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# Changing the significance of argumentation and proof in final secondary school examinations – a comparison between Hungary and Thuringia

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*Argumentation and especially proofs play a crucial role in mathematics as a science. To convey an authentic view of mathematics in school, arguments and proofs should play an important role in mathematics classrooms, too. Analysis of textbooks, curricula and examination tasks provide relevant insight into the role that proofs play in mathematics education. This paper compares the weight and the content of arguing and proving tasks in final secondary school examinations in two educational administrations which share similar recent political, social, and educational history.*

*Keywords: Secondary school mathematics, exit examinations, mathematics tests.*

## **Introduction**

Because of verifying mathematical knowledge, proofs play, without any doubt, an essential role in mathematics as a science (Hanna & Barbeau, 2008). Moreover, Rav (1999) emphasizes the unique value of proofs in mathematics, based on their function as a means for gaining and systemizing mathematical knowledge. He concludes: “Proofs, I maintain, are the heart of mathematics, the royal road to creating analytic tools and catalysing growth.” (ibid., p. 6). In addition, Heintz (2000) declares, proofs would constitute mathematics and, mathematics would define itself by proofs. In any case, demonstrating proofs must be only seen as one of the main activities in mathematics.

Among mathematics-education researchers, there is a consensus about the key role of proofs both in mathematics classrooms and curricula (e.g., Hanna, 2000; Harel, 2008; Mariotti, 2006; Nardi & Knuth, 2017). The reasoning behind this, however, can fundamentally differ. While Harel (2008), for instance, underlines the way of thinking which becomes manifest in proofs, Hanna (2000) emphasizes the function of promoting mathematical understanding. Beyond that, Nardi and Knuth (2017) point out an active view of learners and plead for proofs in school because they are “critically important to knowing and doing mathematics” (p. 267). Different reasoning, indeed, leads to pursuing various objectives as well as to varying implementations when teaching proofs in mathematics classrooms.

In particular, cross-national research on curricular documents, such as educational standards, curricula, and textbooks, can provide an essential contribution to detecting the above-mentioned differences in regard to the objectives and implementations of proofs in mathematics education. This approach was taken by Jones and Fujita (2013), who investigated the implementations of national curricula in the geometry chapters of textbooks in England and Japan. Despite noticing many similarities between the geometry curricula, they identified differences in the two countries regarding the treatment of proofs. Whereas in England a low ratio (6.9 %) of proof-related tasks was found, the ratio was much higher (26.2 %) in Japanese textbooks. This result can be explained by the fact that the Japanese curriculum explicitly stipulates proofs only for geometry, whereas the English

curriculum requires them also in the domains of numbers and algebra. Based on a comparative analysis of textbooks, teacher guides of textbooks and curricula, Miyakawa (2017) identifies differences between France and Japan regarding the nature of proofs to be taught in geometry. He finds that implicit differences in geometry theory as well as in the principal function of proof related to that theory influence the nature of proof in geometry education. In addition, there is a wide range of national analyses of textbooks related to the role of proof, for example, in Australia (Stacey & Vincent, 2009), in the USA (Stylianides, 2009), and in Hong Kong (Wong & Sutherland, 2018).

According to Karp and Shkolnyi (2021), not only textbooks and curricula, but also final exams have a high impact on mathematics education. They also state that the scholarly literature related to final exams in mathematics is not extensive. Moreover, none of the reported studies in their very recent article is explicitly linked to argumentation or proof. With the cross-national study presented in this paper, the author aims to contribute to this research gap.

### **The final secondary school examination in Thuringia and Hungary**

Even if during the last decades, emphases have changed several times in mathematics education, proofs still play an important role in Hungarian mathematics education. A similar development can be seen in Thuringia, these days a German federal state. Hungary and Thuringia share a very analogous recent history on the one hand, while having both experienced substantial changes to their educational systems about the same time, on the other. Following the political changes in 1989, the democratization of the school system took place in the 1990s in both places. For different reasons, a second transformation took place around 2005. In Germany, the Standards for the General Certificate of Secondary Education in Mathematics (Kultusministerkonferenz, 2004) were established, and all federal states started to implement them in their curricula. Meanwhile, Hungary established a new, modernised final secondary school exam in mathematics at that time (Lukács, 2006).

For better understanding of the current research, I describe in the following passage some essential changes, which were applied to the final secondary examination in mathematics in both places. Note that the final secondary exam is carried out in each place at different levels of ability and in different kinds of secondary schools. This paper focuses on the main group addressed by this type of examination: learners in grammar school (Gymnasium, gimnázium) taking a basic course in mathematics and taking the secondary school exam between 2001 and 2020.

Table 1 shows the changing of the main exam characteristics in both places over time. The data demonstrate different tendencies in Hungary and Thuringia: in Hungary, one main structural change took place in 2005, while the characteristics of the examination in Thuringia were changed stepwise. Both places, however, share the characteristics that at the beginning of the 2000s only complex tasks were assigned, whereas in 2020 a mixture of elementary and complex tasks characterize the examination in mathematics, the latter being (partly) elective. Elementary tasks (such as calculating the first derivative of a polynomial function) require only a few cognitive steps. In contrast, complex tasks (such as sketching a curve that models an every-day problem) not only need the use of several cognitive steps but also often require combining different kinds of information. In addition, in Thuringia over the last twenty years, not only have the total exam points available increased twice, but also the length of time for the exam has been significantly extended.

**Table 1: main characteristics of the final secondary school examination in mathematics**

time period	Hungary	Thuringia
2001	180 minutes total score: 80	210 minutes, total score: 60 1 (out of 2) and additional 2 (out of 3) complex tasks
2002 – 2004	6 complex tasks and 1 additional proving task	210 minutes (270' from 2011), total score: 60 4 – 5 elementary tasks and 2 (each out of 2) complex tasks
2005 – 2013	180 minutes total score: 100	
2014 – 2016	12 elementary tasks and 2 (out of 3) additional complex tasks	270 minutes, total score: 60 7 – 8 elementary tasks and 2 (each out of 2) complex tasks
2017 – 2020		(270 in 2017) 300 minutes, total score: 120 7 – 8 elementary tasks 1 complex task and 1 (out of 2) additional complex task

## Research questions

Against the background of the high significance of proofs in mathematics as a science as well as in mathematics education, this paper investigates, to what extent has been changed their weight and content in mathematics education in these two states. Such an analysis can give insight into the claims implemented in mathematics education and addressed to the learners at the end of their secondary education. The notions “argumentation” and “proof” are used in different ways in the related literature. However, in this paper both, mathematical argumentation and proof, are understood as realizations of reasoning in mathematics, based on Brunner (2014). The research presented in this paper was led by the following question: To what extent have the expectations at the end of secondary education related to mathematical argumentation and proof in Hungary and Thuringia been changed in the last two decades? Are the tendencies in those places similar? Which conclusions can be drawn about the role of proofs, based on the identified tendencies?

## Methodology

The methodology used in the study was already successfully applied and detailed described in Szűcs (2021). A short summary of this is to identify temporal alterations based on existing tasks and guidance material for marking and evaluating. Documentary research was chosen as an appropriate method for data collection. In addition, qualitative content analysis, which allows for the systematic and theory-based processing of big textual data, was selected for data examination.

## Documentary research

Original examination tasks, as well as the relevant guidelines for marking and assessment, are the primary sources of the study. *The nature of the source material* in the places that were researched is insignificantly different. Whereas examination tasks, including guidance material, are in the public domain in Hungary, this is not the case in Thuringia. However, materials for the whole period under

analysis could be reconstructed in both places based on the public sources of the Hungarian Ministry of Education, on the unpublished sources of the Thuringian Ministry of Education and, on the information available from task developers (Fried, 2004; Skorsetz, 2005). However, based on the authority of the providers, we can conclude, that all sources are authentic, reliable, and trustworthy.

### Structuring qualitative content analysis

Since models and categories related to argumentation and proof in mathematics already exist, a special type of qualitative content analysis, the so-called structuring qualitative content analysis, was deemed to be appropriate. In addition, this method is particularly useful for identifying temporal tendencies, which is extraordinarily relevant for the current research questions.

**Table 2: excerpts from the encoding manual**

<b>category</b>	<b>reasoning with mathematical tools</b>
definition	Argumentation is based on mathematics, but not necessarily on deductive steps.
standard example	Three books were taken from a bookcase and put back arbitrarily. [...] Demonstrate, that it is not possible that exactly two of the three books are in the right place. (2009, part C, task e)
explanation	Systematic testing of all ways is possible.
encoding rule	Reasoning based on concrete examples is possible and productive.
<b>category</b>	<b>logical reasoning with mathematical tools</b>
definition	Deductive steps are identifiable, but they are not necessarily formal.
standard example	Three points, A (3;0;5), B (6;4;3) and D (0;4;3) are given, in a Cartesian coordinate system. Show that A, B and C explicitly determine a plane $\varepsilon$ . (2001, part 2.1, task a)
explanation	Definition of linear dependence and vectors must be deductively applied.
encoding rule	Deductive application of definitions, theorems and methods is needed, but no formal proof.
<b>category</b>	<b>formal-deductive proof</b>
Definition	formal chain of deductive steps
Standard example	$(a_n)$ is a sequence of numbers with $a_n = \frac{n+1}{n}$ , $n \in \mathbb{N}$ and $n \geq 1$ . Show that $(a_n)$ is not a geometric sequence. (2002, part C, task a)
explanation	Formal notation and, the generality of the claim suggest that a formal proof is expected.
encoding rule	Requirement of a formal proof must be recognized.

To determine *basic material*, it is necessary to define the terms “argumentation” and “proof”. A viable definition of argumentation, which includes the notion “proof”, was given by Schwarzkopf (2000). It means a social interaction in school, in which a need for reasoning is indicated, and afterwards, this need is tested to be satisfied. According to this, all examination tasks have been rated as argumentation tasks, in which a need for reasoning was clearly indicated, meaning tasks contained verbs such as “show”, “reason”, “prove” etc. Note that searching for keywords is a popular method to identify argumentation tasks when analysing textbooks (Mariotti et al., 2018). The set of tasks containing at least one of the listed verbs is called the basic material. A *set of categories* has been specified, according to Brunner (2014). She identifies four categories based on the cognitive level needed for reasoning in mathematics classrooms, which enables a qualitative differentiation between the expected arguments. Excluding the type “everyday-arguing”, the following three types have been applied to the data: reasoning with mathematical tools, logical reasoning with mathematical tools and, formal-deductive proving. The *encoding* was carried out as follows: Each coded text passage was mapped to one of the three categories. This mapping process was supported by the information given in the guidance material. Table 2 shows excerpts from the encoding manual; the tasks have been translated into English by the author of this paper.

After encoding, *results were prepared* as follows: Based on the marking instructions, the score of each argumentation task, and its related mathematical domain were recorded. The total score of each category was calculated for each year and, afterwards, their proportion of the whole exam was determined. Tasks of choice, in the meaning of alternatives, were noted. Scores of the three categories added up to a total argumentation score for each year. Within this total score, the proportion of each mathematical domain was also calculated. Table 3 gives insight into this process, based on the Thuringian data from 2002.

**Table 3: Encoding the arguing tasks from Thuringia in 2002**

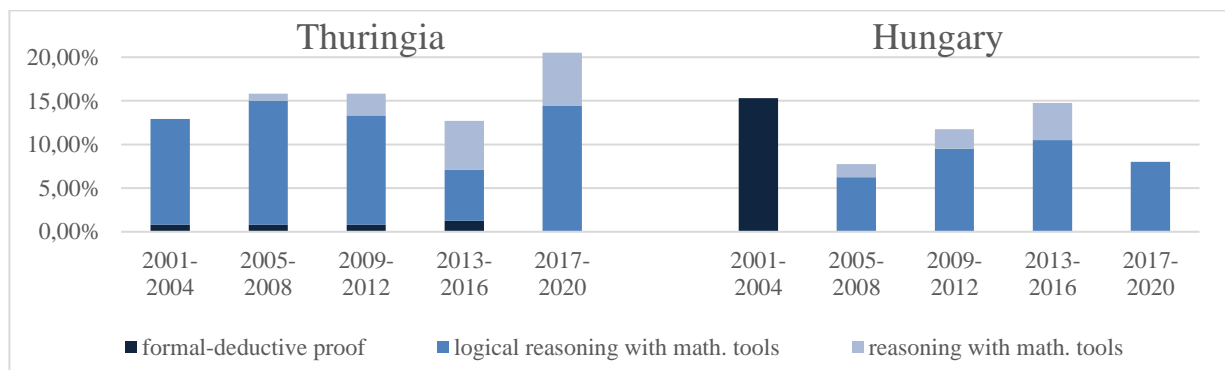
year	task	category	score		proportion of the exam	domain
2002	part A1, task b	log. reasoning with math. tools	1	average: 1.5 (A1 and A2 alternatives)	2.5 %	calculus
	part A2, task a	log. reasoning with math. tools	2			calculus
	part C, task a	formal-deductive proof	2		3.3 %	calculus
	total of argumentation tasks		3.5		5.8 %	

## Results

### Comparison of the changing of quantitative aspects in the final secondary school examination

Tendencies related to the volume of argumentation tasks and the type of reasoning are shown in Figure 1. The data suggest – similarly to the findings of Miyakawa (2017) – that there are differences in the nature of argumentation tasks between Hungary and Thuringia, which can be traced back to

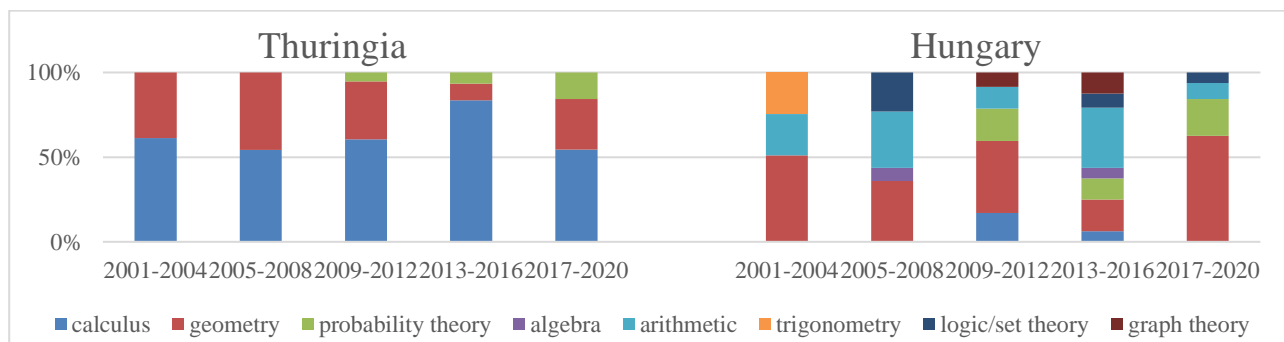
different implicit views on mathematics. In Hungary, before the new examinations in 2005, a high proportion (15.31 %) of argumentation tasks was present and, those tasks required only formal-deductive proofs. However, the introduction of the new exam led not only to the halving of the proportion of these tasks (7.75 %) but also to the complete absence of formal-deductive proofs. Hungary experimented between 2005 and 2016 by addressing argumentation tasks, which do not require deductive reasoning but gave up on that in 2017. Hungary seems to be a country, in which proofs in school mainly have the function of demonstrating a specific, deductive way of thinking (Harel, 2008). In Thuringia, in contrast, formal-deductive proofs did not play an important role in the period under investigation. However, the proportion of argumentation tasks was relatively constant over the time in question (12.7 %–15.8 %) and increased in the last four years to over 20 %. The proportion of tasks requiring logical reasoning with mathematical tools varies between 5.8%–14.5%, but they are complemented by an increasing number of tasks requiring reasoning with mathematical tools. Thus, it could be inferred that argumentation plays an important role in Thuringia, too, but the focus is more on applied mathematics.



**Figure 1: changing of the types of reasoning in final secondary school examinations over time**

### Comparison of the changing of qualitative aspects in the final secondary school examination

The above-described quantitative tendencies regarding the type of reasoning also have a qualitative component. Changes related to the specific mathematical domains are presented in Figure 2. Each percentage expresses the proportion of the score of the domain related to the total score of argumentation tasks.



**Figure 2: changing of the mathematical field of reasoning tasks over time**

Significant differences between the two places in the study can also be observed regarding domains. Whereas only three mathematical domains are used in Thuringia, the argumentation tasks in the final

secondary school examination in Hungary become more varied over time, covering up to seven different domains. Furthermore, calculus dominates constantly in Thuringia, but it is barely evident in Hungary. In addition, while geometry is noticeable but not dominant in Thuringia, it is the main mathematical domain of argumentation in Hungary. Mathematical reasoning is more spread across various domains in Hungary than it is in Thuringia. These findings are similar to the results which compare England and Japan (Jones & Fujita, 2013) and can be traced back to different values of the final secondary school exam in those two places: Whereas this type of examination rounds upper secondary school education in Thuringia, it finishes the entire secondary education in Hungary.

## Summary and open questions

Even if argumentation and proof form the main parts of mathematics as a science leading to the view that they should play a key role in mathematics education, they are only moderately included in final secondary school examinations in Hungary and in Thuringia. Especially alarming is the fact that formal-deductive proofs currently play no role in those examinations. However, slightly different routes led to the current situation: In Hungary, formal-deductive proofs disappeared after the structural change of the exam in 2005, while they have never been focused on in Thuringia. Moreover, mathematical reasoning is spread across more various mathematical domains in Hungary than it is in Thuringia. These results may allow us to infer different views of mathematics in the two places. However, further analysis of curricula, textbooks and classroom activities would be needed to investigate those views and confirm this inference.

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