



HAL
open science

Teachers' perspectives on number and addition in year one: application of FoNS framework in interview analyses

Pernille Bødtker Sunde, Judy Sayers

► **To cite this version:**

Pernille Bødtker Sunde, Judy Sayers. Teachers' perspectives on number and addition in year one: application of FoNS framework in interview analyses. Twelfth Congress of the European Society for Research in Mathematics Education (CERME12), Feb 2022, Bozen-Bolzano, Italy. hal-03746640v1

HAL Id: hal-03746640

<https://hal.science/hal-03746640v1>

Submitted on 5 Aug 2022 (v1), last revised 17 Oct 2022 (v2)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Teachers' perspectives on number and addition in year one: application of FoNS framework in interview analyses

Pernille Bødtker Sunde¹ and Judy Sayers²

¹Katholieke Universiteit Leuven, Belgium and VIA University College, Denmark; pbos@via.dk

²The University of Leeds, England; j.m.Sayers@leeds.ac.uk

This study investigates Danish teachers' perspectives on teaching and learning number and addition in year one through analyses of semi-structured interviews with six year one teachers. The teachers' perspectives are compared with theories on foundations for learning number and addition. The analysis indicates that important foundational factors, estimation, quantity discrimination and number patterns, were either superficially discussed or not addressed at all by the teachers in their interviews.

Keywords: Foundational number sense, teacher perspectives, simple addition, year one students.

Introduction

Learning and teaching of arithmetic has been a major topic for educational research for decades (Nunes et al., 2016) and the importance of students' arithmetic competence for mathematical development in general is widely acknowledged. Thus, numerical and arithmetic competence has been linked to development of mathematics achievement and difficulties (e.g. Feigenson et al., 2013; Geary et al., 2013; Ostad, 1997). Teaching of arithmetic in primary school relies on the implementation of key knowledge to set up strong foundations for a successful development of arithmetic competence, e.g. adaptive flexibility, strategies for mental calculation, and number knowledge. Therefore, it is important to get insight into teachers' perspectives on the teaching of number and arithmetic in the early years of school, where number and basic arithmetic is the primary focus, and research suggests is a crucial stage in children's development of number competencies.

In this paper, we explore six Danish teachers' perspectives on the teaching and learning of number and addition and analyse the extent to which this is aligned with established knowledge on foundations of numerical and arithmetic competence.

Numerical and arithmetic competence

Development of arithmetic competence rely on several components of numerical competence or number sense (Desoete & Grégoire, 2006; Fuson & Burghardt, 2003), for example: symbolic knowledge and number words (e.g. Chu et al., 2015), mapping symbols to quantity (e.g. Geary, 2013), basic counting skills (e.g. Jordan et al., 2009), number comparison skills (e.g. De Smedt et al., 2013), estimation skills (e.g. Booth & Siegler, 2008; Gilmore et al., 2007), and knowledge of base-ten number structure (Laski et al., 2014). The individual components of number sense are all important for further mathematical development, however, the links between them are essential for students' development (Gersten et al., 2005).

Recently, Andrews and Sayers (2015) have proposed a framework for identifying students' opportunities to acquire foundational number sense, FoNS, and demonstrated its strength when analysing lessons and textbooks in different cultural contexts (Andrews & Sayers, 2015; Löwenhielm

et al., 2019; Sayers & Andrews, 2015). The framework is derived from an extensive literature study and comprises eight categories of FoNS (for a thorough description see Andrews and Sayers, 2015): 1) Number recognition, 2) Systematic counting, 3) Relationship between number and quantity, 4) Quantity discrimination, 5) Different representations of number, 6) Estimation, 7) Simple arithmetic competence, and 8) Awareness of number patterns. Each of these categories has been shown to play an important role in students' development in mathematics (Andrews & Sayers, 2015).

The current study

This study investigates six Danish year one teachers' perspectives on the teaching and learning of number and addition in year one through analyses of semi-structured interviews. The aim of the study is to analyse to what extent teachers' perspectives are aligned with established mathematical knowledge of numerical and arithmetic competence.

Methods

Drawing on exploratory Case study methodology (Yin, 2013), this small qualitative study is based on semi-structured interviews with six Danish year one teachers. The teachers' utterances related to different aspects of the teaching and learning of number and addition were analysed and categorised using the FoNS framework (Andrews & Sayers, 2015).

The selection of participants ensured an equal number of male and female teachers, and a cross section of professional experience (2-24 yrs.) and age (30-49 yrs.). The teachers were informed about the project both in writing and at an introductory meeting prior to the interviews took place. By the end of the project, the teachers were offered to read and approve the transcripts of the interview. None of the teachers took advantage of this offer. Pseudonyms have been used throughout.

Teacher interviews

Each interview lasted between 45 and 60 minutes, and the interview focused on the teachers' perspectives on teaching and learning of arithmetic in year one. The semi-structured interview was guided by questions related to the teacher's plans for and reflections on a specific observed lesson on number and arithmetic as well as general questions on the teaching and learning of number and arithmetic (addition and subtraction) in year one. The questions on the observed lesson was related to planning of the lesson: "Why did you choose these specific activities?", carrying out the lesson: "How did you experience the lesson? Did it proceed as you had expected?", Progression: "How will you follow up on this lesson? What will be the next step?", and characteristics of a 'good activity': "what is a good activity and what makes it good? What do the students learn in these activities?".

General questions about learning were asked specifically about teaching addition, for example: "how do you introduce the students to addition" and "what aspects do you emphasize?", prerequisites for learning addition: "What is the prerequisites for learning addition, how do you ensure the students have the prerequisites?", and "Are there aspects of learning addition the students' find especially difficult?", Furthermore, questions were asked about children's progression in teaching addition, for example: "How do you see the progression in teaching addition?" and "What do you expect your students to know or be able to work on by the end of year one?".

Throughout the interview, if the teacher primarily referred to practical aspects, e.g. “a good activity is easy to explain for the students” or “the lesson went well because many students participated in the activity”, the teachers were asked additional questions related to the mathematics of the activities and lesson, e.g. “what aspects of number and addition do you think the students learn through that (particular) activity?”. According to Bryman (2016), this was to ensure teachers had an opportunity to reflect on the aspects of mathematics that were of interest in this project. Only utterances with mathematical content related to number and arithmetic was categorised in the present analysis.

The interviews were transcribed and analysed using NVivo, and excerpts presented here were all translated into English, a process that included transforming Danish idioms into equivalent English expressions without losing the speaker’s intended meaning (Brinkmann & Kvale, 2015).

Results

The FoNS categories were well presented in the teachers’ responses to questions. Table 1 provides an overview of how the categories are distributed over the six teachers’ interviews. In the following we provide examples of utterances by teachers that were mapped directly to each of the FoNS categories. However, a statement or description of an activity can contain several FoNS categories and is then assigned to all relevant categories.

Table 1: Overview of the categories of Foundational Number Sense (Andrews & Sayers, 2015) and their presence in the interviews with the six teachers indicated by X.

A: Allan, B: Bettina, C: Carl, D: Dan, E: Else and F: Frida.

Category	A	B	C	D	E	F
Number recognition	X	X	X	X	X	X
Systematic counting	X	X	X	X	X	X
Number and quantity	X	X	X	X	X	
Quantity discrimination	X				X	
Different representations	X	X	X	X	X	X
Estimation					X	
Simple arithmetic competence	X	X	X	X	X	X
Number patterns			X		X	

FoNS categories in the interviews

Number recognition: Knowing number symbols and number names were emphasised by all teachers but explicated very differently. The teachers expressed very different levels of necessary knowledge for the students. One teacher, Else, said that the students “need to know the numbers, their value and

be able to write and read them, and recognise them”. Frida exemplifies by saying “they should know what twelve looks like”. Allan specifies a number range: “the students need to learn the numbers to 100” and Dan explicated that the students do not have to know the number name as long as they “can write it” although he also emphasised knowing the names of the tens. Knowing the names of the tens was also mentioned by Carl as a help to find the number names of two-digit numbers. Bettina, talking about the base ten number system, explained how she focused on “enhancing the students’ competencies of naming number”. Naming two-digit numbers are something that many students find difficult because of the Danish number names. Carl mentioned this and explains how he addressed this in activities where the students “have to find the number 13 or 17 so they practise finding the correct symbol for the correct number name”.

Systematic counting: Counting skills are addressed by all teachers. Dan said that “they need to know the number sequence” and Carl stated “Early maths is mostly about counting”. Else said, when asked what she thought should be the focus in year 0 (a preschool class) she said “it’s important that they just count and count and count”. All the teachers provided many examples of counting procedures, often performed by the use of manipulatives or other representations e.g. a number line. The number line was mentioned by all the teachers in relation to activities of ordering numbers or “find the number” and when performing counting procedures. However, Carl and Else were the only teachers directly referring to knowledge of “the number before and after”. All teachers referred to skip counting, often by ten. Skip counting was used together with references to times tables and the teachers thus referred to “knowing the ten times table” when they taught the students to count in tens in order to find the name of a two-digit number.

Number and quantity: The relationship between number and quantity was addressed by four of the teachers but with different levels of articulation. Allan emphasised that “they need to recognise that quantity and number kind of go together”. Bettina said “the students need to understand the symbols and the naming of quantity”. Carl, elaborating on “the translation between number and quantity”, underlined that “they need to understand quantity; the symbol 4 equals four things”. Likewise, Else emphasised the understanding of the relationship between number and quantity. “They need to have an idea of what value is and what is worth more (...) so many dots or centicubes, what is the size and quantity of that”.

Quantity discrimination: Comparing quantity was addressed very briefly by only two teachers and only in a single statement form each. Else described an activity where students construct two-digit numbers by combining two playing cards, write the number and compare with the next number they construct. She reflects: “I don’t know if they just write some numbers or if they actually understand which is bigger and which is smaller”. Allan also addressed comparing numbers in relation to doing addition and comparing possible results: “seeing this result is one bigger and this is one smaller than the other”.

Representations of number: All teachers mentioned several different representations of numbers, both concrete materials like money, centicubes and fingers, but also partitioning in tens and ones, friends of ten and the number line. In two of the classes all students had a tablet, and the two teachers also

mentioned and app, Number Pieces, where students can represent numbers using ones, tens etc. and partition numbers.

Estimation was only mentioned by one teacher, Else. She very briefly referred to estimation of quantity by mentioning an activity of “how many in the jar”. This is an activity, where the students have different containers with an unknown number of items. The students then guess how many items are in the container and afterwards they count the exact number of items. However, she did not explicitly use the expression estimation or to estimate.

Simple arithmetic: Given the teaching and learning of number and arithmetic in year one was the focus of the interviews, simple arithmetic was mentioned by all teachers. However, the teachers differed substantially with regard to their focus on different calculation methods and strategies, the number range, bridging ten and level of fluency with single digit addition.

Number patterns: Several teachers talked about knowing the sequence of numbers (categorised as systematic counting), but only two explicitly referred to putting in correct order. Else referred to a specific evaluation activity where the students have to put some number cards in the correct order. Carl described an equivalent activity where students are given a number card and then has to line up according to the number sequence.

Discussion

The aim of this paper was to investigate to what extent teachers’ perspectives are aligned with established mathematical knowledge of numerical and arithmetic competence working with year one pupils. The discussion provides insights into how year one teachers in six Danish schools perceive key teaching and learning number and arithmetic attributes, but also how a simple framework can be used to identify these.

The analysis of the interviews revealed that all components of FoNS were addressed explicitly or implicitly in the interviews, but not equally by all teachers. A single teacher, Else, addressed all eight categories, whereas the teacher Frida only addressed four. The remaining four teachers addressed five or six of the categories. However, what is perhaps more important is to what extent the different components of FoNS was addressed by the teachers and how this aligns with established number knowledge of importance for further mathematical development.

Although all teachers explicitly emphasized number symbols and number names (Chu et al., 2015) as the most important aspects of “knowing number” not all teachers mentioned quantity and creating the link between symbols and quantity (Geary, 2013). However, all teachers mentioned the importance and relevance of using manipulatives, which implicitly provides learning opportunities for students to create the link between number and quantity, and other number representations. Counting skills (Jordan et al., 2009) was explicitly mentioned by all teachers. Thus, some of the basic components of FoNS and prerequisites for doing arithmetic is explicitly or implicitly part of the teachers’ perspectives on the teaching and learning of number in year one.

Three of the components found to be of special importance in the early years, quantity discrimination (De Smedt et al., 2013), estimation (Booth & Siegler, 2008; Gersten et al., 2005; Gilmore et al., 2007) and number patterns (Gersten et al., 2005), was only addressed implicitly by one (estimation) or two

teachers (quantity discrimination and number patterns), and in all cases these categories was only addressed superficially and implicitly by the teachers. These findings resonate with a cross-cultural study by Sayers and Andrews (2015) on the opportunities to learn different aspects of FoNS in different activities observed in six different European classrooms. Across countries, they found no episodes where teachers encouraged students to estimate and only 2 of 18 episodes where a single teacher introduced quantity discrimination.

Although estimation is considered to be one of the most important mathematical competences along with proportional reasoning and problem solving (Sriraman & Knott, 2009) it is remarkably absent in both classrooms (Andrews & Sayers, 2015; Sayers et al., 2016), textbooks (Sayers et al., 2021), and curricula (Andrews et al., 2021; Sunde et al., 2021). In this study we have shown that teachers do not explicate estimation or estimation related activities as an important part of their year one teaching.

With regard to the application of the FoNS framework to teacher interviews we found it successful on two key points: 1) The framework provided an easy to use categorisation of components of number sense known to be of importance for further development in mathematics (Andrews & Sayers, 2015). 2) The use of the framework also highlighted the differences in the number sense components the different teachers addressed.

In conclusion, the findings indicate that teachers are cognisant of a wide range of the important foundations for developing FoNS and arithmetic competence. However, it is also apparent that three crucial aspects, quantity discrimination, estimation skills and number patterns, were only mentioned implicitly by one or two teachers in the interviews.

We have shown that the FoNS framework is easy to apply in analysing teacher interviews, and can successfully reveal patterns of teachers' perspectives on the learning of number in year one. The analysis shows the differences between teachers with respect to the number of FoNS categories and it highlights the underrepresented categories.

This study cannot provide insight in how teachers actually teach. The analysis can only give an indication of what teachers emphasise in their classroom practice. A teacher's description of an activity cannot provide the full picture of the complete range of FoNS categories that the activity would cover when actually performed by the teacher in interaction with students in the classroom. Thus, the actual learning opportunities for the students might be richer than the interview would suggest. However, it would be reasonable to expect that what the teachers emphasise in the interviews is what they would also emphasise during teaching. Further research on video observations will show to what extent the findings of the lack of awareness on quantity discrimination, number patterns and estimation skills are accentuated in the actual teaching and learning in the classroom.

References

Andrews, P., & Sayers, J. (2015). Identifying Opportunities for Grade One Children to Acquire Foundational Number Sense: Developing a Framework for Cross Cultural Classroom Analyses. *Early Childhood Education Journal*, 43(4), 257–267. <https://doi.org/10.1007/s10643-014-0653-6>

Andrews, P., Xenofontos, C., & Sayers, J. (2021). Estimation in the primary mathematics curricula

of the United Kingdom: Ambivalent expectations of an essential competence. *International Journal of Mathematical Education in Science and Technology*. <https://doi.org/10.1080/0020739X.2020.1868591>

- Booth, J. L., & Siegler, R. S. (2008). Numerical Magnitude Representations Influence Arithmetic Learning. *Child Development*, 79(4), 1016–1031. <https://doi.org/10.1111/j.1467-8624.2008.01173.x>
- Brinkmann, S., & Kvale, S. (2015). *InterViews: Learning the craft of qualitative research interviewing* (3rd ed.). SAGE.
- Bryman, A. (2016). *Social research methods* (4. edition). Oxford University Press.
- Chu, F. W., VanMarle, K., & Geary, D. C. (2015). Early numerical foundations of young children's mathematical development. *Journal of Experimental Child Psychology*, 132, 205–212. <https://doi.org/10.1016/j.jecp.2015.01.006>
- De Smedt, B., Noël, M.-P., Gilmore, C., & Ansari, D. (2013). How do symbolic and non-symbolic numerical magnitude processing skills relate to individual differences in children's mathematical skills? A review of evidence from brain and behavior. *Trends in Neuroscience and Education*, 2(2), 48–55. <https://doi.org/10.1016/j.tine.2013.06.001>
- Desoete, A., & Grégoire, J. (2006). Numerical competence in young children and in children with mathematics learning disabilities. *Learning and Individual Differences*, 16(4), 351–367. <https://doi.org/10.1016/j.lindif.2006.12.006>
- Feigenson, L., Libertus, M. E., & Halberda, J. (2013). Links Between the Intuitive Sense of Number and Formal Mathematics Ability. *Child Development Perspectives*, 7(2), 74–79. <https://doi.org/10.1111/cdep.12019>
- Fuson, K. C., & Burghardt, B. H. (2003). Multidigit addition and subtraction methods invented in small groups and teacher support of problem solving and reflection. In A. J. Baroody & A. Dowker (Eds.), *The Development of Arithmetic Concepts and Skills: Constructive Adaptive Expertise* (pp. 267–307). Routledge. <https://doi.org/10.4324/9781410607218>
- Geary, D. C. (2013). Early Foundations for Mathematics Learning and Their Relations to Learning Disabilities. *Current Directions in Psychological Science*, 22(1), 23–27. <https://doi.org/10.1177/0963721412469398>
- Geary, D. C., Hoard, M. K., Nugent, L., & Bailey, D. H. (2013). Adolescents' Functional Numeracy Is Predicted by Their School Entry Number System Knowledge. *PLOS ONE*, 8(1), e54651. <https://doi.org/10.1371/journal.pone.0054651>
- Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities*, 38(4), 293–304. <https://doi.org/10.1177/00222194050380040301>
- Gilmore, C. K., McCarthy, S. E., & Spelke, E. S. (2007). Symbolic arithmetic knowledge without instruction. *Nature*, 447, 589–591. <https://doi.org/10.1038/nature05850>
- Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology*, 45(3), 850–867. <https://doi.org/10.1037/a0014939>
- Laski, E. V., Ermakova, A., & Vasilyeva, M. (2014). Early use of decomposition for addition and its relation to base-10 knowledge. *Journal of Applied Developmental Psychology*, 35(5), 444–454. <https://doi.org/10.1016/j.appdev.2014.07.002>

- Löwenhielm, A., Marschall, G., Sayers, J., & ... (2019). Opportunities to acquire foundational number sense: A quantitative comparison of popular English and Swedish textbooks. *Proceedings of the ...* <https://hal.archives-ouvertes.fr/hal-01873468>
- Nunes, T., Dorneles, B. V., Lin, P.-J., & Rathgeb-Schnierer, E. (2016). *Teaching and Learning About Whole Numbers in Primary School*. Springer International Publishing. https://doi.org/10.1007/978-3-319-45113-8_1
- Ostad, S. A. (1997). Developmental differences in addition strategies: A comparison of mathematically disabled and mathematically normal children. *British Journal of Educational Psychology*, 67(3), 345–357. <https://doi.org/10.1111/j.2044-8279.1997.tb01249.x>
- Sayers, J., & Andrews, P. (2015). Foundational Number Sense : Summarising the Development of an Analytical framework. In N. V. Konrad Krainer & N. V. Konrad Krainer (Eds.), *CERME 9 - Ninth Congress of the European Society for Research in Mathematics Education, Prague, Czech Republic, February 2015* (pp. 361–367). <https://hal.archives-ouvertes.fr/hal-01286842>
- Sayers, J., Andrews, P., & Björklund Boistrup, L. (2016). The Role of Conceptual Subitising in the Development of Foundational Number Sense. *Mathematics Education in the Early Years*, 371–394. https://doi.org/10.1007/978-3-319-23935-4_21
- Sayers, J., Petersson, J., Rosenqvist, E., & Andrews, P. (2021). Opportunities to learn foundational number sense in three Swedish year one textbooks: implications for the importation of overseas-authored materials. *International Journal of Mathematical Education in Science and Technology*, 52(4), 506–526. <https://doi.org/10.1080/0020739X.2019.1688406>
- Sriraman, B., & Knott, L. (2009). The Mathematics of Estimation: Possibilities for Interdisciplinary Pedagogy and Social Consciousness. *Interchange 2009 40:2, 40(2), 205–223*. <https://doi.org/10.1007/S10780-009-9090-7>
- Sunde, P. B., Petersson, J., Nosrati, M., Rosenqvist, E., & Andrews, P. (2021). Estimation in the Mathematics Curricula of Denmark, Norway and Sweden: Inadequate Conceptualisations of an Essential Competence. *Scandinavian Journal of Educational Research*, 1–16. <https://doi.org/10.1080/00313831.2021.1897881>
- Yin, R. K. (2013). *Case study research: design and methods* (5th ed.). SAGE.