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How preservice teachers enact mathematical argumentation and proof in class – an activity-theoretical perspective

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Argumentation and proof as core activities in mathematics should be staged continuously and meaningfully in mathematics lessons. But to what extent are preservice teachers at the end of their studies able to adequately introduce mathematical argumentation and proof as activities into their classroom planning and staging? Activity theory makes a valuable contribution to answering this question by emphasizing the importance of prospective teachers' development of motives and goals, corresponding modes of action, subjective constructions of meaning and the ability to identify appropriate objects for argumentation and proving activities in the classroom. In this work-in-progress paper, we outline an activity-theoretical framework and present empirical research tools based on it for analyzing prospective teachers' classroom enactments. We apply these to case studies from an exploratory, qualitative study with preservice teachers in their final year of study. We present first results and draw conclusions towards future work.

Keywords: Argumentation, proof, activity theory, preservice teacher education, second discontinuity.

Introduction

In preservice teacher education, one way to address the *issue of developing preservice teachers' ability to adequately introduce mathematical argumentation and proof as activities into their classroom planning and staging* is to foster students' own argumentation competence, e.g., via proving tasks, and to get them acquainted with didactical models and theories on developing argumentation competence in class. However, it is unclear whether this kind of study suffices, within the bounds of possibility, to prepare preservice teachers well for teaching mathematical argumentation. One source of such doubt is the so-called phenomenon of double discontinuity (Klein, 1908), which concerns two difficult transitions: first, the secondary-tertiary transition, when students enter university (see Gueudet, 2008); second, the transition from university to teaching at school. For the latter transition, the crucial question is to what extent teachers are able to make effective use of academic knowledge in their teaching. There is evidence in practice that teachers do not make full use of their content knowledge and pedagogical content knowledge when designing lessons. An activity-theoretical approach can help to theoretically ground such practical impressions regarding the second discontinuity and to substantiate and qualify them through empirical research. It goes beyond the consideration of the role of affect and beliefs for teaching argumentation and proof, but can also function as an interface to it. It offers starting points for the development of suitable formats for preservice teacher training courses in order to effectively address this issue.

Activity-theoretical framework

Activity-theoretic perspectives have already proven helpful in the teaching and learning of mathematical argumentation and proof in connection with the role of tools and cultural artifacts (e.g.

Cerulli, Pedemonte, & Robotti, 2005). In our work we shift the focus to the role of motives, goals, and constructions of meaning in the teaching of mathematical argumentation. We use the conceptual framework of Leontjew (1982), as developed further by Lompscher and Giest (see Bruder & Schmitt, 2016). From the perspective of activity theory, the constitutive elements of human activities are a superordinate *motive*, the *objects* of activity, and *ways or means of action* to act on and with the objects. The motive drives actions directed towards an object of activity, dependent on the repertoire of ways of action and available means. Concrete *goals* of such actions realize the motive in various ways. In individual activity, superordinate motives usually are unconsciously or subconsciously behind consciously set goals for actions. In the process leading from the superordinate motive to the concrete goals of action, *individual constructions of meaning* emerge.

The activity-theoretical framework enables us to analyze differences and commonalities between mathematical practices at university vs. school – they manifest themselves in all components of the activity-theoretical framework: a) While one of the central motives of university practice lies in the argumentative justification and explanation of the deductive derivability of a statement within the framework of a mathematical theory, a related motive in school mathematics would focus on truth or general validity of a statement rather than its derivability. b) While at university the objects of activity are explicitly stated (as in propositions, conjectures or proofs), the objects in school mathematics are often more implicit, “hidden” opportunities for argumentation (such as the comparison of different solutions or the justification of calculation rules). c) At university, the ways and means of action consist (in the context of justification) in valid reasoning within the framework of globally ordered mathematical theories. In school mathematics, we rather find plausible and example-based as well as heuristic and generic argumentation, and (more informal) deductions in locally ordered propositional systems. Prominent goals realizing the motives of argumentation activity in both practices are the well-known “proof functions” according to Villiers (1990).

Research question for the exploratory study

From the perspective of activity-theory just presented, we can now formulate an initial hypothesis with regard to issue raised at the beginning. *We hypothesize that preservice teachers often do not adequately develop suitable motives and corresponding constructions of meaning in their studies, as well as develop an inadequate repertoire of actions and dismiss possible objects for argumentation and proof in mathematics lessons. Consequently, they are often not able to give space and shape to argumentation and proof in their own school teaching in a way that in principle accommodates the profound and multifaceted meaning of these activities for relevant mathematical practices.* The hypothesis is motivated by the observation from university teaching practice that preservice teachers develop a highly reduced image of mathematical argumentation and proof during the mathematics lessons that they experienced at school, which is only put into perspective in de facto little mediated ways at university. Moreover, constructions of meaning for argumentation and proof, developed in university and school mathematics practice, are primarily shaped by actual experience: “Meaning is educated” (Leontjew, 1982). These experiences can be quite one-sided in both of the respective practices. For example, preservice teachers at university increasingly experience that the meaning of proving is systematization, whereas in the school practice they experience it may at best mean verification. Possible objects of argumentation and proof as well as appropriate ways of acting are

often not perceived very much. At school, for example, objects of proof appear only singularly, objects of argumentation rather covertly, and generic argumentations are often not (fully) recognized as argumentations. Furthermore, the respective practices are partly experienced under other, more dominant overriding motives, for example as “learning practices” under the motive of solving set tasks according to certain standards. In school, for instance, an emphasis on application, when experienced as dominant, may overshadow meaningful motives of argumentation and proof. We consider such discontinuity experiences of naturally existing differences between mathematical practices on the part of preservice teachers as an additional cause of the circumstances claimed in our hypothesis, and pose the following, *open research question* for our explorative study:

In which sense do preservice teachers lack effective motives and corresponding constructions of meaning, appropriate ways of acting or access to suitable objects, in order to stage argumentation and proof activities in a meaningful way in the mathematics lessons they plan and conduct?

Methodology of the exploratory study

Using the activity-theoretical perspective described above, we deductively developed an observation and analysis framework for teaching productions by preservice teachers. In our study, we applied this instrument in the context of a course in the final year of study, in which six pairs of preservice teachers each plan one classroom session on a mathematical subject of their choice, carry it out as a teaching experiment with their fellow students as peer experts, and receive professional and peer feedback. Hence, we observed and analyzed a total of six different classroom sessions. The chosen topics for the sessions were: *area of triangles* (1), *power functions* (2), *half-life* (3), *zeros* (4), *binomial formulas* (5), *scalar product* (6). The two authors worked independently of each other with a semi-structured observation sheet and compared their observations in follow-up discussions. Descriptions of observed, argumentative or argumentation-related actions of the teacher, of requests for such actions to the learners, as well as related formulations of goals, motives or object designations made by the teacher were noted in the sheet. The time and phase of the lesson or the phase transition were also recorded, as well as optional comments by the observers, both descriptive and interpretive in nature, for example on the actual actions of the learners. In addition to the completed, semi-structured observation sheets, the written plans, the classroom materials and the preservice teachers’ written post-lesson reflections form the data basis of our study.

As a first step of evaluation of the observational data and the planning and reflection documents, we describe stably occurring phenomena and patterns and propose an activity-theoretical analysis and explanation. The framework categories of *motive*, *object*, (*way of*) *action*, *goal*, and *construction of meaning* we use are obtained deductively from activity-theory. We supplement these inductively with intended or actual motives, goals and ways of action that can be recognized in the data.

Results

In a first review of the observational data, we were able to identify three overarching phenomena and associated stable patterns as specific manifestations of the phenomena in the teaching productions of the preservice teachers, which contribute to further differentiate our initial hypothesis with regard to our research question. In the following, we describe each phenomenon and its patterns, give concrete examples of the patterns and propose an activity-theoretical explanation for the phenomenon.

Phenomenon 1: Missing out on opportunities for argumentation

Results and answers of the learners are not questioned further in class, sometimes not even checked. In addition to “how did you arrive at this result?”, questions like “why does it work that way?” and “what is good about this way?” are missing, i.e., questions that are fundamental for mathematical argumentation as an activity.

Phenomenon 1 occurs in three different patterns and shows up both on the situational-spontaneous level of action and on the level of reflexive planning action. As we illustrate in the following, these observations can be understood from an activity theoretical perspective as an indication that the availability or accessibility of *objects of argumentation* has an impact on two essential professional competence areas of teachers: “reflective competence” and “action-related competence”, which are defined and measured through the corresponding action (Lindmeier, 2011).

We first describe the patterns and concretize them through examples from the staging observations. Then we add suitable excerpts from the planning observations.

Pattern 1.1: Receiving results and moving on. *Learners’ answers are received and rated, but they are not questioned further or confronted with each other.*

Answers and results are not used as potential objects of argumentation activities. This applies both to planning (“symptoms”: discussion phases are planned far too briefly, possible variants for solutions are not considered in advance) and to situational ad hoc action in lessons.

Pattern 1.2: Leaving questions from students behind. *Unexpected questions from learners are acknowledged as an element of classroom interaction, but they are left behind as objects of argumentation.*

This pattern primarily concerns situational ad hoc action in class.

Pattern 1.3: Leaving opportunities unused in task construction. *The argumentative potential is not exploited in task construction, the staging does not focus on argumentation.*

This pattern primarily concerns the planning process, when during task construction possible objects of argumentation are not realized and hence do not become effective in staging.

We choose examples for the patterns from session 6 (“scalar product”) because all three patterns occur in this session. An overview of the patterns that were recognized in agreement by both observers (regarding all phenomena and sessions) is provided in Table 1. The learning content of session 6 are four basic mental models for the scalar product, relating it to *projection*, *orthogonality*, *product*, and *angle*. Small groups of learners go through four learning stations, each assigned to one of the basic mental models. Patterns 1.1-1.3 can be recognized in the staging observations in the following places:

[1.1: Receiving results and moving on] At station 1 (*projection*) the learners spend much time with calculations, which are then only looked at. Later in plenary, a pure checking of results is done.

[1.2: Leaving questions from students behind] In plenary after the station work, a learner reports purely procedurally, which the teacher acknowledges with “OK”. The question of whether the

scalar product can be negative is raised by a learner and answered by the teacher, but only with the brief mention of an inappropriate technical term. It was obviously not foreseen in the planning.

[1.3: Leaving opportunities unused in task construction] At station 2 (*orthogonality*) the teacher asks about “commonalities” among the given cases of vector pairs. In return to the answer “are perpendicular”, the teacher asks “why”. This could be a good attempt to go into depth argumentatively. Unfortunately, the teacher’s question remains unanswered and is then not pursued further. The conceptual aspect of relative coordinates, which would have been part of such an argumentation, remains excluded throughout the whole session.

Table 1: Phenomena and patterns (X = pattern recognized in class or found in planning documents)

Phenomena	Patterns	Sessions					
		1	2	3	4	5	6
(1) Missing out on opportunities for argumentation	1.1: Receiving results and moving on	X	X	X	X	X	X
	1.2: Leaving questions from students behind		X			X	X
	1.3: Leaving opportunities unused in task construction	X		X	X		X
(2) Missing focus on content and conceptual core	2.1: Strong emphasis on methodological side of teaching	X		X		X	
	2.2: Missing the conceptual core		X	X	X		X
(3) General structure and discursive character of argumentation not exemplified.	3.1: Teaching by preparing written tasks or task sequences without a (local or global) argumentation-oriented dramaturgy	X	X	X	X	X	X

The additional planning observations can be used on the one hand to support that the occurrence of the three patterns in class is consistent with the planning: The planning statement “The educational content of the lesson is the recognition of a new operation and the application of the arithmetic operation with vectors” is consistent with extended phases of mechanical calculations as described, in contrast with short phases for in-depth comparison of results (Pattern 1.1). The planning statements “The pupils are able to *experience* the scalar product in its various forms and effects in group work” and “In addition, it can have a motivating function to *show* the practical benefits of the new operation” (our emphasis) indicate Pattern 1.3 inasmuch as they focus on the phenomenological rather than on the argumentative aspect. On the other hand, both the staging observations and the planning observations reported so far appear to be in contrast with goals that the preservice teachers set, partly with explicit reference to German core standards K1-6 and levels of cognitive complexity AB1-3:

“The learning objectives are as follows. [...] The pupils *explain the effect* of the scalar product [...] and *explain the connection* between the scalar product and the cosine (K1, K6, AB3). [...] [They

realize that the angle between the vectors plays a *crucial role*. [T]hey *discuss why* zero comes out for the orthogonal vectors and not for the other vectors.” (our emphasis)

We explain phenomenon 1 from an activity-theoretical perspective in a more general way on the level of *objects*: Our findings suggest that, even though goals were set that are appropriate for argumentation activities, in task construction preservice teachers do not succeed in connecting these goals to suitable objects of activity. In our example, they do not recognize the basic mental models of the scalar product as objects of argumentation activity, but rather as phenomena to be experienced.

Phenomenon 2: Missing focus on content and on the conceptual core

A lack of focus on content and on the conceptual core within the planning manifests itself in a conspicuous accumulation of learning activities that are not properly related to the learning content of the session. Accordingly, there is no focused content-related activation of the learners; in particular, activities that can be beneficial to argumentation (e.g. observation) lose the content focus.

Phenomenon 2 occurs in two patterns that fit a distinction developed in Renkl & Atkinson (2007) from the viewpoint of educational psychology: active responding, active processing, and focused processing. *Active responding* merely refers to a visible engagement of the learner with the learning environment. *Active processing* refers to actual processing of the content, beyond overt action and interactivity. Finally, the stance of *focused processing* emphasizes that it may be crucial that learners activities are focused on the central concepts and principles to be learned.

Pattern 2.1: Strong emphasis on the methodological side of teaching. *The staging is methodically (and sometimes technically) overloaded with actions that are not related to the mathematical learning content. As a result, learners' engagement contributes little to their understanding of the content.*

The occurrence of this pattern only leads to active responding of the learners in the sense of Renkl & Atkinson (2007). Pattern 1.1 particularly concerns the level of reflective planning actions.

Pattern 2.2: Missing the conceptual core. *The staging contains mathematical actions related to the mathematical learning content, but these do not reach its mathematical core. As a result, learners' engagement is not focused on the content core. In particular, argumentation-related activities appear not to be “conceived from an explanatory warrant” with a view to foster learners' deeper mathematical understanding.*

In Pattern 2.2 active processing of the learners can be observed, but their mathematical engagement does not constitute focused processing. The pattern concerns planning as well as staging.

The staging in session 3 (“half-life”) exemplifies both patterns (see Table 1). The learning content is the half-life in the context of exponential functions, which is concretized regarding the real-life phenomena of beer foam decay and dice throwing. The individual patterns can be recognized in the staging observations in the following places:

[3.1: Strong emphasis on the methodological side of teaching] The foam measuring activities or the implementation of the dice throwing experiment dominate the staging of the group phases.

[3.2: Missing the conceptual core] In working with the experimental data, learners are asked to plug in and calculate in the first place. Modelling work including discussions about the exponential

behavior (as core of the matter) is neither visible during the group phases nor addressed in the follow-up plenary. In both phases we observe active processing, but no focused processing.

The planning documents reveal a certain tension: on the one hand, the aim of the session appears to be the *application* of existing knowledge about exponential processes and half-life to self-conducted, real-life experiments, presupposing that it is *already known* that the core processes involved are exponential. However, the planning of the concrete implementation is geared towards *argumentation* – but it is unclear from which premises and to which conclusion the argument leads:

“Pupils use the example of an everyday phenomenon to apply their already learned knowledge and skills about half-lives. They determine experimentally the half-life of dice throwing and beer foam decay by conducting experiments.”

“In the case of the dice experiment, arguments can be made mathematically or with the help of exponential correlations. In the case of beer foam, [...] the learners should argue that the half-life does not change. Here, they could, for example, argue with prior knowledge from the previous lesson or the exponential equation. [...]”

All in all, the planning fluctuates between a focus on argumentation about exponential behavior on the one hand and experimentation and application on the other. In the implementation, we saw no argumentation-related activities, but a number of unfocused technical or instrumental activities like measuring, plugging-in, and calculating instead. We explain phenomenon 2 on the level of *motives*: Regarding Pattern 2.1, one possible explanation of our findings could be that “active” learning (in a naive interpretation of “being active” as “doing”) and application to the real world are effective as superordinate motives that override specific motives for mathematical argumentation activities. Regarding Pattern 2.2, the preservice teachers could be guided by rather nonspecific motives like “doing mathematics” (not specifying the content focus) or “doing argumentation” (unaware of concrete assignments of functional roles within the arguments).

Phenomenon 3: General structure and discursive character of argumentation are not exemplified

There is a lack of exemplification of mathematical argumentation by the teacher as a “living model” and as a knowledgeable navigator in argumentative classroom discourse. Such discourse hardly takes place, and if at all, the structural elements of argumentation remain hidden.

We observed one stable pattern which occurred in all six sessions (see Table 1):

Pattern 3.1: Teaching by preparing written tasks or task sequences without a (local or global) argumentation-oriented dramaturgy. *The teacher prepares pre-formulated work assignments for individual or group work, and then largely fades into the background in the production. Neither the work assignments nor the classroom discussion of the results guide argumentation activities in a structural or discursive sense and clarify the argumentative dramaturgy.*

This pattern concerns planning activities in the first place, but can also be instantiated, e.g. in the form of an ad hoc decision of the teacher to withdraw from an active role in a class discussion. Due to limited space, we have to dismiss more detailed example illustrations. We explain phenomenon 3 on the level of *sense constructions*: The planning documents show that the preservice teachers are

somehow aware of the general motives of mathematical argumentation activity and also concretize these in part in suitable goals, such as exploration, conjecture and systematization. However, the chosen ways of action are often either not appropriate for pursuing the selected goals or they are not implemented as part of an effective argumentative dramaturgy. A reason for this could be that the preservice teachers lack the corresponding meaningful experience that could serve as a source of meaning constructions. Hence, they lack a basis to link goals and suitable ways of acting in a meaningful way and to concretize the motives of argumentation activity in the lesson.

Summary and Outlook

We showed how an activity-theoretical framework can be used to analyze prospective teachers' classroom enactments. Our analysis of preservice teachers' lesson planning and staging exhibits phenomena and stable patterns that can be explained in terms of motives, objects, goals, ways of action, and meaning construction. Due to the exploratory nature of our study, we obviously cannot draw general conclusions. Regarding specific limitations, we point out that the lessons were conducted in a university seminar (in digital format) with peers as learners. While one could argue that the participants might act differently in a real-life setting, we conjecture that core elements of analysis (i.e., motives and objects) are not affected substantially by the setting. Our observations differentiate the hypothesis formulated at the beginning and illuminate it as a general issue in teacher education from a new perspective. Our activity-theoretical analysis of the three phenomena interprets them as specific variants of the overarching phenomenon of the 'second discontinuity': Developing preservice teachers' own argumentation competence and didactical knowledge alone might not be sufficient for them to successfully enact mathematical argumentation in class. From a developmental perspective, an important objective for future work is the design and exploration of appropriate formats for teacher training that sustainably address the observed discontinuity phenomena. Of particular interest might be in how far the phenomena identified in our observational data can be developed into explicit guiding principles for teaching mathematical reasoning and proof (e.g., Buchbinder & McCrone, 2022) and be incorporated into preservice teacher courses.

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