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# Counting strategies in a proficient grade 1 student: the case of Petra

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*A case study design was used to explore a foundational number-sense (FoNS) proficient student's strategy use in counting. The student, referred to as Petra, participated in a task-based semi-structured systematic counting interview. The interview was analysed qualitatively exploring part-whole reasoning in counting strategies in interrelation with FoNS, which was measured using a digital assessment. The interrelations were discussed as reflecting strands that Kilpatrick et al. (2001) found important for mathematical proficiency. Petra showed the ability to use combinations of one-to-one and relational strategies and high levels of adaptive reasoning and productive disposition. Petra also showed that a number-sense proficient student may be flexible in counting in the 0-10 range but not necessarily in the 10-20 range. Results are discussed as part-whole reasoning on the mental number line.*

*Keywords: Part-whole reasoning, mental number line, counting strategies, number sense, grade 1.*

## Introduction

Counting is a central to number sense, which is the ability to work flexibly with numbers and quantities predicting students' mathematical proficiency (Andrews & Sayers, 2015). Flexibility is essential in definitions of number sense. Developing mental representations of numbers and strategies represented on a mental number line enables flexible mental calculations, part-whole reasoning and relational strategies (Dehaene, 2011; Hunting, 2003; Rathgeb-Schnierer & Green, 2019).

Variation in counting and counting strategies is traditionally described in terms of Gelman and Gallistel's (1986) three how-to-count principles: the one-to-one principle of correspondence between numerals and items, the stable-order principle of numerals in any count, and the cardinal principle of the final counted item's numeral representing the number of items in the set. Students with mathematical difficulties are found to exhibit immature, inflexible and inefficient counting strategies and having problems shifting between strategies compared to proficient students (Gelman & Gallistel, 1986). Alternatively, or complementarily, variation in counting strategies can be considered as included in arithmetical problems as part-whole structures supporting relational strategies and part-whole reasoning (e.g. Hunting, 2003).

Mental flexibility is critical to the efficiency of mental strategies (Rathgeb-Schnierer & Green, 2019). Carpenter and his colleagues (e.g. Carpenter et al., 1996) has emphasised understanding students' thinking as a knowledge base for cognitively guided instruction promoting mathematical development. However, what mental processes contribute to mental flexibility? Kilpatrick et al. (2001) claimed that five interwoven and interdependent strands represent the flexible interrelations needed for mathematical proficiency. Extended knowledge of the continuum of number-sense proficiency and flexibility in counting is needed to develop characteristics of various proficiency, and improve the discovery and teaching of typical and atypical variations to support mathematical development in all students. The paper discuss flexibility as part-whole reasoning by exploring a

grade 1 number-sense proficient student's systematic counting strategies considered in interrelation with foundational number sense (FoNS) concepts. The paper interprets interrelations to promote counting flexibility and raises the following research questions:

RQI: What strategies does a number-sense proficient grade 1 student use in counting?

RQII: How may strategy use be interrelated with part-whole reasoning on the mental number line?

The paper reports on data from a Ph.D. project studying 75 Norwegian grade 1 students' variations in counting and patterning strategies in relation to FoNS and verbal and nonverbal reasoning.

## **Theoretical framework**

Andrews and Sayers' (2015) described systematic counting as being able to count forwards and backwards between 0 and 20, understanding ordinality and being able to start counting from an arbitrary starting point between 0 and 20. Systematic counting is integrated into Andrews and Sayers' (2015) FoNS definition and interrelates with the seven other components: number identification, number and quantity, quantity discrimination, representing numbers, estimation, arithmetic competence, and number patterns (Andrews & Sayers, 2015). Counting strategies develop from count-all one-to-one correspondence strategies to the more efficient retrieval strategies, such as count-on, which partly depend on one-to-one strategies and relational strategies. Finally, interrelations between counting and arithmetic emerge in the most abstract derived fact relational strategies based on commutativity and the inverse principle. Such mental representations enable part-whole reasoning about the relations between parts constituting the whole and decomposition of the whole into parts of the mental number line and efficient relational strategies (Dowker, 2014; Hunting, 2003).

Flexible counting strategies, conceptual understanding of numbers and the ability to count systematically have been found to depend on estimation abilities and the ability to compare small quantities without counting, also called subitising (Andrews & Sayers, 2015; Dehaene, 2011). Derived fact strategies might be a combination of counting and subitising (Dowker, 2014).

Understanding structures of patterns develops generalisations and promotes part-whole reasoning, which is critical for commutativity, number sense, algebra, and counting (Hunting, 2003). Counting repeating patterns (e.g. ABABAB,  $\square\Delta\Delta\square\Delta\Delta\square\Delta\Delta$ ) and growing patterns (e.g. 1 3 5,  $\square\square\square\square\square$ , ABAABAAAB) enable the use of both one-to-one building-up strategies and more advanced unit factor and scalar relational strategies, which consider a repeating unit or multiplicative relations between quantities in two or more measure spaces, but with different advanced levels.

Concrete counting operations with counting blocks or other manipulatives develop flexible and efficient mental counting and calculations (Rathgeb-Schnierer & Green, 2019). The mental number line enables the use of part-whole reasoning and is the cognitive system underlying number sense (Dehaene, 2011) serving mental representations of the ordinality, cardinality and magnitudes of numbers as well as the relations between numbers, thus enabling flexible operation with numbers.

Kilpatrick et al. (2001) holistically described flexibility as the following five strands that make up mathematical proficiency: (1) conceptual understanding interrelations between mathematical ideas

and their mental representations, including knowledge of why and when mathematical ideas are important and useful, (2) strategic competence to formulate, represent and solve mathematical problems, (3) procedural fluency and knowing when procedures are appropriate, which is shown as flexible, accurate, and efficient procedures, (4) adaptive reasoning as the capacity to think logically about interrelations between concepts and situations, and (5) productive disposition involving the ability to see mathematics as useful and understanding that steady effort will pay off in combination with a view of oneself as an effective learner and doer of mathematics (Kilpatrick et al., 2001). The strands include students' attitudes and metacognition as it concerns knowledge and reflection on one's reasoning and problem-solving abilities.

## **Methods**

A task-based semi-structured interview was designed to explore Petra's strategies for systematic counting: forwards and backwards in counting on 1s, 2s and 3s in the 0-20 range. Petra's counting and counting strategies were explored using a case study design (Yin, 2014).

### **Recruitment, sample, and case**

Following informed parental consent, Petra, aged 5 years and 11 months, was purposefully chosen from the larger study sample as the student with the highest score on the digital number sense test of 75 grade 1 students. Petra is thus referred to as a number-sense proficient student.

### **Assessments and procedures**

A task-based semi-structured counting interview was developed based on Andrews and Sayers' (2015) definition of systematic counting. A toy frog named Mr Minus asked ten questions to facilitate observation of Petra's counting strategies. Counting blocks were made available. If needed, Mr Minus modelled counting by jumping on a number line made of counting blocks. No time limits were given, the interview was video-recorded and lasted approximately 10 minutes. The author was Mr Minus's voice and initiated a dialogue by asking the following questions: (1) 'What number do we start counting at?' (2) 'How far is it possible to count?' (3) 'Is it possible to count backwards as well?' (4) 'Do we need to start counting at 1? Is it possible to count forwards from 7?' (5) 'Is it possible to count backwards from 9?' (6) 'Can we count from 20 to 0 too?' (7) 'I have heard some adults count in a weird way. They counted 2-4-6... Have you heard such counting? I am wondering how to continue to count if I am to continue to count the same way. Can you help me?' (8) 'Is it also possible to count backwards this way?' (9) 'I have heard some adults count in a weird way. They counted 1-3-5... Have you heard such counting? I am wondering how to continue to count if I am to continue to count the same way. Can you help me?' and (10) 'Is it also possible to count backwards this way?'. Mr Minus (test-administrator) gave verbal and nonverbal prompts if needed.

A digital FoNS assessment (Saksvik-Raanes & Solstad, in press) measured all FoNS components except for representing numbers, following Andrews and Sayers' (2015) number sense definition. Subitising was included as an eight component: (1) number identification, (2) systematic counting, (3) number and quantity, (4) quantity discrimination, (5) estimation, (6) arithmetic competence, (7) subitising, and (8) number patterns. Subitising tasks were timed.

In a 30-minute session, Petra dragged, dropped and organised objects on the screen or tapped the appropriate multiple-choice item to solve tasks. Figure 1 provides an example of each of the different task designs and the verbal instructions given in the systematic counting component tasks. Petra scored 62 out of 69 possible points.

Figure 1. The tree task design of the systematic counting component



A) Put the numbers in order, B) Place the star on the third star in the line, and C) Place the correct number into the box.

### Analytical procedures

Petra's counting response and strategies in tasks 1 to 6 (see description in the "assessments and procedures" section) were coded and qualitatively analysed according to Andrews and Sayers' (2015) description of systematic counting and Gelman and Gallistel's (1986) how-to-count principles using NVivo. In addition, Petra's strategy use in tasks 7 to 10 was coded as one-to-one building-up/correspondence strategies, and as part-whole relational strategies depending on multiplicative relations between two or three units in counting. Number sense was measured using a digital assessment based on Andrews and Sayers' (2015) eight components of FoNS on an individual tablet. Petra's responses to the tasks in the digital FoNS assessment were scored dichotomously.

Petra's strategy use in relation to her FoNS was discussed to reflect part-whole reasoning on her mental number line, using Kilpatrick et al.'s (2001) strands of mathematical proficiency as guidelines.

### Results and analyses

#### Semi-structured systematic counting interview

- Mr Minus: What is the smallest number we can start to count from?  
 Petra: 1.  
 Mr Minus: How far is it possible to count?  
 Petra: Am I to count to hundred? 1-2-3-4-5-6-7-8-9-10-11-12-13-14-15-16-17-(...)-100.

At every -9 from 29 to 99, she prolonged pronunciation of the /i:/ vowel before the 10 transitions.

- Petra: I also manage to count to hundred like this: 10-20-30-40-50-60-70-80-90-100. Now I counted only the tens.

Her thumb represented 10 and her index finger represented 20. No fingers were used for subsequent counting.

- Mr Minus: Is it possible to start counting backwards?  
 Petra: I manage to count from hundred to eighty, I think. 99-98-97-96-95-94-93-92-91-80-89-88-87-86-85-84-83-82-81.

She prolonged the pronunciation of the vowels in 95 and 91 and quickly said 80 without self-correction.

- Mr Minus: What comes next, after 81?  
 Petra: Seventy!

Petra finger-counted on her left hand and correspondently whispered every 10th number word from 10 in the additive direction for each finger, which was the opposite direction from the subtracting counting she had started using. She said 70 loudly and stopped counting.

Mr Minus: Can we count from twenty to zero too?

Petra: I do not think I know much about that! Sometimes I need to do like this.

She illustrated finger-counting corresponding to lip movements pronouncing 1-2-3-4-5.

Mr Minus: That is okay if you need to.

Petra: 20-90-no-19-17-15-14-13-12-11-10-9-8-7-6-5-4-3-2-1-0.

Petra self-corrected 90 to 19. The three fist phonemes of Norwegian number words for 90 and 19 are identical and perhaps phonologically distracted her. After she said 19, she started finger-counting on her left thumb and whispered 1 before she said 10 while still holding up only her thumb, representing the 10<sup>th</sup> addend. As such, she combined a retrieval relational and a one-to-one strategy, as adjacent fingers one-by-one represented the consecutive addends while she whispered 11-12-13-14-15-16 before answering 17 loudly. She counted forwards from 10 and stopped when she saw there was one less finger on her hand than the number of fingers she remembered seeing on her hand from the preceding answer. Then she repeated her strategy of the thumb representing 10. Because she whispered 11 without showing the index fingers before she whispered 11, she answered 15. She used the same strategy to find the subsequent answers, including 11. Petra stopped using her fingers when she continued to count down from 10 to 0.

Mr Minus: High five! You made it! Is it possible to counting forwards from seven to twenty?

Petra: I think so. That is almost what I already counted. 8-9-10-11-12-13-14-15-16-17-18-19-20. That was easy, you only start at eight.

Mr Minus: Cool! Is it possible to start counting from 14 as well?

Petra: 14-15-16-17-18-19-20.

Mr Minus: Is it so that we can count backwards from an arbitrary point as well, say from 9?

Petra: 9-8-7-6-5-4-3-2-1-0.

Mr Minus: I have heard some adults count in a weird way. They counted 2-4-6... Have you heard that? I wonder how to count on from 6 if I were to continue counting the same way. Do you know?

Petra: Yes, 1-2-3-4. And that's only two! The next numbers are 8-10-12. And you know what? I also manage from three to nine.

Fingers one-to-one corresponded to the number words. Then she grouped two and two fingers on her left hand. She counted 3-6-9 while she showed one and one finger.

Mr Minus: WOW! What is the next number when counting like that?

Petra: 12-15-18-20-23-26-29.

Petra one-to-one correspondently finger-counted and whispered numbers between 9 and 12. She continued to whisper-count with the same three fingers and said the number on the third finger loudly. Incorrect one-to-one correspondence made her answer 20 without self-correction.

Mr Minus: You teach me lots of things! Is it possible to count backwards in the same way as you counted 2-4-6 forwards?

Petra: Yes, I can try that. 10-8-6-4-2-0.

Mr Minus: Okay. I have heard some adults count in a weird way. They counted 1-3-5... Have you heard that? I wonder how to count on from 5 if I am to continue counting the same way. Do you know?

Petra: 1-... Am I to do addition?

Mr Minus: I do not know. I just heard someone count that way. 1-3-5, and I do not understand what to say next if I am to continue in the same pattern.  
Petra: 1-3-5...? 6?  
Mr Minus: Maybe? I want to try something! I want to jump like they counted. 1-3-5... Now I jump once more. What am I to say when I land here? And here?

The interviewer lined up counting blocks, making a number line from 1-10. The frog jumped on every other block and said the number names corresponding to the number line position on which he landed on. The frog jumped on every other block and said the number names corresponding to the number line position on which he landed.

Petra: 7-10

This was Petra' answer for the two positions Mr Minus jumped on after 5.

Mr Minus: How did you do that?  
Petra: I counted inside myself. I counted 2-4-6-8-10.

She correspondently pointed at the second, fourth, sixth, eight and tenth block.

Mr Minus: Okay. But I said 1-3-5 and you said seven was the next number. How did you know that?  
Petra: I added two more.  
Mr Minus: Okay, that was what happened! Is it possible to jump back and count in this way as well?  
Petra: Do you mean 7-5-3?

She tapped her fingers on the table in the opposite downwards or subtractive direction.

Mr Minus: Yes! And now I think you showed me that it is possible!  
Petra: I know how to make a rocket with my fingers! I do like this. Do you manage?

She ended the dialogue, making a rocket with both hands and laughed.

### **Foundational number sense (FoNS)**

Petra mastered all number sense components in the digital assessment, expect for estimation and arithmetic. For numbers between 10 and 20, she showed difficulties estimating a number's position on the number line and doing arithmetical operations.

### **One-to-one and relational strategies**

For tasks 1 to 6, Petra successfully used the required one-to-one strategy with correspondence between the number name and the counting procedure. Tasks 7 to 10 served as opportunities for part-whole reasoning about multiplicative relations between numbers and processes on the mental number line. Therefore, the tasks enabled the use of both one-to- one and relational strategies. As the results showed, Petra successfully and unsuccessfully used a combination of counting-all, counting-on and retrieval strategies. That is, both one-to-one and relational strategies. Success seemed to depend on the number range in which she counted. Petra did not use the one-to-one count-all strategy alone. Despite various success, she considered the appropriateness of more advanced relational strategies and showed emergent part-whole reasoning (Dowker, 2014). She used these strategies in combination with one-to-one backup strategies for support. The digital assessment also supported the assumption that she used relational fact retrieval strategies for numbers between 1 and 10 in forwards counting but used a combination of one-to-one and relational strategies in backwards counting above 10.

## **Discussion**

### **Indications of conceptual understanding, procedural fluency, and strategic competence**

Petra met some of Andrews and Sayers' (2015) systematic counting criteria. She counted correctly forwards to 100 and met their criteria of forwards counting from 0-20, but she did not meet the criteria of backwards counting from 20 to 0. She mastered forwards and backwards counting in patterns of 2s on even numbers in the 0–20 range but mastered only backwards counting on odd numbers in the 10–0 range. She successfully counted backwards from an arbitrary starting point between 10 and 0, but not between 20 and 10. There is compliance between Petra's counting, estimation, and arithmetic competence. Estimation is important for mentally representing the number line (Dehaene, 2011). Petra met the one-to-one principle of Gelman and Gallistel's (1986) in a count-all backup strategy but not in a combined backup and retrieval strategy counting-on from 10. The stable order principle was met in forwards counting in 1s and in patterns of 2s and 3s. She was challenged in backwards counting by 1s in the 20-10 range, omitting two numbers but not in the 10-0 range. The cardinal principle was confirmed in the number and quantity component of the digital FoNS assessment. Petra's strong subitising abilities may have played a role when her thumb represented 10 and when she reasoned in multiplicative relations, counted on 2s and explored counting on 3s.

Self-correction and efficient strategy use were implemented as procedural fluency. Self-correction occurred inconsistently. It is unknown whether incorrect counting from 99-80 resulted from number name or ordinality problems and a not fully established mental number line for the eighth or ninth 10 range, something her strategy use in the 20-10 range supports. If so, subtracting perhaps distracted her. These findings underline the interrelations between conceptual understanding and procedural fluency. Petra used efficient strategies, that is, when she counted to 100 by counting by 10s and when she counted in 2s and tried to count in 3s. Additionally, she showed a high level of strategic competence as she represented and solved problems verbally, explained her mental strategies and utilised manipulatives. The findings show the mutually supportive interrelations between strategic competence, conceptual understanding and procedural fluency (Kilpatrick et al., 2001).

### **Indications of metacognition through adaptive reasoning and productive disposition**

Petra searched for similarities and differences or patterns in the counting and gave generalised explanations to show accountability of her strategy use in other situations. For example, she argued for transferring the use of the strategy, which made her master counting from 1 to 20 when she was asked to count-on from 7 to 20, and that she recognised relational patterns and explained she had to count by 2s to correctly continue the number sequence 2-4-6. Petra initiated, argued, explored, and demonstrated different ways and patterns of counting. For example, she counted by 10s to 100 and she tried to count by 3s after counting by 2s. Her interest in exploring unknown counting overcame her insecurity of succeeding or meeting an expectation of mastery without finger-counting.

Kilpatrick et al. (2001) claimed that knowledge about one's own thinking and ability to monitor one's own understanding and problem-solving contribute to strategic competence and adaptive reasoning, and is known as metacognition. Petra showed indications of a high metacognitive level. One is that she explained how she was strategically going to approach new tasks, which, according to Kilpatrick et al. (2001), reflected her motivation or productive disposition, which is important for learning.



## Counting strategies as part-whole reasoning on the mental number line?

Exploring Petra's counting raises the question of whether and how mental number lines representing the different 10s interrelate and how to develop an understanding of the pattern regularity linking them. Petra seemed to have mental representations for the tens' group structure and the positioning system of -1 to -9 range. Still, ten-transitioning was challenging in backwards counting and in the 10-20 range. The phonological structure perhaps distracted her. Exploring and considering strategy use in an integrated approach of the area model part-whole reasoning and the linear model the mental number line and, as such, combined the mental map of numerosity and the mental number line needs further examination (Dehaene, 2011).

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