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► **To cite this version:**

Anders Månsson. Categorization reliability of preservice elementary teachers' use of mental computation addition strategies on natural numbers using a written questionnaire. Twelfth Congress of the European Society for Research in Mathematics Education (CERME12), Feb 2022, Bozen-Bolzano, Italy. hal-03746623v2

**HAL Id: hal-03746623**

**<https://hal.science/hal-03746623v2>**

Submitted on 14 Oct 2022

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# **Categorization reliability of preservice elementary teachers' use of mental computation addition strategies on natural numbers using a written questionnaire**

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*Preservice elementary teachers' use of mental computation strategies in addition on two-digit natural numbers is analyzed using a written questionnaire. The inter-coder reliability of three researchers in their categorization of the preservice elementary teachers' explanations as mental computation strategies is investigated. The results show that the coders are to a large degree in agreement, strengthening the reliability of the method.*

*Keywords: Mental computation, addition, questionnaires, interrater reliability.*

## **Introduction and theory**

There exist variations in the definition of mental computation in the research literature, but a general trait among the definitions is calculating without using any equipment (Baranyai et al, 2019; Hartnett, 2007; Heirdsfield, 2011; Maclellan, 2001; Thompson, 1999). There are many advantages of becoming better at mental computation; for example, it improves number sense, it gives a better understanding of the place value system and elementary calculation rules, and it is often involved in everyday use of mathematics (Hope & Sherill, 1987; Maclellan, 2001; Thompson, 2010). Mental computation is part of the elementary curricular content globally, and in several countries there have in recent years been an increased focus on mental computation in teacher education (Hartnett, 2007).

Mental computation strategies are different ways that arithmetic problems are solved mentally (Hartnett, 2007; Threlfall, 2000). To do mental computation efficiently, one needs to be flexible, learn several different strategies, and know when to use which strategy (Rechtsteiner, 2019). Some strategies are more general, and others are more dependent on coincidences in the calculation. Thompson (2009) stresses the importance of teaching and using mental calculation strategies, since the traditional methods are generally not effective enough to improve students' numeracy proficiency. However, even though mental strategies are a desired focus for computational instruction in schools, Hartnett (2007) suggests that teachers have been slow to adopt such changes in their classrooms, and that a possible block to adopting this approach is the teachers' lack of knowledge about possible computation strategies. There is evidence to suggest that pupils are often not directly exposed to mental computation strategies in school, but rather are left to devise for themselves more or less efficient strategies (McIntosh et al, 1995). Many mental computation strategies are possible for pupils to discover on their own, but one cannot presume that all pupils will be able to do so (Murphy, 2004). Some pupils get stuck in unwieldy mental computation strategies, such as doing the standard algorithm mentally, and therefore need to learn more efficient strategies in an organized and systematic way (Heirdsfield & Cooper, 2004).

Mental computation strategies in connection to elementary pupils has been well researched, but less so in connection to preservice elementary teachers (PETs). Since PETs are the next generation of teachers, it is important that they know and master mental computation strategies. They need a strong foundation of the mathematics of mental computation including an ability to use efficient strategies of their own (Heirdsfield & Cooper, 2004; Thompson, 2009). It is therefore important to know the current knowledge and proficiency base of PETs on mental computation. Knowing which strategies PETs are aware of and use, gives valuable information for continuing professional development on how to address mental computation strategies in teacher education and related research, to improve teacher content knowledge on mental computation (Hartnett, 2007; Thompson, 2009; Valenta & Enge, 2013).

To investigate the mental computation strategy use of PETs, researchers need to categorize the PETs' current strategies. In research literature a common method to do this is through interviews, where one can ask the PETs follow up questions, an alternative that is not available in the same way for a written questionnaire. However, a written questionnaire is a more efficient tool than an interview when it comes to gathering larger sets of data in statistical investigations (Mastrothanas et al, 2018).

There is not a single, consensus approach regarding how to categorize mental computation strategies (Whitacre, 2015). In this paper, I focus on strategy categorization using as a theoretical lens a comprehensive list of mental computation strategies on addition on two-digit natural numbers (Table 1). The list is the result of an exhaustive search of the mental computation strategies that occur in the literature. After each strategy is given a reference to where the definition can be found in the literature.

**Table 1: Mental computation strategies for addition on two-digit natural numbers**

<b>Strategy</b>	<b>Definition</b>	<b>Source</b>
<b>1010</b> [ten-ten]	$46 + 23 \rightarrow 40 + 20 = 60 \rightarrow 6 + 3 = 9$ $\rightarrow 60 + 9 = 69$	(Beishuizen, 1993)
<b>10s</b> [1010 stepwise]	$45 + 39 = ((40 + 30) + 5) + 9$ $= (70 + 5) + 9 = 75 + 9 = 84$	(Reys et al, 1995)
<b>A10</b> [adding-on]	$35 + 29 = (35 + 5) + 24 = 40 + 24 = 64$	(Blöte et al, 2000)
<b>AUTO</b> [Automatic calculation]	Retrieve the answer automatically or from memory.	(Lucangeli et al, 2003)
<b>B</b> [balancing]	$89 + 24 = (89 + 1) + (24 - 1) = 90 + 23$ $= 113$	(Heirdsfield & Cooper, 1997)
<b>Counting</b>	$3 + 5$ : 4, 5, 6, 7, <u>8</u>	(McIntosh & Dole, 2005)
<b>Doubles and near doubles</b>	$6 + 7$ : $6 + 6$ is 12, so it is one more.	(McIntosh & Dole, 2005)
<b>N10</b> [stringing]	$46 + 23 \rightarrow 46 + 20 = 66 \rightarrow 66 + 3 = 69$	(Beishuizen, 1993)

<b>N10C</b> [stringing with compensation]	$52 + 79 \rightarrow 52 + 80 = 132 \rightarrow 132 - 1 = 131$	(Baranyai et al, 2019)
<b>Round one or both addends to multiple of ten, then adjust</b>	$79 + 26 \rightarrow 80 + 30 = 110 \rightarrow 110 - 1 - 4 = 105$	(Reys et al, 1995)
<b>Round to multiples of five</b>	$79 + 26 = (75 + 25) + 4 + 1 = 100 + 5 = 105$	(Reys et al, 1995)
<b>SA</b> [standard algorithm done mentally]	Mental image of pen and paper algorithm, placing numbers under each other, as on paper, and carrying out the operation, right to left.	(Heirdsfield, 2001)
<b>u-1010</b> [1010 right to left]	$46 + 23 \rightarrow 6 + 3 = 9 \rightarrow 40 + 20 = 60 \rightarrow 60 + 9 = 69$	(Beishuizen, 1993)
<b>u-N10</b> [N10 right to left]	$46 + 23 \rightarrow 46 + 3 = 49 \rightarrow 49 + 20 = 69$	(Beishuizen, 1993)
<b>Using tens as the unit</b>	$80 + 50 = 8 \text{ tens} + 5 \text{ tens} = 13 \text{ tens} = 130$	(McIntosh & Dole, 2005)

When categorizing there is the question of inter-coder reliability (Lange, 2011), that is if different researchers agree in their categorizations. Evaluating the inter-coder reliability is recommended as good practice in qualitative analysis, although this is a somewhat controversial topic in the qualitative research community, with some arguing that it is an inappropriate or unnecessary step within the goals of qualitative analysis (O'Connor & Joffe, 2020). Team coding is a good inter-coder reliability check (Miles & Huberman, 1994), and a standard way of doing this is using percentage agreement or Cohen's Kappa (Cohen, 1960).

## Method

### Purpose of article and research question

This paper is the first article in a planned series of articles where I utilize written questionnaires to conduct research into PETs' mental computation strategy use, making use of the comprehensive list of strategies in Table 1. In general, research articles categorizing students' and PETs' strategy use only draw on a subset of these strategies and different subsets in each article, thus limiting the degree of comparisons that can be done across the research literature. Before using the comprehensive list of strategies in Table 1 to categorize data from the PET questionnaires, I propose that it is important to first investigate the inter-coder reliability of researchers. (Some researchers may not think this is necessary in qualitative research, so alternatively the investigation can also be seen as out of curiosity or interest to understand how different teacher educator colleagues categorize the same data.) If different researchers' categorizations are in agreement that strengthens the reliability of the method. Therefore, a relevant and interesting research question is:

*Using a written questionnaire and a comprehensive list of mental computation strategies occurring in the literature, how do different researchers differ in their categorizations of preservice elementary teachers' mental computation strategy use in addition on two-digit natural numbers?*

To limit the scope of the investigation only addition on two-digit natural numbers is considered. If the categorizations are in agreement in this particular case, that increases the confidence in using written questionnaires to analyze PETs' mental computation strategy use in general.

### **Research participants**

A written mental computation strategy questionnaire was in 2020 given to two different classes of PETs at a mid-sized university in Norway:

- I. 31 second year PETs with 30 ECTS credits of university mathematics.
- II. 15 third year PETs with a mix of 0, 30 or 60 ECTS credits of university mathematics.

### **Measures**

The PETs' use of mental computation strategies was measured with a written questionnaire consisting of fifteen exercises on addition on two-digit natural numbers. Figure 1 shows the questionnaire instructions and how each exercise was presented to the PETs.

<p><b><u>Important instructions! For every exercise do these steps in order:</u></b></p> <ol style="list-style-type: none"><li><b>1. Calculate the exercise in your head.</b><ul style="list-style-type: none"><li>- The calculation must be done solely in your head. You are <b>not</b> allowed to write anything on the paper.</li><li>- If you do not know the answer after you have been thinking for a while, do not write anything and instead go to the next exercise.</li></ul></li><li><b>2. Write down the answer.</b></li><li><b>3. Explain mathematically how you were thinking.</b></li></ol> <p><b>Exercise 1</b></p> <p>11 + 13 = _____</p> <p>Explanation: _____</p> <p>_____</p>
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**Figure 1: Questionnaire instructions together with the first exercise**

The exercises were constructed by the author so that many different strategies would be used by the PETs:

- 1.** 11 + 13, **2.** 17 + 18, **3.** 62 + 27, **4.** 80 + 50, **5.** 76 + 58, **6.** 44 + 33, **7.** 38 + 76, **8.** 60 + 37, **9.** 47 + 45, **10.** 64 + 46, **11.** 70 + 67, **12.** 97 + 86, **13.** 88 + 88, **14.** 45 + 79, **15.** 99 + 9

The PETs' written explanations were categorized as mental computation strategies by three different researchers (of which one was the author) according to the list of strategy definitions given in Table 1. The explanations could also be categorized as "Other" (O) or as an "Unclear strategy" (US). In

addition to being instructed to use the list of mental computation strategies, the researchers were directed to base their categorization solely on a PET's own written explanation (not speculating on how a PET was "really thinking" when calculating).

The categorization inter-coder reliability was measured by comparing (1) the percentage agreement (that is the proportion of the exercises that the coders agreed on) and (2) the average strategy distribution of the coders. Note that in (1) percentage agreement was chosen instead of Cohen's Kappa (Cohen, 1960), because the difference between them is negligible since there are many categories of strategies and the probability for random agreement when categorizing is thus small.

## Data collection

The questionnaire was administered as part of a normal lecture in two classes (referred to here as Class I and Class II) at the university. The PETs' participation in the questionnaire was voluntary and anonymous. They were not informed beforehand that they would take a questionnaire, so they had no way of preparing for it. There was no time limit to the questionnaire.

The PETs' explanations in Class I were categorized as strategies by the author and another mathematics teacher educator (called instructor A). Both the author and instructor A are associate professors in mathematic didactics. The PETs' explanations in Class II were categorized by the author and another mathematics teacher educator (called instructor B). Instructor B is a university lecturer in mathematic didactics with several years of experience as an elementary school teacher.

## Results

### Class I categorized by author and instructor A

The author and instructor A were in agreement in 89.9 % and disagreement in 4.7 % of the 465 (=  $31 \times 15$ ) exercises. In 5.4 % of the exercises their categorizations were overlapping (but not inclusion in set theory terms). One can conclude that the two persons categorizations are to a high degree in agreement, strengthening the reliability of the method.

Figures 1 & 2 show the averages of the 15 exercises for Class I as categorized by the author (Figure 1) and instructor A (Figure 2), where similar strategies have been grouped. (The strategies were consider separately when considering if categorizations were in agreement. Each strategy in the category Remaining have a small relative frequency ( $\leq 1$  %).)

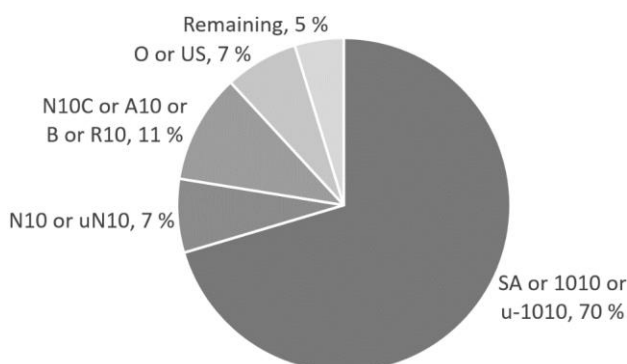


Figure 1: Class I categorized by author

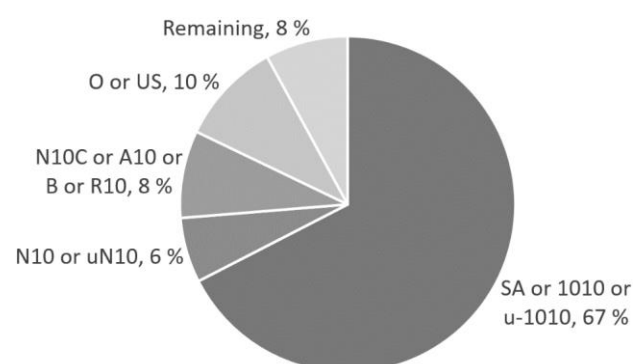


Figure 2: Class I categorized by instructor A

Generally seen the averages in Figures 1 & 2 reflect the relative frequencies in each of the 15 exercises. One exception is exercise 4 (that is  $80 + 50$ ). Most of the PETs have calculated exercise 4 by adding  $8 + 5$  and then attaching a zero. The author has categorized this strategy as “Other” (O) whereas instructor A categorized it as an “Unclear strategy” (US).

### Class II categorized by author and instructor B

The author and instructor B were in agreement in 80.4 % and disagreement in 11.6 % of the 225 (=  $15 \times 15$ ) exercises. In 8,0 % of the exercises the categorizations were overlapping (but not inclusion in set theory terms). Although the author’s agreement with instructor B was somewhat smaller than with instructor A, the overall agreement of two categorizations is good also in this case.

The diagrams in Figures 3 & 4 show the averages of the 15 exercises for Class II as categorized by the author (Figure 3) and instructor B (Figure 4). The grouping of the categorizations is here the same as for Class I.

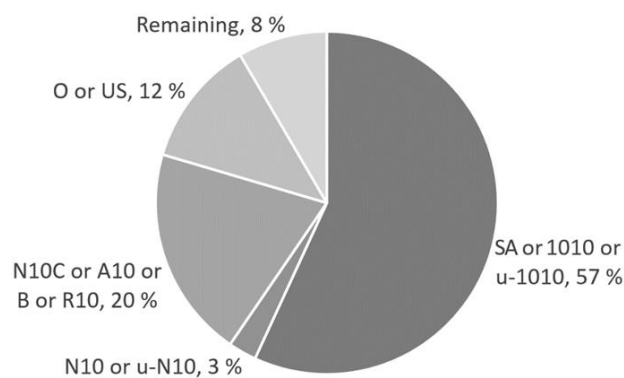


Figure 3: Class II categorized by author

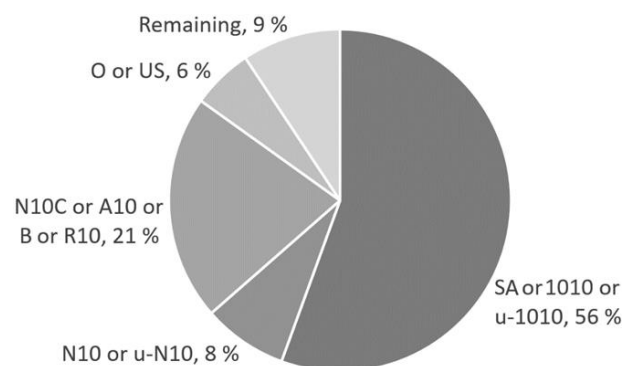


Figure 4: Class II categorized by instructor B

## Discussions and conclusions

Exploring how different researchers categorize preservice elementary teachers’ mental computation strategy use, utilizing the comprehensive list of strategies as a theoretical lens to analyze their work, is an important first step for me before moving into the all-important task of understanding more, in general, about PETs’ strategy use. We have in this paper seen that, using a written questionnaire and a comprehensive list of mental computation strategies occurring in literature, there is good agreement between how three different researchers categorize PETs’ explanations as mental computation strategies. This is a valuable result since it means that written questionnaires and the comprehensive list of strategies in table 1 can be used more reliably to analyze PETs’ use of mental computation strategies.

## Acknowledgement

I would like to thank Kathy Nolan and Elisabeta Eriksen for useful feedback.

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