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Proving as multimodal argumentation: an investigation based on Toulmin's scheme

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In this theoretical paper, we discuss mathematical argumentation and proving as multimodal text. It is argued that such a perspective is in line with the everyday experiences of mathematics and school mathematics. The PROving-as-Multimodal-Text analytical tool (ProMoTe) is presented, which draws upon Toulmin's scheme to address proving and argumentation as multimodal text. Two exemplary implementations and their implications on research and practice are discussed.

Keywords: Argumentation, proof, Toulmin, multimodal argumentation, multimodality.

Arguments, proofs, mathematics, and school mathematics in a multimodal reality

Mathematical argumentation is at the crux of everyday mathematical practice, whilst the notion of mathematical proof lies at the heart of modern mathematics. Specific forms and norms of reasoning about well-defined objects with fundamental properties constitute axiomatic systems that need to be consistent, independent, and complete. Within these constructions, mathematical proof serves as the gatekeeper, assigning to a mathematical conjecture the status of mathematical knowledge (Mariotti et al., 2018). It seems reasonable to expect that the importance of both mathematical argumentation and proof would be also evident in the school mathematics curricula and the mathematics teacher education programmes. Nevertheless, mathematics education researchers reveal that opportunities for reasoning and proving are limited in the mathematics textbooks (Davis, 2013; Hanna, 2000; Stylianides & Silver, 2004). Importantly, in school mathematics, proof seems to act as a dichotomizing, rather than a unifying, practice that crucially affects the way that each mathematics learner positions himself/herself with respect to mathematics. For example, it seems that these opportunities are often linked with those tasks that are more demanding or with proofs to (already known) theorems (Hanna & de Bruyn, 1999; Nordström & Löfwall, 2005). Moreover, everyday mathematics teaching practices, include an amalgam of verbal, non-verbal, emotional, and embodied aspects, which are also part of mathematical argumentation and proving. Diverse semiotic systems (for example, linguistic, visual, audio, gestural etc) are present in the mathematics classroom and the students are required to make meaning from and amongst them, as they are engaged with mathematics. The complex relationships amongst the meanings related to the different semiotic choices (for example, mutually reinforcing, antagonistic, or complementary) that sometimes co-exist during the utterance of a mathematical argument are of the interest of this paper.

In this paper, mathematics argumentation is conceptualised to be communicated as a multimodal text, in the sense that it involves the employment of more than one semiotic system (cf. Kress, 2010; O'Halloran, 2008); including, verbal and non-verbal aspects in diverse semiotic systems, as well as implicit, explicit, cognitive and affective aspects. This led to the development of an analytical tool (Proof-as-Multimodal-Text; ProMoTe), with the purpose to gain deeper understanding of the complexity of the multifaceted communicational interactions that are at work when teachers and students are engaged in argumentation and proving.

Toulmin's scheme in mathematics education

Toulmin, in his seminal book *The Uses of Argument* (Toulmin, 1958), developed a scheme to analyse the logical micro-structure of an argument. According to the scheme, each argument includes the following components (see Figure 1): Data, Warrant, Backing, Qualifier, Rebuttal, Claim. A Claim is drawn upon some facts (the Data), based on a rule (a hypothetical statement; the Warrant) that associate such facts to this claim. This relationship is valid to a degree of certainty (the Qualifier) unless there is a case of refuting this relationship (the Rebuttal). The applicability of employing a warrant in the specific argument is supported by a categorical statement (the Backing) that identifies the broader system within this warrant may be utilised.

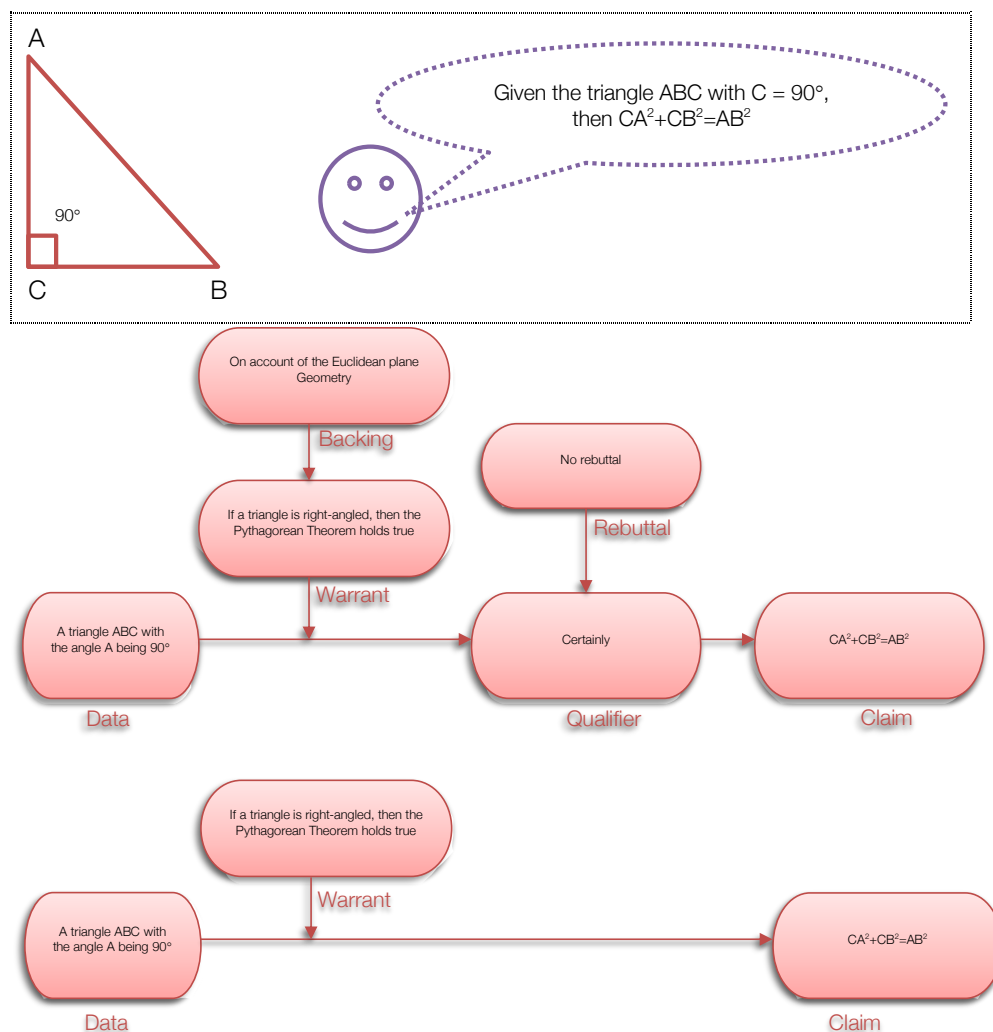


Figure 1: An example of applying Toulmin's scheme; full (up) and a restricted version (below)

In Figure 1, two versions of Toulmin's scheme are applied to analyse the argument posed by a student who saw the red triangle: "Given a triangle ABC with $C = 90^\circ$, then $CA^2 + CB^2 = AB^2$." Considering the full scheme, the Claim is an algebraic relationship with geometrical meaning that links the lengths of the sides of the right-angled triangle ABC, given the Warrant of the Pythagorean Theorem which holds true for all (no Rebuttal, absolute Qualifier) right-angled triangles within the system of the Euclidean Geometry (the Backing). Note that not all components of the analysis are explicit (verbally

or else); the Warrant, the Backing, the Qualifier, and the Rebuttal are usually implicit and are hypothesised by the analyst (for example, the researcher or the teacher). Moreover, in our example, the data provided to this student draw upon different semiotic systems (numerical, figural), whilst the students' answer combines (natural) language with algebraic symbolism.

Though the scheme was not designed specifically for mathematical arguments, it has been widely utilised in mathematics education research. Initially, it was employed in restricted versions (Krummheuer, 1995). Nevertheless, Inglis et al. (2007) argued for the necessity to utilise the full version of the scheme to gain deeper understanding of in-class mathematical argumentation and its development towards a proof. They showed that students faced difficulties due to their inappropriately linking qualifiers and warrants (for example, an inductive warrant with an absolute qualifier). Various mathematics education researchers agree with employing the full version of the scheme, whilst others propose expansions of the scheme drawing upon other theoretical tools (Aberdein, 2005; Conner et al., 2014; Krummheuer, 1995; Pedemonte & Balacheff, 2016; Simpson, 2015). Moreover, researchers have employed Toulmin's scheme to investigate collective argumentation (Knipping & Reid, 2013), the teachers' supporting the students' mathematical argumentation (Conner et al., 2014), and the development of teaching tools about argumentation and proof (Hein & Prediger, 2017; Moutsios-Rentzos & Micha, 2018).

The fact that mathematical argumentation and proving involves both verbal and non-verbal aspects in diverse semiotic systems, as well as implicit and explicit aspects led to the development an analytical tool (ProMoTe) that acknowledges the mathematical argument as a multimodal text.

The PROof-as-MultimOdal-TExt analytical tool

A series of small-scale studies have been conducted to incorporate Toulmin's scheme within a tool to analyse the micro-structure of multimodal mathematical argumentation. The Proof-as-Multimodal-Text (ProMoTe) tool was designed to analyse multimodal arguments according to Toulmin's scheme, also identifying the *explicit* and the *implicit* elements of the arguments (see Figure 2).

ProMoTe Elements	Data	Warrant	Claim	Qualifier	Rebuttal	Backing
Explicit						
Verbal						
<i>natural language</i>						
<i>symbols (numbers, symbols etc)</i>						
Non-verbal						
<i>written (figures etc)</i>						
<i>embodied -sensory (gestures etc)</i>						
Implicit						
Verbal						
<i>mental (non-explicitly communicated)</i>						
<i>verbal warrants etc)</i>						
Non-verbal						
<i>mental (non-explicitly communicated)</i>						
<i>non-verbal warrants etc)</i>						
<i>embodied -bio-metric (heart-rate etc)</i>						

Figure 2: The Proof-As-Multimodal-Text (ProMoTe) analytical tool.

The explicit/implicit differentiation reflects what is considered to be overtly present for those engaged in the mathematical argumentation. Explicit elements include those verbal and non-verbal aspects of the mathematical argumentation that are in some way sensory-present to those who are engaged in the communication. Explicit verbal elements may include natural or mathematical language, whilst explicit non-verbal elements may be classified as written or as embodied-sensory (for example, gestures, posture, voice pitch etc). On the other hand, implicit elements include aspects of the argumentation that are only mentally present (for example, a non-explicitly communicated warrant; the Pythagorean Theorem in Figure 1), or they are not consciously present (for example, biometric data, such as heart rate or salivary cortisol measurements). Moreover, element types that are hypothesised by the analyst (for example, researcher or a reflective practitioner) may be also included in this category. Notably, ProMoTe allows for the identification of both cognitive and affective aspects of mathematical argumentation.

The analysis of each argument according to ProMoTe is designed to be conducted in three complementary stages/layers. First, the argument is decomposed according to Toulmin's scheme with respect to its explicit verbal elements. Subsequently, the non-verbal explicit elements are considered. The third stage/layer concerns the identification and/or assumption of the implicit elements.

In this paper, two exemplary implementations of ProMoTe are presented from a study¹ conducted in 2018 in a Greek high school (14 years old). Video-recorded data (only the teachers were visible; informed consents were obtained) of eight teaching hours were collected and analysed. Two restricted versions of ProMoTe were utilised (respectively, one including Data, Warrant and Claim, and one including Data, Warrant, Claim and Qualifier) to analyse one teacher's multimodal mathematical argumentation focusing on both cognitive and affective aspects.

The first example (outlined in Figure 3) concerns a teacher's argument regarding the slope of a linear function. He attempts to exemplify the property that when the slope of a linear function is positive, then the function is strictly increasing. He utilizes the example of $f(x)=3x$ to argue that "when α is positive [referring to the general form $f(x)=\alpha x$], the line goes upwards". The analysis reveals that the teacher's arguments are essentially two completely different ones that are co-developed simultaneously: a) a deductive argument discussing the graph of the specific function as a special case of the general rule he had presented a few moments earlier, and b) an inductive argument about the general rule that is verified by the specific example. At the same time, the teacher explicitly non-verbally shows the specific example and verbally utters the general rule. The students are implicitly required to differentiate between these two arguments, which warrants link which data with which claim. Importantly, not all the elements of the ongoing argumentation are legitimatised to be temporary stable; that is to be written on the blackboard.

¹ Stamataki, A. (2019). *Non-verbal affective aspects in education* [Mi Lektikes Opseis tou Thymikou sti Didaktiki] (Unpublished Master's dissertation). Athens, Greece: National and Kapodistrian University of Athens.

ProMoTe Elements	Data	Warrant	Claim
Explicit			
Verbal			
natural language	"...when a is positive..."	-	"... the line goes upwards"
symbols			
Non-verbal			
written	$a=3$ of the graph of the function $f(x)=3x$ as depicted on the blackboard	Visual inspection of the graph	The graph of the function $f(x)=3x$ as depicted on the blackboard
embodied -sensory			Upwards gestures over the straight line Followed by a movement of the hands from left to right on the horizontal axis
Implicit			
Verbal			
mental	The graph of $f(x)=3x$ is a line going upwards, with $a=3>0$	Inductive	In a function $f(x)=ax$, when $a>0$ then the graph is an increasing straight line
	$a=3>0$	When the slope of a straight line is positive, the angle of the line with the horizontal axis is less than 90°	The line is strictly increasing
Non-verbal			
mental			

Figure 3: Implementing the ProMoTe analytical tool (example 1)

In the second example, the same teacher, teaching in another cohort of students of the same grade, presents a textbook exercise and asks the students to create the graph of the function $y=3x$. He argues: "The function $y=3x$ is of the type [raises his voice a little, makes intense hand-movements, his eyes were bulging out] ... $y= ax$, then it is a straight line". The application of ProMoTe (see Figure 4) contrasts a standard verbal analysis which would identify only one argument, by revealing two arguments that work in parallel: the first is "The function $y=3x$ is of the type [raises his voice a little, makes intense hand-movements, his eyes were bulging out] ... $y= ax$ " and the second argument is "then it is a straight line".

The decomposition of the second argument is in line with the standard analysis, as it involves only verbal explicit and implicit elements (see Figure 4, below). However, regarding the first argument, ProMoTe allowed us to identify the role of non-verbal affective elements in argumentation in the micro-structure of argumentation (see Figure 4, up). It appears that the teacher amplifies the presence (cf. Perelman & Olbrechts-Tyteca, 1969) of the whole argument by providing three implicit non-verbal affective warrants, as depicted in the ProMoTe tool. At the same time, the argument is also based on the implicit data that "Raised voice, intense hand-movements and eyes bulging out imply that emphasis is given and attention is required", as well as on the implicit warrant of the teacher's authority, on the implicit claim that "When the teacher emphasises something and/or draws attention to something, then this is correct and/or important", and on the implicit qualifier "certainly".

Notice that the teacher's argument draws simultaneously on cognitive and affective aspects, though they appear to inform different aspects of the mathematical argumentation. There seems to be an implicit dimension, essentially forming an implicit argument about the affective warrants. Explicitly and implicitly the teacher's verbal argumentation seems to promote deductive argumentation. At the same time, his implicit and explicit non-verbal argumentation appears to legitimatise his authority as

means for the validation of mathematical argument, as the embodied-sensory authority-derived warrants together with the deductive implicit warrants are linked with the qualifier “certainly”, which may communicate inappropriate warrant-qualifier couples as acceptable (Inglis et al., 2007).

ProMoTe Elements	Data	Warrant	Claim	Qualifier
Explicit Verbal				
natural language	"The function $y=3x$ is	-	" $y= ax$ "	
symbols	of the type"			
Non-verbal				
written	-	-	-	-
embodied -sensory	-	raised voice	-	-
		intense hand-movements		
		eyes bulging out		
Implicit Verbal				
mental		The graph of the function $f(x)=ax$ is a straight line		certainly
		The teacher's authority		
Non-verbal				
mental	Raised voice, intense hand-movements and eyes bulging out imply that emphasis is given and attention is required		When the teacher emphasises something and/or draws attention to something, then this is correct and/or important	certainly

ProMoTe Elements	Data	Warrant	Claim	Qualifier
Explicit Verbal				
natural language	-	-	"then it is a straight line"	-
symbols				
Non-verbal				
written	-	-	-	-
embodied -sensory	-	-	-	-
Implicit Verbal				
mental	The function $y=3x$ is of the type	The graph of the function $f(x)=ax$ is a straight line		certainly
Non-verbal				
mental				

Figure 4: Implementing the ProMoTe analytical tool (example 2)

Concluding remarks

In this paper, we approached argumentation and proof as multimodal text, with the purpose to design a tool that may prove to be helpful in analysing the complexities of everyday mathematics

argumentation and proving. We presented the implementation of the ProMoTe tool on a teacher's multimodal argumentation. The ProMoTe tool appeared to be helpful in identifying the specific ways that different verbal and non-verbal semiotic systems are explicitly and implicitly orchestrated in the different elements of mathematical argumentation as decomposed according to Toulmin's scheme. The ProMoTe tool helped in revealing aspects of the teachers' argumentation that may allow for the construction of inappropriate sociomathematical norms (Yackel & Cobb, 1996) about argumentation and proving (cf. Moutsios-Rentzos et al, 2019). Though researchers have considered multimodal aspects of argumentation (Krummheuer, 1995), ProMoTe allows for the different lines of argumentation that may be developed within and/or across different semiotic systems to be visible (cf. O'Halloran, 2008). Considering the rapidly evolving and technologically expanded mathematics classroom (cf. Hanna et al., 2019), which includes the digital, the hybrid and the virtual, and the relevant radical transformations of the educational processes, settings, and power-relationships, it is posited the multimodal mathematical argumentation (further amplified by technology) is at the heart of mathematical practices, thus rendering crucial the development of appropriate analytical tools that explicitly address multimodality to gain deeper understanding of the complex phenomena. Ongoing research is focusing on utilising the ProMoTe tool in mathematical multimodal argumentation about tasks concerning measurement and area, on including implicit non-verbal embodied-biometric and digital data, as well as on investigating the incorporation of ProMoTe in a teacher education programme.

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