



# Mathematical reasoning at the age of four

Ieva Kilienė

## ► To cite this version:

Ieva Kilienė. Mathematical reasoning at the age of four. Twelfth Congress of the European Society for Research in Mathematics Education (CERME12), Feb 2022, Bozen-Bolzano, Italy. hal-03746826v2

**HAL Id: hal-03746826**

**<https://hal.science/hal-03746826v2>**

Submitted on 17 Oct 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Mathematical reasoning at the age of four

Ieva Kiliene<sup>1</sup>

<sup>1</sup>Vilnius University, Lithuania; ieva.kiliene@mif.vu.lt

*Keywords: Mathematics education, reasoning skills, word problems (mathematics).*

## Word problems for developing mathematical reasoning

Mathematical reasoning is a widely analyzed topic amongst secondary and high school students. It is widely believed that mathematical reasoning is only possible for students once they are aware of various mathematical concepts, so we decided to check whether 4-year-olds are capable of mathematical reasoning. We used word problems that have been shown to develop mathematical reasoning for 8–10-year-old students in Schliemann et al. (2002). In this study, we adapted the methodology to test the word problem with a 4-year-old child and older solvers.

We analyze ideas about mathematical reasoning from Thompson et al. (2012), Arbaugh et al. (2018) and Stylianides et al. (2016) and define word problem solving characteristics that develop mathematical reasoning.

*A word problem develops mathematical reasoning*, if solving includes argumentation, justification or patterns identification and if it uses the statements known to a presenter and a class community.

## Experiment with a 4-year-old student

We adapt the word problem for 4-year-old students, who may not know how to read. We illustrate the problem and change the numbers to smaller ones. Our aim is to check if we can develop mathematical reasoning from early years and to see how students of different ages solve the same problem.

A word problem from Schliemann et al. (2002) is used:

Table A has 3 candy bars and 2 chairs. Table B has 6 candy bars and 4 chairs. Table C has 5 candy bars and 2 chairs. Show how you would divide the candy bars equally among those at each table.

Which table would you sit at? Convince us that you chose the best table. Are any tables the same?

The numbers in the original problem are such that at the first and second tables the subject gets 1 and a half candy bars, and at the third table there are 2 and a half candy bars. Our word problem (illustrated in Figure 1), using smaller numbers so younger kids do not make mistakes in computation. The solver must choose between tables where the bear character gets less than 1, more than 1 or 1 cupcake. The question is formulated in the following way: All bears are going to sit on chairs. If the bear wants to eat as many cupcakes as possible, at which table he should sit? Could you explain why?

A 4-year-old girl attending kindergarten solved this word problem. During the interview, she expressed ideas such as:

Girl: At the first table, the bear would get one cupcake.

Interviewer: Really? How many bears will sit there?

Girl: Four bears. Then we need to cut this cupcake into two parts. This bear will get one part; another bear will get second part (points to chairs). Let us cut another cupcake

into two parts. This bear will get one part; another bear will get the second part (points to chairs).

Interviewer: How much would each teddy bear get?

Girl: By four parts<sup>1</sup>.

Interviewer: How much would each bear get sitting at the middle table?

Girl: He would get two.

Interviewer: Even two? Why?

Girl: There are two chairs, three cupcakes.

Interviewer: One would get two, and the other?

Girl: Then we can cut one cupcake. One would get two and another the other two.

Interviewer: But not two whole ones?

Girl: By one and part<sup>2</sup>.

Interviewer: And at the third table.

Girl: They will get one each.

Interviewer: Where is it best to sit?

Girl: At this table (shows to the middle table), there they will get two cupcakes each.



**Figure 1: Mathematical reasoning word problem in image**

## Results

We interviewed different age groups with the same task. The justification was different, but correct amongst all interviewees, and we can identify that the argumentation changes during the learning period. The 4-year-old solver makes mistakes in the concepts she used, but shows correct reasoning. Our results show that we can use word problems that develop mathematical reasoning for younger kids.

## References

- Arbaugh, F., Boyle, J., Stylianides, G. J., & Steele, M. (2018). *We Reason & We Prove for ALL Mathematics: Building Students' Critical Thinking, Grades 6-12*. Corwin Press.
- Schliemann, A. D., & Carraher, D. W. (2002). The evolution of mathematical reasoning: Everyday versus idealized understandings. *Developmental Review*, 22(2), 242–266. <https://doi.org/10.1006/drev.2002.0547>
- Stylianides, A. J. (2016). *Proving in the elementary mathematics classroom*. Oxford University Press.
- Thompson, D. R., Senk, S. L., & Johnson, G. J. (2012). Opportunities to learn reasoning and proof in high school mathematics textbooks. *Journal for Research in Mathematics Education*, 43(3), 253–295. <https://doi.org/10.5951/jresmetheduc.43.3.0253>

---

<sup>1</sup> The 4-year-old student indicated that each bear would receive  $\frac{1}{4}$  of two cupcakes. She did not say this explicitly because she has not learned fractions and could therefore not articulate the answer in those terms.

<sup>2</sup> Same comment as above. The 4-year-old student indicated each bear would receive  $1\frac{1}{2}$  cupcakes.