the crack tip 240 experiences more stress, compared to the open end of the crack. This is in agreement 241 with experimental observations reported in previous research studies [17]. 242

Apart from the acoustic spectrum, the multiphysics model also allows to produce a map of the thermal response as an output to study. It is expected that the temperature increases as a result of energy dissipated from friction at the defect's interface. Figure 7 displays the temperature increase on the planar surface of the defect (i.e. the partial part of the y-z plane at x = 0.15 m, covering only the defected area and no healty material). Again, the crack tip reveals the highest temperature increases. In addition, some smaller



**Figure 4.** Generation of higher harmonics of the applied frequency, as a result of the friction and clapping of the interfaces of a zero-thickness, planar cut.



**Figure 5.** Source of the harmonics of the excitation frequency in a rectangular aluminum block containing a vertical cut at x = 0.15 m extending between y = 0 m and y = 0.033 m. Figure 4a displays the amplitude of the tangential displacement, with relation to the defect, at each location on the surface (z = 0 m), calculated by selecting the Fourier transform magnitude at the excitation frequency. Figures 4b and 4c shows the magnitudes of the same signals for the second and the third harmonic respectively.

temperature effect is present near the open edge of the crack. Within the small total 249 simulation time, the temperature distribution is only affected close to the defect's region, 250 as the timescale of the overall thermal response is much larger compared to the acoustic 251 timescale and the timescale of the contact physics. To illustrate this, Figure 8 shows the 252 region of the front surface (z = 0 m near the defect, revealing only a thin dissipation 253 zone of the generated heat through the sample. Following Equation 4, temperature increases can be achieved as a result of either a relative large displacement or/and a 255 large stress. The region near the crack tip experiences a large amount of friction, as the 256 higher harmonics are originating from that location, and therefore also temperature will 257 increase from this local heat source. At the crack tip itself however, the asperities of 258 the rough surfaces will be locked, making it unable to displace them and to experience 259 rubbing. Therefore, heating as a result of friction does not take place at the very tip of 260 the defect itself, but only near that area. In addition, the part of the defect that opens 261 up near the free side of the sample also appears to engender an increase of the local 262 temperature and of its surrounding. The component from Equation 4 responsible for this 263 is the relative tangential displacement, as this part of the defect can move more freely. 264

#### 265 3.2. Defect with a complex geometry

Again, the amplitudes at the fundamental frequency and its higher harmonics were 266 examined on the surface of the sample and their distribution is visualized in Figure 9 for 267 the displacement tangential to the defect and in Figure 10 for the displacement in the 268 normal direction to the defect. The results reveal a concentration of the magnitudes of 269 the second and third harmonic in the neighbourhood of the defect and confirm that the 270 origin of these generated frequencies is to be attributed to the interaction of its interfaces: 271 friction (Figure 9) and clapping (Figure 10). Friction is more present at the closed part 272 of the defect, whereas clapping is more present at the open part, as is retrieved from 273 both figures. From Figures 9 and 10a, b and c it can be concluded that the harmonics are 274 generated along the length of the defect, the reason being that the region of the crack 275 tip experiences more stress, compared to the open end of the crack, but the open end 276 experiences more movement compared to the closed part. This is in agreement with 277 experimental observations reported in previous research studies [17]. 278



**Figure 6.** Source of the harmonics of the excitation frequency in a rectangular aluminum block containing a vertical cut at x = 0.15 m extending between y = 0 m and y = 0.033 m. Figure 5a displays the amplitude of the normal displacement, with relation to the defect, at each location on the surface (z = 0 m), calculated by selecting the Fourier transform magnitude at the excitation frequency. Figures 5b and 5c shows the magnitudes of the same signals for the second and the third harmonic respectively.



**Figure 7.** Temperature increase at the defect's surface (partial y-z plane at x = 0.15 m over 4 time instances, covering the full defected area without any undamaged part of the sample. The highest temperature increase occurs at surfaces z = 0 m and z = 0.1 m near the crack tip (y = 0.033 m) with some smaller temperature increase also being present at the open edge of the crack.



**Figure 8.** Thermal view from the surface (z = 0) of the specimen zoomed in on the region of the defect, showing that the increase in temperature is highest at the edges (tip and open end of the cut) and increases over time.

Apart from the acoustic spectrum, the multiphysics model also allows to produce a 279 map of the thermal response as an output to study. It is expected that the temperature 280 increases as a result of energy dissipated from friction at the defect's interface. Within 281 the small total simulation time, the temperature distribution is only affected close to the 282 defect's region, as the timescale of the overall thermal response is much larger compared 283 to the acoustic timescale and the timescale of the contact physics. To illustrate this, 284 Figure 11 shows the region of the front surface (z = 0 m near the defect, revealing only a 285 thin dissipation zone of the generated heat through the sample. Following Equation 4, 286 temperature increases can be achieved as a result of either a relative large displacement 287



**Figure 9.** Source of the harmonics of the excitation frequency in a rectangular aluminum block containing a vertical cut at x = 0.15 m extending between y = 0 m and y = 0.033 m. Figure 8a displays the amplitude of the tangential displacement, with relation to the defect, at each location on the surface (z = 0 m), calculated by selecting the Fourier transform magnitude at the excitation frequency. Figures 8b and 8c shows the magnitudes of the same signals for the second and the third harmonic respectively.



**Figure 10.** Source of the harmonics of the excitation frequency in a rectangular aluminum block containing a vertical cut at x = 0.15 m extending between y = 0 m and y = 0.033 m. Figure 9a displays the amplitude of the normal displacement, with relation to the defect, at each location on the surface (z = 0 m), calculated by selecting the Fourier transform magnitude at the excitation frequency. Figures 9b and 9c shows the magnitudes of the same signals for the second and the third harmonic respectively.



**Figure 11.** Thermal view from the surface (z = 0) of the specimen zoomed in on the region of the defect, showing that the increase in temperature is highest at the edges (tip and open end of the cut) and increases over time.

or/and a large stress. In addition, the part of the defect that opens up near the free
side of the sample also appears to engender an increase of the local temperature and
of its surrounding. The component from Equation 4 responsible for this is the relative
tangential displacement, as this part of the defect can move more freely.

#### 292 4. Discussion and conclusion

The results of the above qualitative study confirm that the presence of higher 293 harmonics of the excitation frequency is related to the nonlinear behaviour of the defect 294 when it experiences friction and clapping as a result from an acoustic excitation [18– 295 20]. The appearance of a rather large defect (3.3 cm-by-1 cm) can be revealed through 296 the mapping of these nonlinearities in the acoustic spectrum, as these harmonics are 297 originating from the defect, cfr. Figure 5, 6, 9 and 10 [17,21,22]. In addition, the zone of 298 the surface breaking crack that activates these higher harmonics is similar to literature 200 reportings. For instance, Fierro at al. performed experiments on a fatigue surface 300 breaking crack in an aluminium plate and demonstrated that the zones that generated 301 the second harmonic were located near the crack tip [23]. 302

With respect to the thermal response of a dynamical action, Renshaw et al. reported that heating due to dissipation of energy when surfaces experience friction is one of the most significant heating mechanisms during vibrothermography [24]. Rizi et al. developed a phenomenological 3D model to describe the energy dissipated by friction [25]. In both works, the authors communicated that the temperature primarily increases in a region close to the crack tip, where the aspereties are not locked, far more than compared to the crack region near the open part of the specimen or the crack tip itself. These findings were also confirmed in the above mentioned study of Fierro et al. [23].

Analyzing the results obtained with our computational model, it can this be con-311 cluded that qualitatively, the presently developped 3D model successfully accounts for 312 the generation of higher harmonics and the origin of heat within both a planar defect and 313 a more complex defect at locations that are in agreement with pioneering reports found 314 in literature, handling both 2D computational models and experiments. Even though the 315 input parameters of the model were not specifically tuned to match a real experiment, 316 the actual temperature increases lay within the expected range. More quantitative results 317 can be obtained on a case by case basis. Currently, the main drawback of the simulation 318 tool is that the three dimensional computational model requires a lot of computational 319 time. For instance, for the specific model discussed here, it took three days to run the 320 0.3 ms simulation on a desktop. 321

- In the future, we will address more quantitative studies of the developed model, including anisotropic materials, and investigate adaptations to reduce the computational time of the model to acceptable execution times.
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- software, K.T.; validation, K.T.; formal analysis, K.T.; investigation, K.T.; resources, K.T. and K.VDA;
- 327 data curation, K.T.; writing—original draft preparation, K.T.; writing—review and editing, K.VDA.;
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