



**HAL**  
open science

## Toulmin and beyond: structuring and analyzing argumentation

Jenny Cramer, Leander Kempen

► **To cite this version:**

Jenny Cramer, Leander Kempen. Toulmin and beyond: structuring and analyzing argumentation. Twelfth Congress of the European Society for Research in Mathematics Education (CERME12), Feb 2022, Bozen-Bolzano, Italy. hal-03746876v2

**HAL Id: hal-03746876**

**<https://hal.science/hal-03746876v2>**

Submitted on 17 Oct 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Toulmin and beyond: structuring and analyzing argumentation

Jenny Cramer<sup>1</sup> and Leander Kempen<sup>2</sup>

<sup>1</sup>University of Bremen, Bremen; [j.cramer3@schule.bremen.de](mailto:j.cramer3@schule.bremen.de)

<sup>2</sup>TU Dortmund University, Germany; [leander.kempen@tu-dortmund.de](mailto:leander.kempen@tu-dortmund.de)

*In recent years, the Toulmin model seems to have become a “silver bullet” when analyzing argumentation (and proof) in mathematics education. While the model and its adaptations are well-fitted especially to reveal the structure of arguments, it is not suitable to grasp all aspects of argumentation which are relevant to the mathematics classroom. In this paper, we outline both, possibilities and limitations of the Toulmin model and present some additional considerations concerning this model undertaken in the last decades. Furthermore, we present and discuss some alternative approaches to argumentation and highlight their additional benefits.*

*Keywords: Argumentation, Toulmin model, Rationality, Logic of inquiry, Walton’s dialogue theory*

## Introduction

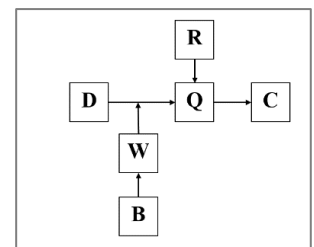
In this article, we would like to follow up on a question that arose in the context of the TWG Argumentation and Proof at CERME11: Which models can be used besides the Toulmin model to analyze (mathematical) argumentation? Indeed, the Toulmin model has proven and established itself for structuring and presenting argumentation processes in a mathematical context. However, the associated disadvantages and 'blind spots' of the model are often ignored or disregarded. Moreover, the model appears to be so widespread that young researchers in particular fail to identify alternatives that might be better suited to address their research questions.

Our first aim in this article is to show which possibilities the original Toulmin model offers for structuring mathematical argumentation processes, which advantages and disadvantages are associated with it, and which conceptual extensions of the model are discussed in mathematics education. Furthermore, we would like to discuss other perspectives on argumentation that go beyond structural analysis and present alternative approaches to investigating argumentation.

## The Toulmin-Model

Toulmin (1958) proposed a model for structuring argumentation in general. Inglis et al. (2007, p. 4; emphasis in original) summarize this scheme as follows (see also Figure 1):

Toulmin’s (1985) scheme has six basic types of statement, each of which plays a different role in an argument. The *conclusion* (C) is the statement of which the arguer wishes to convince their audience. The *data* (D) is the foundations on which the argument is based, the relevant evidence for the claim. The *warrant* (W) justifies the connection between data and conclusion by, for example, appealing to a rule, a definition or by making an analogy. The warrant is supported by the *backing* (B) which presents further evidence. The *modal qualifier* (Q) qualifies the conclusion by expressing degrees of confidence; and the *rebuttal* (R) potentially refutes the conclusion by stating the conditions under which it would not hold.



**Figure 1: Toulmin’s model of a general argument (similar to Inglis et al. (2007, p. 4))**

As Nussbaum (2011, p. 86) points out, Toulmin did not originally intend to propose a model of argument. He originally wanted to counter formal logic with an alternative foundation for argument. Nevertheless, the model has been misinterpreted as normative. Toulmin was not concerned with the completeness of arguments, for example, to evaluate their quality. Especially in non-mathematical contexts, the warrants are often not made explicit. Nor was it Toulmin's concern to use the model to teach students how to propose an argument. It has been shown that the Toulmin model can be used to reveal argumentation structures in mathematical contexts (e.g., Inglis et al., 2007; Knipping, 2008). Here, the Toulmin model is often reduced to the somehow core aspects of Data, Warrant, and Claim. However, as stressed by Inglis et al. (2007) and Jahnke (2008), the whole model should be preferred to cover essential aspects of argumentations such as the backing, which specifies the warrant, or likewise the modal qualifier that serves to qualify the conclusion.

### **Limits of the Toulmin model and opening a discussion**

The Toulmin model has its benefits when trying to reveal an argument's structure. However, some limitations of the model have to be considered, too. On the structural level, Nussbaum (2011, p. 86) discusses that in some situations, a reliable distinction between warrant and backing is hard to undertake, and there are some arguments in real life (such as arguments with a number of separate reasons) that do not fit the components and structure of the model. Aberdein (2013) criticizes that the Toulmin model is weak for the representation of instructions which may be part of mathematical arguments in particular. Due to its structural-analytical orientation, the model is less suitable for highlighting how people argue and why, and there is no focus on the correctness of arguments or their acceptance by others. In a collective argumentation setting, the Toulmin model does not specify who produces (parts of) the argument, and the context is not taken into consideration.

The Toulmin model furthermore does not capture the way in which an argument is presented to an audience, although as Gabel and Dreyfus (2019) show with regard to Perelman's new rhetoric, the audience is a major factor in proof teaching. This holds true not only in situations in which a teacher guides the argumentation, but also in collective argumentation settings. Motives and reasons with which intention which person contributes which reasons with which rhetoric are not part of the Toulmin model's structural focus but they are of importance when looking at argumentation and proof in the mathematics classroom.

### **Extensions of the Toulmin model**

Inglis et al. (2007, p. 9–16) distinguish several warrant-types as categories of warrants with similar properties. In an argumentation, the check of some concrete examples or an apparent absence of a counterexample for the statement in focus might reduce the uncertainty about the conclusion of an argument. Such warrants are considered the *inductive warrant type* and might typically lead to modal qualifiers like “it seems that” or “it is plausible that”. The *structural-intuitive warrant-type* is about using some kind of observation (it might be of an intuitive type or not) that might persuade the individual to accept the validity of a conclusion. Again, corresponding modal qualifiers might be “it seems that” or “it is plausible that”. Finally, the deductive warrant-type is concerned with those warrants used in a valid mathematical proof (axioms, statements, algebraic manipulations, etc.). For professional mathematics, using a *deductive warrant* should normally lead to a modal qualifier like “with necessity”. However, it has been shown in the literature that this need not be the case for

students (see Reid & Knipping 2010, p. 62 for an overview). Kempen (in press) analyzed high school graduates' proof construction and extended the discussion of the observed warrants by the aspect of epistemic value. "The epistemic value is the degree of certainty or conviction assigned to a proposition" (Duval, 1991, p. 254; authors' translation). Accordingly, it can take on values such as obviously, likely, absurd, necessary, etc., and it is closely connected to the individuals' understanding of the content (Duval, 2007, p. 138). This extension is based on the idea that a warrant is linked with an individual level of conviction and validity. Kempen showed that the epistemic values attributed to the warrants involved have an impact on the conclusion's modal qualifiers.

Besides focusing on a single argument leading from data to conclusion, it is also possible to consider longer chains of arguments. Here, a conclusion becomes new data for the application of further warrants. Such chains of arguments have been described as "argumentation stream" (Knipping & Reid, 2019), "Recyclage" (Duval, 1995), or "Sequential" (Aberdein, 2006).

The Toulmin model is not only used as a tool to explore the local structure of an argument, but also its global structure (the structure as a whole) as described by Knipping and Reid (2019) when discussing classroom argumentations. In their analysis of global argumentation structures which transcend individual arguments, these authors added an element called "refutation" and explain:

A refutation completely negates some part of the argument. In a finished argumentation refuted conclusions would have no place, but as we are concerned with representing the entire argumentation that occurred, it is important for us to include refutations and the arguments they refute, as part of the context of the remainder of the argumentation, even if there is no direct link to be made between the refuted argument and other parts of the argumentation. (Knipping & Reid, 2019, p. 5)

When analyzing complex arguments, e.g., when different people contribute to an argument, it has been valuable to use a schematic representation that enables the description of argumentation at different levels of detail, like using different shades, colours, or forms (see Knipping, 2003). Knipping and Reid (2013) show how teacher actions can shape argumentation processes in the mathematics classroom by identifying different global argumentation structures. They distinguish differently structured discourses that become visible in the shape of the global argumentation structure and thereby show how the Toulmin model may add to a deeper understanding of classroom interaction. Kopperschmidt (1989) also links the local and the global structure by analyzing the micro and macro structures. The micro structure contains the *functional* analysis (role and functions of the different parts involved in an argument with respect to the Toulmin model), the *material* analysis (linguistic interaction), and the *formal* analysis (structural patterns involved). At the macro level, a distinction is first made between single-step and multi-step argumentation. Here, a multi-step argumentation can be convergent (all elements involved fully support or refute a particular claim) or controversial (the elements involved only partially support or refute a particular claim).

### **Further models and ideas for considering argumentation**

In the following, we will present and discuss other perspectives on argumentation that enable further aspects and views on the topic and thus open up new perspectives for corresponding analyses. A comprehensive overview on developments and different perspectives in argumentation theory in

general may be found in Van Eemeren et al. (2014). In selecting the perspectives listed below, we have tried to choose those that have experienced a certain spread in mathematics education and exemplarily focus on various aspects of argumentation beyond structure considerations.

### **Considering the purpose of argumentation: communicative versus strategic action**

Arzarello and Soldano (2019) criticize the jurisprudential nature of the Toulmin model. According to them, in the type of argument considered by Toulmin “the goal is to convince the adversary, not the search/explanation of the truth, which is instead the goal of scientific argument, in particular in mathematics.” (Arzarello & Soldano, 2019, p. 227). We do not necessarily share this criticism of the Toulmin model as being too narrowly focused on jurisprudential arguments. However, Arzarello and Soldano’s critique raises an important issue: it might be necessary to consider the purpose of an argumentation when looking at argumentation in a classroom context.

One way to distinguish between different purposes in which argumentation may play a role is described by Habermas (1981). He postulates a distinction between two different types of action: communicative action towards reaching agreement, and strategic action focused on securing consent.

Moreover the action theoretical approaches [i.e. communicative vs. strategic action] differ in whether for coordinating actions they postulate *agreement* [german: „Einverständnis“; authors’ comment], that means *common* knowledge [german: „gemeinsames Wissen“; authors’ comment], or just an external *influence* [german: „Einflussnahme“; authors’ comment] on another. “Common” knowledge needs to satisfy challenging conditions. It is not enough if participants share some opinions; not even if they know that they share those opinions. *Common* knowledge is a knowledge that constitutes *agreement*, while agreement terminates in the intersubjective recognition of criticisable claims. Agreement means that participants accept knowledge as valid, i.e. intersubjectively binding. [...] The external exertion of influence (in the sense of a causal effect) on the convictions of another participant in the interaction, on the other hand, keeps a one-sided character. (Habermas, 1981, p. 574–575; emphasis in original; authors’ translation)

Mathematical argumentation should be a conjoint search for the truth and thus oriented towards reaching common knowledge. However, it is doubtful whether all classroom situations fulfil these noble goals. A study examining the prevalence of communicative and strategic behaviour in the classroom might provide interesting insights.

Within Habermas’ framework of communicative action, Boero (2006) identified the construct of rationality as an especially useful tool for grasping the complexity of a classroom situation. Boero (2006, p. 189–190; emphasis in original) paraphrases Habermas’ perspective on three types of rationality:

The *epistemic rationality* is related to the fact that we know something only when we know why the statements about it are true or false (otherwise our knowledge remains at an intuitive or implicit-pragmatic level). [...] The *teleologic rationality* is related to the intended character of the activity, and to the awareness in choosing suitable tools to perform the activity and orient it to the aim to be achieved. The *communicative rationality* is related to communication practices in a community whose members can establish communication amongst them.

This threefold view on classroom situations shows that there is more to argumentation than its mere structural components. An individual or a group of individuals producing a mathematical argument need an awareness of shared knowledge which they can presume, they need to choose their tools and strategies wisely, and they need to choose means of communication which can be understood by other participants in the argumentation. Thus, rational behaviour presupposes an orientation towards communicative action.

Boero et al. (2010) have shown how supplementing the structural analysis of argumentation with the Toulmin model with Habermas' perspective on rationality opens up the possibility to consider a speaker's intentions and consciousness within the argumentation. Whether an argument is accepted by a community or not is not simply defined by its logical soundness, its correctness, or its adherence to a certain structure. To put it in a different way: What counts as proof in a 5th year classroom may differ significantly from what is considered an acceptable proof at university level. This is captured well in a quote from Habermas highlighted by Boero and Planas (2014, p. 205): "The rationality of a judgment does not imply its truth but merely its justified acceptability in a given context". Looking at argumentation from an epistemic, a teleological and a communicative perspective simultaneously may help us understand more about how an argument becomes a foundation of newly established common knowledge.

#### **"Logic of inquiry" to account for discoveries in argumentation**

Arzarello and Soldano (2019) point out that a perspective on argumentation with a narrow focus on deductive reasoning cannot account well for surprising discoveries. They refer to an example from the story *Silver Blaze* by Sir Arthur Conan Doyle. In this story, the fictional detective Sherlock Holmes concludes that a missing horse had not been stolen as assumed, but had indeed been taken by the stable owner, because the watch dog had not barked in the night. To account for the arising of such new theories and conjectures, Arzarello and Soldano introduce Hintikka's "logic of inquiry", to which they ascribe three characteristics: (i) the dialectic between questions and answers; (ii) the deep link with game theory; (iii) the functional interpretation of connectives and quantifiers (Arzarello & Soldano, 2019, p. 230).

According to the logic of inquiry, deductions such as the one made by Sherlock Holmes in the story can be rewritten into a chain of questions and answers. The argumentation is guided by definitory rules framing the deductive steps on the one hand, and by strategic principles generating the inquiry steps on the other hand. This interplay of rules and strategies forms a connection to game theory. Arzarello and Soldano (2019) show how mathematical proving processes can be regarded as (semantic) games. They introduce an example from a mathematics classroom in which students fulfil roles as verifiers or falsifiers in their joint search for the truth. In this model, the existence of a proof is tantamount to the existence of a strategy with which the verifier will always win.

#### **Teaching and evaluation of argumentation: Walton's Dialogue Theory and a Bayesian approach to argument evaluation**

Nussbaum (2011, p. 85) points out that argumentation models may serve three different purposes: analytical (with a focus on revealing the structure of an argument), normative (for judging an argument's quality or to determining what an argument should look like), and descriptive (to identify

how people actually argue). He points out that the Toulmin model is primarily analytical, even though its components are not always clearly identifiable. Besides, also the adaptations towards the representation of global argumentation structures by Knipping (2003) serve a descriptive purpose. However, normative aspects are not covered by the Toulmin approach. Nussbaum (2011) therefore suggests two alternative frameworks for looking at argumentation in educational contexts: Walton's Dialogue Theory and Bayesian models of everyday arguments. In this contribution, we only take the former into consideration. The Bayesian model works with probabilistic calculations to help determine posterior odds; it does not help in evaluating mathematical arguments whose premises are usually either correct or incorrect.

Walton's Dialogue Theory, according to Nussbaum (2011), is a complex argumentation framework consisting of several levels: types of argumentation dialogues, specific argument schemes and critical questions, degrees of plausibility, and criteria for moral and aesthetic argumentation. Due to the limited space in this contribution, we focus on two aspects which appear particularly promising to address the shortcomings of the Toulmin model laid out above: argumentation schemes and posing critical questions, which may be applied both to the instruction and evaluation of argumentation.

One way to improve students' argumentation skills could be direct instruction in how to use different argumentation schemes. This has been a strategy in mathematics education, e.g. when teaching students two-column proofs or methods for proving by induction. However, while students might benefit from a scheme on which they can rely, it has been criticized in the past that students tend to apply such schemes without a deeper understanding of why the proof they are creating is, in fact, a proof. Regardless of whether argumentation schemes are explicitly taught to students, they can also serve to evaluate argumentation quality.

A second possible application of Walton's Dialogue Theory, according to Nussbaum (2011), is to use direct instruction on critical questions and *stratagems* useful for argumentation. "A stratagem is a generic type of brief argument expressed in discourse [such as...] I think [POSITION] because [REASON]" (Nussbaum, 2011, p. 93). Both, critical questions and stratagems, serve to inform students of evaluative criteria, which may in turn serve to assess argumentation quality from a teacher or researcher perspective.

Besides schemes and critical questions, Nussbaum (2011, p. 94) also presents a list of criteria for the assessment of argument quality. However, several criteria on the list appear ill-fitted for the mathematics classroom, as they rely on an assessment of the depth of a discussion. The quality of a mathematical argument is not necessarily dependent on the number of arguments brought forth, or by how many of the arguments brought forth were defeated. A closer look at Walton's schemes may, however, be beneficial also for evaluating and assessing mathematical argumentation with regard to quality.

### **Conclusion: Widen the scope – there is more to arguments than structure**

We started out from the question "Which models can be used beside the Toulmin model to analyze (mathematical) argumentation?". Our thorough consultation of alternative approaches to and perspectives on argumentation primarily shows the necessity of specifying the question further: *what exactly do we want to analyze? I.e., what is the focus and the purpose of our research?* The Toulmin

model can be suitable or well-fitted to reveal the structure of an argument. This approach, however, makes the structure of an argument (in Toulmin's sense) visible; no more, no less.

As we have seen above, there are several attempts to extend the traditional Toulmin model to gain more information about the different parts involved in the model. Besides, also the connections between these parts have been (theoretically and empirically) analyzed further. It is, however, important to notice that an analysis of the different parts of an argumentation's structure does not imply any information about the soundness of the argumentation steps, the plausibility for the participants in the argumentation, or the generation of common knowledge.

In this paper, other aspects that might be focused on when discussing argumentation became visible that go beyond the (original) intention of the Toulmin model. These aspects can be described as considering the purpose of an argumentation (communicative versus strategic action), accounting for discoveries in the course of the argumentation, and the discussion and conceptualization of normative aspects and educational purposes.

From our point of view, we want to highlight the following conclusions. For structural analysis of the different parts of argumentation, we have not yet discovered a more suitable model than the Toulmin model with its extensions. However, a danger must also be pointed out here: If argumentation is theoretically defined according to the Toulmin model's structural components, the Toulmin model becomes the sole instrument of analysis. One should beware of this redundant relationship because it obscures the view of a field of research that, as we have shown here, is quite broad. Argumentation in the mathematics classroom is more than the sum of its (structural) parts. It involves awareness, validity, intentions, strategies, questions, rhetoric and rationality.

We have tried to broaden the view of the research field of argumentation in mathematics education. The examples we have chosen and brought up are well suited to point out horizons for further research on the topic.

## References

- Aberdein, A. (2006). Managing Informal Mathematical Knowledge: Techniques from Informal Logic. In J. M. Borwein & W. M. Farmer (Eds.), *Mathematical Knowledge Management* (pp. 208–221). Springer
- Aberdein, A. (2013). The parallel structure of mathematical reasoning. In A. Aberdein & I. J. Dove (Eds.), *The argument of mathematics* (pp. 361–380). Springer.
- Arzarello, F., & Soldano, C. (2019). Approaching Proof in the Classroom Through the Logic of Inquiry. In G. Kaiser & N. Presmeg (Eds.), *Compendium for Early Career Researchers in Mathematics Education* (pp. 221–243). Springer.
- Boero, P. (2006). Habermas' theory of rationality as a comprehensive frame for conjecturing and proving in school. In J. Novotná, H. Moraová, M. Krátká & N. Stehlíková (Eds.), *Proceedings of PME30* (Vol. 2, pp. 185–192). PME.
- Boero, P., Douek, N., Morselli, F., & Pedemonte, B. (2010). Argumentation and proof: A contribution to theoretical perspectives and their classroom implementation. In M. Pinto & T. Kawasaki (Eds.), *Proceedings of PME34* (Vol. 1, p. 179–205). PME.



- Boero, P., & Planas, N. (2014). Habermas' construct of rational behavior in mathematics education: New advances and research questions. In D. Allan, P. Liljedahl, C. Nicol, & S. Oesterle (Eds.) *Proceedings of PME38* (Vol. 1, pp. 205–208). PME & UBC.
- Duval, R. (1995). *Sémiosis et pensée humaine. Registres sémiotiques et apprentissages intellectuels*. Peter Lang.
- Gabel, M., & Dreyfus, T. (2019). To whom do we speak when we teach proofs?. In U. Jankvist, M. van den Heuvel-Panhuizen & M. Verlhuis (Eds.), *Eleventh Congress of the European Society for Research in Mathematics Education* (pp. 2494–2501). ERME.
- Habermas, J. (1981). *Theorie des kommunikativen Handelns, Band 1 Handlungsrationalität und gesellschaftliche Rationalisierung*. Suhrkamp.
- Inglis, M., Mejia-Ramos, J. P., & Simpson, A. (2007). Modelling mathematical argumentation: the importance of qualification. *Educational Studies in Mathematics*, 66(1), 3–21. <https://doi.org/10.1007/s10649-006-9059-8>
- Jahnke, H. (2008). Theorems that admit exceptions, including a remark on Toulmin. *ZDM*, 40(3), 363–371. <https://doi.org/10.1007/s11858-008-0097-9>
- Kempen, L. (in press). Investigating High School Graduates' Basis for Argumentation: Considering Local Organisation, Epistemic Value and Modal Qualifier when Analysing Proof Constructions. In R. Biehler, G. Gueudet, M. Liebendörfer, C. Rasmussen & C. Winsløw (Eds.), *Practice-Oriented Research in Tertiary Mathematics Education: New Directions*. Springer.
- Knipping, C. (2008). A method for revealing structures of argumentations in classroom proving processes. *ZDM*, 40(3), 427–441. <https://doi.org/10.1007/s11858-008-0095-y>
- Knipping, C. & Reid, D. A. (2013). Revealing Structures of Argumentations in Classroom Proving Processes. In A. Aberdein & I. Dove (Eds.), *The argument of mathematics* (pp. 119–146). Springer.
- Knipping, C., & Reid, D. A. (2019). Argumentation Analysis for Early Career Researchers. In G. Kaiser & N. Presmeg (Eds.), *Compendium for Early Career Researchers in Mathematics Education* (pp. 3–31). Springer International Publishing. [https://doi.org/10.1007/978-3-030-15636-7\\_1](https://doi.org/10.1007/978-3-030-15636-7_1)
- Kopperschmidt, J. (1989). *Methodik der Argumentationsanalyse (Problemata)*. Frommann-Holzboog.
- Nussbaum, E. M. (2011). Argumentation, Dialogue Theory, and Probability Modeling: Alternative Frameworks for Argumentation Research in Education. *Educational Psychologist*, 46(2), 84–106. <https://doi.org/10.1080/00461520.2011.558816>
- Reid, D. A., & Knipping, C. (2010). *Proof in Mathematics Education. Research, Learning and Teaching*. Sense. <https://doi.org/10.1163/9789460912467>
- Toulmin, S. (1958). *The uses of argument*. Cambridge University Press.
- Van Eemeren, F., Garssen, B., Krabbe, E., Snoeck Henkemans, F., Verheij, B., & Wagemans, J. (2014). *Handbook of Argumentation Theory*. Springer.