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Categorization reliability of teacher students' use of mental computation addition strategies on natural numbers using written test

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Teacher students' use of mental computation strategies in addition on two-digit natural numbers is analyzed using a written test. To test the reliability of the categorization of the students' explanations as mental computation strategies the categorization is done by three persons and their categorizations are then compared. The test result show that the categorizations are to a large degree in agreement, strengthening the reliability of the method.

Keywords: Mental computation, strategies, addition, written test.

Introduction and theory

There exist variations in the definition of mental computation in the research literature, but a general trait among the definitions is calculating without using any equipment (Baranyai et al, 2019; Hartnett, 2007; Heirdsfield, 2011; Lemonidis, 2016; Lopez, 2014; Maclellan, 2001; QCA, 1999; Thompson, 1999; Threlfall, 2000). There are many advantages of becoming better at mental computation, for example that it improves number sense, it gives a better understanding of the place value system and elementary calculation rules, and it is often involved in everyday use of mathematics (Hope & Sherill, 1987; Maclellan, 2001; Thompson, 2010). Mental computation is part of the curricular content globally, and in several countries there have in recent years been an increased focus on mental computation in teacher education (Hartnett, 2007; Lemonidis et al, 2014).

Mental computation strategies are different ways that arithmetic problems are solved mentally (Hartnett, 2007; Threlfall, 2000). To do mental computation efficiently, one needs to be flexible, learn several different strategies, and know when to use which strategy (QCA, 1999; Rechtsteiner, 2019). Some strategies are more general, and others are more dependent on coincidences in the calculation. Thompson (2009) stresses the importance of teaching and using mental calculation strategies, since the traditional methods are not effective enough to improve students' numeracy proficiency. However, even though mental strategies are a desired focus for computational instruction in schools, Hartnett (2007) suggests that teachers have been slow to adopt such changes in their classroom, and that a possible block to adopting this approach is the teachers' lack of knowledge about possible computation strategies. There is evidence to suggest that pupils are often not directly exposed to mental computation strategies in school, but are rather left to devise for themselves more or less efficient strategies (McIntosh et al, 1995). Many mental computation strategies are possible for pupils to discover on their own, but one cannot presume that all pupils will be able to do so (Murphy, 2004). Some pupils get stuck in unwieldy mental computation strategies, such as doing the standard algorithm mentally, and therefore need to learn more efficient strategies in an organized and systematic way (Heirdsfield & Cooper, 2004).

Mental computation strategies in connection to pupils has been well researched, but less so in connection to preservice teachers. Since teacher students are the next generation of teachers, it is important that they know and master mental computation strategies. They need a strong foundation of the mathematics of mental computation and the ability to use this important calculation method and use efficient strategies of their own (Heirdsfield & Cooper, 2004; Thompson, 2009). It is therefore important to know the current knowledge and proficiency base of teacher students on mental computation. Knowing which strategies teacher students are aware of and use, gives valuable information for continuing professional development and how to address mental computation strategies in teacher education and related research, to improve teacher content knowledge on mental computation (Hartnett, 2007; Thompson, 2009; Valenta & Enge, 2013).

Method

Purpose of article and research question

To investigate the mental computation strategy use of teacher students one needs to categorize their approaches and strategy accounting. When categorizing student explanations as mental computation strategies there is the question of reliability (Bryman, 2016), that is if other researchers agree with the categorization. When using interviews one can ask students follow up questions, but this alternative is not available in the same way for a written test. Since a written test is a more practical tool than an interview when it comes to gather larger set of data in statistical investigations, it is of interest to test the reliability of strategy categorizations using a written test. To test the reliability I let university mathematics educators and researchers categorize the data. If their categorizations are in agreement that strengthens the reliability of the method. Therefore a relevant and interesting research question, which to my knowledge has not before been addressed in the research literature, is the following:

Do different university mathematics educators and researchers agree in their categorizations of teacher students' mental computation strategy use in addition on two-digit natural numbers using a written test?

To limit the scope of the investigation only addition on two-digit natural numbers is considered. If the categorizations are in agreement in this particular case, that increases the confidence in using written tests to analyze teacher students' mental computation strategy use in general.

Research participants

A written mental computation strategy test was in 2020 given to two different classes of teacher students at the University college of Southeast Norway:

- I. 31 second year students with 30 ECTS credits of university mathematics.
- II. 15 third year students with a mix of 0, 30 or 60 ECTS credits of university mathematics.

The student explanations in class I were categorized as strategies by the author and another person (called A). Both the author and the other person A are associate professors in mathematics and mathematic didactics. The student explanations in class II were categorized by the author and another person (called B). Person B is a university lector in mathematics and mathematic didactics with several years of experience as an elementary school teacher.

Measures

The students' use of mental computation strategies was measured with a written test consisting of fifteen exercises on addition on two-digit natural numbers. The exercises were constructed so that many different strategies would be used by the students:

1. $11 + 13$, **2.** $17 + 18$, **3.** $62 + 27$, **4.** $80 + 50$, **5.** $76 + 58$, **6.** $44 + 33$, **7.** $38 + 76$, **8.** $60 + 37$, **9.** $47 + 45$, **10.** $64 + 46$, **11.** $70 + 67$, **12.** $97 + 86$, **13.** $88 + 88$, **14.** $45 + 79$, **15.** $99 + 9$

The student written explanations were categorized as mental computation strategies according to the list of strategy definitions given below. This is an operational approach, basing the categorization solely on the student's own written explanation and not speculating on how the student was "really thinking" when calculating. Person A and B were given a minimum of instructions on how to categorize other than that they should use the list below. It is an exhaustive list of the mental computation strategies on addition on two-digit natural numbers that occur in the research literature. The categorization reliability was measured by comparing (1) the number of exercises the different persons agreed on the categorization and (2) the average strategy distribution of the different persons.

Mental computation strategies for addition on two-digit natural numbers

SA [standard algorithm done mentally]

Mental image of pen and paper algorithm, placing numbers under each other, as on paper, and carrying out the operation, right to left.

$$\begin{array}{r} 1 \\ 67 \\ +58 \\ \hline 125 \end{array}$$

1010 [splitting]

Add the tens, then add the ones, then add the sums.

$$67 + 58 \rightarrow 60 + 50 = 110 \rightarrow 7 + 8 = 15 \rightarrow 110 + 15 = 125$$

u-1010 [1010 units first]

Add the ones, then add the tens (possibly with an exchange from the ones), then add the sums.

$$67 + 58 \rightarrow 7 + 8 = 15 \rightarrow 60 + 50 = 110 \rightarrow 110 + 15 = 125$$

10s [1010 stepwise]

First add the tens, then add the ones one by one.

$$67 + 58 = ((60 + 50) + 7) + 8 = (110 + 7) + 8 = 117 + 8 = 125$$

u-10s [10s units first]

First add the ones, then add the tens one by one.

$$67 + 58 = ((7 + 8) + 60) + 50 = (15 + 60) + 50 = 75 + 50 = 125$$

N10 [stringing]

Split one of the terms into tens and ones. Then add to the other term first the tens, then the ones.

$$67 + 58 = 67 + 50 + 8 = 117 + 8 = 125$$

u-N10 [N10 units first]

Split one of the terms into tens and ones. Then add to the other term first the ones, then the tens.

$$67 + 58 = 67 + 8 + 50 = 75 + 50 = 125$$

N10C [N10 with compensation]

Same as N10 but instead of splitting e.g. $58 = 50 + 8$ one instead “splits” it as $58 = 60 - 2$.

$$67 + 58 = 67 + 60 - 2 = 127 - 2 = 125$$

A10 [bridging through ten]

Split one of the terms into two parts, such that when one of the parts is added to the other term the sum becomes a multiple of ten. Then add the other part to the sum.

$$67 + 58 = 67 + 3 + 55 = 70 + 55 = 125$$

B [balancing]

Add/Subtract a number from one of the terms so it becomes a multiple of ten, and subtract/add the same number to the other term. Then add the sum and the difference.

$$67 + 58 = (67 + 3) + (58 - 3) = 70 + 55 = 125$$

Round to multiples of ten [R10]

Round up or down at least one of the terms to a multiple of ten. Compensate by adding or subtracting with that which was rounded up or down. If only one term is rounded up or down to the nearest multiple of ten, it is a N10C.

$$67 + 58 = (70 + 60) - 3 - 2 = 130 - 5 = 125$$

Round to multiples of five [R5]

Round terms up or down so that both terms are a multiple of five (not both a multiple of ten). Compensate by adding or subtracting with that which was rounded up or down.

$$67 + 54 = (65 + 55) + 2 - 1 = 120 + 1 = 121$$

Doubles and near doubles [DND]

Calculate the sum as a double (i.e. multiplication by a factor of two), possibly by adjusting the terms up or down so that they become equal.

$$13 + 13 = 2 \cdot 13 = 26$$

Counting [C]

$$4 + 5: \quad 5, 6, 7, 8, \underline{9}$$

Memory [M]

Direct retrieval of result from memory.

Other [O]

Using a strategy not included in the list.

Combination of two or more strategies [$X \cap Y$, $X \cap Y \cap Z$,...]:

Using a combination of for example 1010 and DND write “ $1010 \cap DND$ ”; and if using a combination of C and R5 and M write “ $C \cap R5 \cap M$ ”.

Uncertain [$X \cup Y$, $X \cup Y \cup Z$,...]

About equally uncertain which of two or more strategies that has been used. If for example about equally uncertain if it is 1010 or SA, write “ $1010 \cup SA$ ”; and if about equally uncertain if it is 1010 or SA or DND write “ $1010 \cup SA \cup DND$ ”.

Unclear [US]

Unclear which strategy has been used, or no explanation given.

Wrong answer [WA]

Wrong answer or no answer given.

Data collection

The students did the tests anonymously as part of a normal lecture at the university. All students attending the lectures participated in the tests. The students were not informed beforehand that they would take a test on mental computation, so they had no way of preparing for the test. The students had not been introduced to mental computation strategies by the author before the tests. There was no time limit to the test. The students did not use any equipment other than pen and paper.

Results

Class I categorized by author and person A

The diagrams in figures 1-2 show the averages of the 15 exercises for class I as categorized by the author (figure 1) and person A (figure 2). (The student success rate in the exercises was more than 99 %.) For practical and theoretical reasons I have chosen to group similar strategies. Other groupings are of course possible, but these groups are natural both theoretically and based on the data.

Using the categorization groups in figure 1 and 2 the author and person A were in agreement in 89.9 % and disagreement in 4.7 % of the 465 (= 31 × 15) exercises. In 5.8 % of the exercises their categorizations were overlapping (but not inclusion in set theory terms). [The strategies in the category “Remaining” were consider separately when considering if categorizations were in agreement. Each strategy in the category Remaining have a small relative frequency (≤ 1 %).]

One can conclude that the two persons categorizations are to a high degree in agreement, strengthening the reliability of the method.

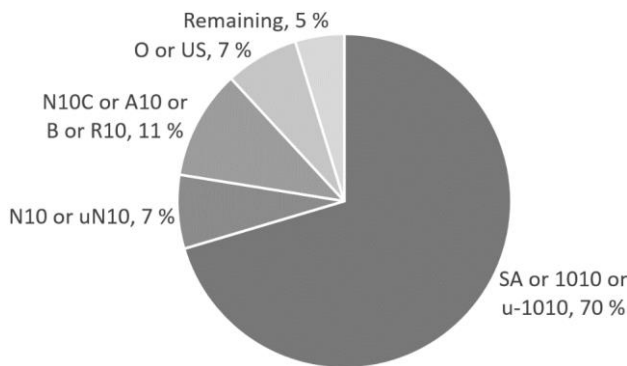


Figure 1: Class I categorized by author

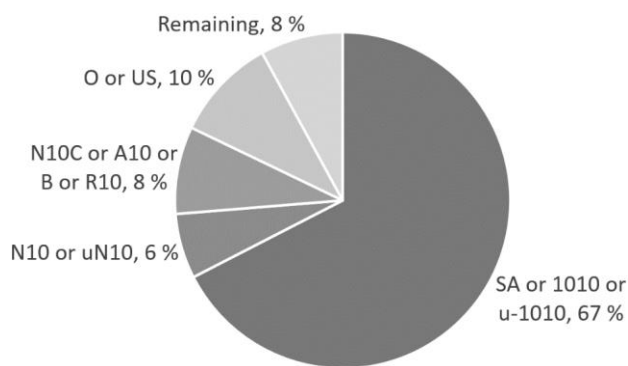


Figure 2: Class I categorized by person A

Generally seen the averages in figures 1 and 2 reflect the relative frequencies in each of the 15 exercises. One exception is exercise 4 (that is $80 + 50$). Most of the students have calculated exercise 4 by adding $8 + 5$ and then attaching a zero. The author has categorized this strategy as “Other” (O) whereas person A categorized it as an “Unclear strategy” (US).

Class II categorized by author and person B

The grouping of the categorizations is here the same as for class I. For these categorization groups the author and person B were in agreement in 80.4 % and disagreement in 11.6 % of the 225 (= 15×15) exercises. In 8,0 % of the exercises the categorizations were overlapping (but not inclusion in set theory terms). Although the authors agreement with person B was somewhat smaller than with person A, the overall agreement of two categorizations is good also in this case. The diagrams in figures 3-4 show the averages of the 15 exercises for class II as categorized by the author (figure 3) and person B (figure 4). (The success rate was more than 99 %.)

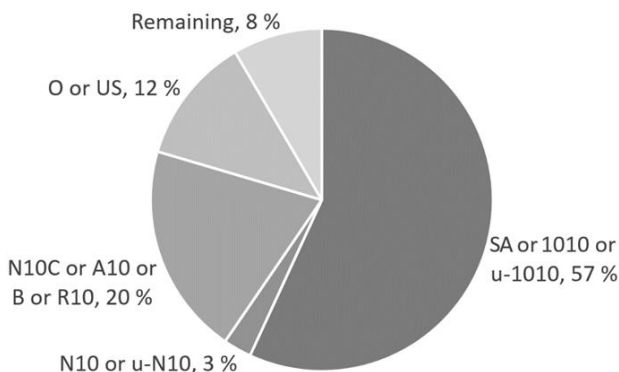


Figure 3: Class II categorized by author

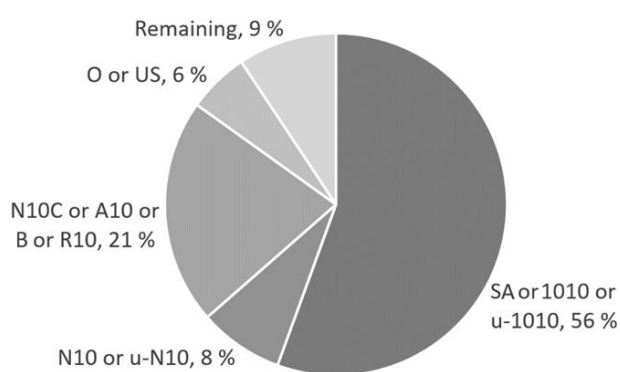


Figure 4: Class II categorized by person B

Discussions and conclusions

We have in this article seen that there is good agreement between how three different mathematicians categorize the teacher students’ explanations as mental computation strategies. This is an important result since it means that written tests can be used more reliably to analyze teacher students’ use of mental computation strategies. Written tests are better suited than interviews for collecting large amount of data and more data means better statistics.

When constructing a written test on mental computation strategies one should keep in mind that the test results can be highly dependent on the exact form of the exercises in the test. Therefore it is important to use identical tests or a tests with several different types of exercises. Using too many or too detailed strategies can also be a problem when categorizing. Grouping similar strategies that are difficult to differentiate principally and practically is a good idea.

For both classes tested we have seen that the most commonly used mental computation strategy used by the teacher students is “1010 \cup u-10101 \cup SA”. Two other common types of strategies are “N10 \cup u-N10” and “N10C \cup A10 \cup B \cup R10”. Then there are also student explanations that are more difficult to categorize as strategies, because the student explanations are unclear or because the explanations involve characteristics of several different strategies. These problems could for example be addressed by combining written test with interviews and follow-up questions, perhaps also letting students themselves categorize their own explanations.

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