



**HAL**  
open science

# How to map larger parts of the mathematics curriculum? The case of primary school arithmetic in Japan

Mayu Aoki, Carl Winsløw

## ► To cite this version:

Mayu Aoki, Carl Winsløw. How to map larger parts of the mathematics curriculum? The case of primary school arithmetic in Japan. Twelfth Congress of the European Society for Research in Mathematics Education (CERME12), Feb 2022, Bozen-Bolzano, Italy. hal-03746496v2

**HAL Id: hal-03746496**

**<https://hal.science/hal-03746496v2>**

Submitted on 17 Oct 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# How to map larger parts of the mathematics curriculum? The case of primary school arithmetic in Japan

Mayu Aoki<sup>1</sup> and Carl Winsløw<sup>2</sup>

<sup>1</sup>University of Copenhagen, Denmark; [mayu13ak@gmail.com](mailto:mayu13ak@gmail.com)

<sup>2</sup>University of Copenhagen, Denmark; [winslow@ind.ku.dk](mailto:winslow@ind.ku.dk)

*We present a methodological discussion of how to elaborate different scale epistemological reference models and in particular, present a large-scale model for the domain of arithmetic in Japanese primary school, based on the national program and selected textbooks. Our theoretical base is the Anthropological Theory of Didactic (ATD), specifically the notion of praxeology and the levels of didactic codetermination. We also briefly discuss our motivation for creating such models, namely to analyse how students experience shifts between mathematics in different school systems.*

*Keywords: Japanese arithmetic curriculum, An epistemological reference model, The Anthropological Theory of Didactic*

## Introduction

When we observe mathematics teaching in a classroom, we observe teachers and students interacting on tasks like “Let us think ways of calculating  $12+3$ ” or “Let us calculate  $29 \times 3$ ” etc., normally introduced by the teacher; we may want to understand how the activity promotes various types of learning. Other questions may be difficult to answer from what we observe in the class, like: where do the specific mathematical tasks come from? Why do the students encounter difficulties? etc. In my doctoral research project, I am interested in pupils who attend (or attended) mathematics teaching in different school systems (Japanese, Swedish and Danish). For these and many other questions in educational research, we need to include information on conditions and constraints in a wider context, even if we just want to analyse the phenomena observed within certain classroom situations.

A metaphor may be useful here. The world map gives us a precise idea of the location of the countries we want to look for. For instance, we can see that Germany is located on the European continent and Kenya on the African continent. With a more detailed map, we can find out more details (e.g., what cities are there in Germany?). In other words, we are able to look up and compare since we have a map. Even when preparing a holiday in a small beach town, we need to know how it is situated in relation to other locations like airports and so on.

In the same way, when analysing a classroom episode on the teaching of addition of fractions in grade 3, to understand the actions of teachers and students, a wider “map” of the knowledge at stake in the subject (or even in the school institution) may be needed. The main point of the present paper is to show how the anthropological theory of the didactic (ATD) can be used to define different scales of such a “school knowledge maps” (also called epistemological reference models), and to develop a methodology for the specific purpose of creating “large scale” models that shows how a mathematical domain is structured over several school years within a specific school institution. Concretely, we present an epistemological reference model for the domain of arithmetic in Japanese primary school, along with a methodological discussion of how that model was produced, based on certain official

documents along with textbook material. It is also a main point that the ATD allows us to connect such large-scale models directly with more detailed models of specific parts of the domain which are more commonly used in ATD analysis of classroom episodes.

### **Theoretical Framework and research question**

Our study adopts the anthropological theory of the didactics (ATD) proposed by Yves Chevallard from the early 1980s (Bosch & Gascòn, 2006), more precisely, the notion of praxeology and the levels of didactic codetermination. According to ATD, any human knowledge and practice can be described by using the notion of praxeology. Praxeologies are made of a practical block: a type of task (T) and a technique ( $\tau$ ) and a knowledge block: a technology ( $\theta$ ) and a theory ( $\Theta$ ). In addition, Chevallard proposed the levels of didactic codetermination for researchers to identify conditions and constraints at different levels. The scale of levels of didactic codetermination are constituted of nine different levels: 1. *Subject* 2. *Theme* 3. *Sector* 4. *Domain* 5. *Discipline* 6. *Pedagogy* 7. *School* 8. *Society* 9. *Civilisation* (Bosch & Gascòn, 2006). Concretely, for instance, the quadratic formula, polynomial equations, polynomials, and algebra constitute a subject, theme, sector and domain, respectively (Artigue & Winsløw, 2010). The school subject of mathematics is a discipline. Higher levels determine all disciplines and will not be modeled here. The first three levels (1-3) are defined by praxeologies (Artigue & Winsløw, 2010). A subject corresponds to a few praxis blocks, while a theme and a sector are determined by a shared technology and a shared theory, respectively.

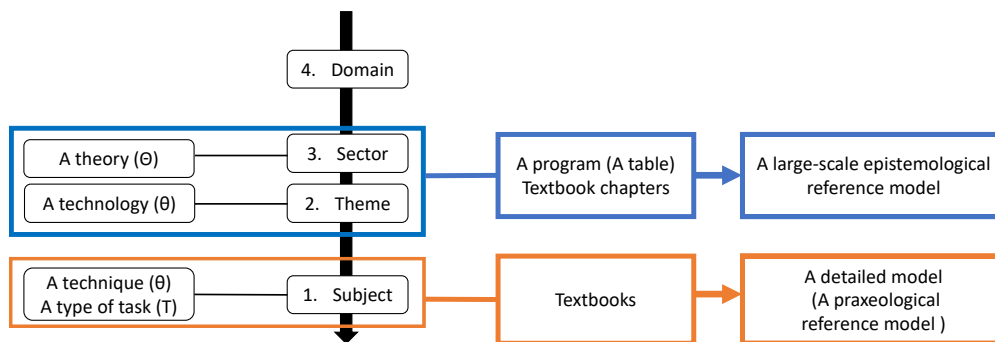
We can now formulate our research question for this study: How to elaborate an epistemological reference model for an entire domain of school mathematics, in the sense of the official domain to be taught in a given school system, when the model is required to cover several years of teaching, and still with a level of detail that would make it useable to situate a concrete teaching situation and to compare with how the domain is taught in other school systems? As a case (considered here), how to do this for the domain of arithmetic and grade 1-6 in Japanese primary school?

Note that this paper is mainly about how to develop a method based on a strong theoretical paradigm, and methodology is therefore more important in this paper than the products we exhibit from the concrete case mentioned in the research questions, in order to illustrate it our methodological approach.

### **Methodology 1: choice of data (and how to get the data)**

Before describing the process of elaborating an epistemological reference model, we explain the selection of materials for the case, which could be similar (yet different) according to the context. There are two levels of official programs in Japan: a general course of study for primary school (SHOGAKKO GAKUSHU SHIDO YORYO) and a primary school *teaching guide* for the Japanese course of study in mathematics (SHOGAKKO GAKUSHU SHIDO YORYO KAISETSU SANSU-HEN). Both are issued by the Ministry of Education, Culture, Sports, Science and Technology (MEXT). The former includes the basic act of education, general educational goals, and an outline of contents for teaching in each discipline. By contrast, the latter is published in each discipline and contains more detail than the course of study. When more detail is sought, we consider also mathematics textbooks authorized by MEXT. These are published by commercial textbook companies, based on the first mentioned documents, and are selected and distributed by schools. The

three levels of material are interconnected, and tightly aligned. The primary sources of our modeling approach are the teaching guide for mathematics (referred to in the sequel as the *program*), and one selected textbook series. The textbook is published by TOKYO SHYOSEKI publishing company, and is widely used both in Japan, and in the supplementary schools that will later be a main focus of our research. From the program, we mainly use a table that summarizes mathematical contents for teaching in each grade. This table is extremely useful for elaborating large-scale models since it specifies that, for instance, teaching addition and subtraction of 2-digit numbers occurs in grade 1. One can say that the table specifies the location of themes and sectors. Then, the textbook can be used to develop more detailed, local models (example given in table 2), in other words, at the subject level in terms of types of task and techniques. This is why we use the program rather than the general course of study to develop our models. It is also easier to model because what is shown in the general course of study school is similarly explained in the programme, with additional detail. In fact, we can elaborate the large scale model based on the program to some extent; however, textbooks are needed to specify more closely the types of task and techniques to be taught.



**Figure 1: How to elaborate different scale models**

The process of elaborating a large-scale epistemological reference model is then as follow:

1. Browse through the table for the domain of arithmetic, dividing it into sectors.
2. Identify themes within each sector from the table, and to clarify and substantiate these further, refer to the detailed description in the program and to practices exhibited in relevant textbook chapters.

The method of constructing detailed models of themes is then as follow:

1. Browse through relevant textbook chapters, and analyse all examples and exercises to identify types of tasks and the corresponding techniques; in conjunction with that, when we identify types of tasks and analyse techniques. (In Japan, as textbooks are strongly aligned with the program, it is justified for many purposes to base the model on just one textbook system.)
2. Whenever a task is encountered which does not belong to a type of task already identified, a new type is added to the model. The end result covers the theme in question.

The second part of our approach is similar to the one employed in the study by Wijayanti and Winslōw (2017). They elaborate praxeological reference models for certain themes related to proportion, as they appear in a range of different Indonesian textbooks. However, our method to categorize and situate different themes and sectors, is new.

## Methodology 2: analysis of data (examples and overall outcomes)

We now present a large-scale epistemological reference model for the domain of arithmetic in Japanese primary school in grades 1-6 (Table 1), based on the above methodology. After the long table presenting the result, we provide further details of how the method was applied.

**Table 1: An epistemological reference model 1 for the domain of arithmetic**

Sector <sub>1</sub> : Representation of (positive) rational numbers	Grade					
	G1	G2	G3	G4	G5	G6
Theme <sub>1-1</sub> : Compare the size given objects						
Th.1-2: Count the size given objects						
Th.1-3: Decompositions and compositions of a given numbers						
Th.1-4: Cardinal numbers						
Th.1-5: Ordinal numbers						
Th.1-6: Representation of 2-digit numbers						
Th.1-7: Representation of 3-digit numbers						
Th. 1-8: The principle of the base-10 numeration system						
Th. 1-9: Representation of 4-digit numbers						
Th. 1-10: Representation of unit fractions						
Th. 1-11: Representation of unit of ten thousand $\cong N \cong$ one hundred million						
Th. 1-12: Representation of fraction						
Th. 1-13: 10 times, 100 times, 1000 times and 1/10 times of whole numbers						
Th. 1-14: Representation of decimal numbers (0.1)						
Th. 1-15: Representation of unit one hundred million and trillion						
Th. 1-16: Representation of approximate numbers						
Th.1-17: Representation of proper fraction, improper fraction, and mixed fraction						
Th. 1-18: Representation of decimal numbers (0.01, 0.001)						



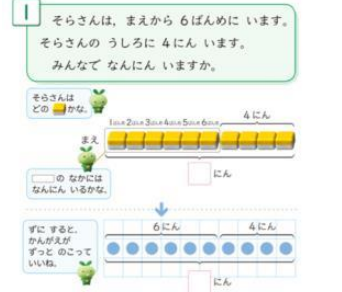
Th. 1-19: Representation of even and odd numbers						
Th. 1-20: Representation of divisors and multiples						
Th. 1-21: 10 times, 100 times, 1000 times, 1/10 times and 1/100 times of decimal numbers						
<b>Sector2: Operations with (positive) rational numbers</b>	G1	G2	G3	G4	G5	G6
Theme2-1: Addition of 1-digit numbers						
Th.2-2: Addition of simple 2-digit numbers						
Th.2-3: Addition of 2-digit numbers						
Th.2-4: Addition of simple 3-digit numbers						
Th.2-5: Addition of 3-digit numbers						
Th.2-6: Addition of 4-digit numbers						
Th.2-7: Addition with decimal numbers (until 1/10)						
Th.2-8: Addition with decimal numbers (until 1/100)						
Th.2-9: Addition with fractions of the same denominator (total is less than 1)						
Th.2-10: Addition with fractions of the same denominator (total is more than 1)						
Th.2-11: Addition with fractions of the different denominator						
Th.2-12: Addition using approximate numbers						
Th.2-13: Addition using letters such as a, x						
Th.2-14: Subtraction of 1-digit numbers						
Th.2-15: Subtraction of simple 2-digit numbers						
Th.2-16: Subtraction of 2-digit numbers						
Th.2-17: Subtraction of simple 3-digit numbers						
Th.2-18: Subtraction of 3-digit numbers						
Th.2-19: Subtraction of 4-digit numbers						
Th.2-20: Subtraction with decimal numbers (until 1/10)						

Th.2-21: Subtraction with decimal numbers (until 1/100)						
Th.2-22: Subtraction with fractions with the same denominator (total is less than 1)						
Th.2-23: Subtraction with fractions with the same denominator (total is more than 1)						
Th.2-24: Subtraction with fractions of the different denominator						
Th.2-25: Subtraction using approximate numbers						
Th.2-26: Subtraction using letter such as a, x						
Th.2-27: Multiplication of 1-digit numbers (multiplication table)						
Th.2-28: Multiplication of simple 2-digit numbers						
Th.2-29: Multiplication (2-digit numbers or 3-digit numbers $\times$ 1-digit numbers or 2-digit numbers)						
Th.2-30: Multiplication using approximate numbers						
Th.2-31: Multiplication of decimal numbers in which a multiplier is whole numbers						
Th.2-32: Multiplication of decimal numbers						
Th.2-33: Multiplication of fractions						
Th.2-34: Multiplication using letters such as a, x						
Th.2-35: Division in which a divisor and a quotient are 1-digit numbers						
Th.2-36: Division (simple calculation in which a divisor is 1-digit numbers, and a quotient is 2-digit numbers)						
Th.2-37: Division using approximate numbers						
Th.2-38: Division (calculation in which a divisor is 1-digit numbers or 2-digit numbers, and a dividend is 2-digit numbers or 3- digit numbers)						
Th.2-39: Division of decimal numbers in which a divisor is whole numbers						
Th.2-40: Division of decimal numbers						
Th.2-41: Division of fractions						
Th.2-42: Division using letters such as a, x						

In our model, representation with rationale numbers and operation with rational numbers were identified as the two sectors, corresponding to theories developed throughout the six years of primary school. This is because the domain of arithmetic is roughly divided into two parts in the table of the program: number and operations, and it is also justified by the details (roughly, themes) appearing in these parts. In Japan, negative numbers are taught only in secondary school. Hence, rational numbers do not include negative numbers in this context. Within the sector1 and 2, we identified twenty-one themes (for instance, representation of 3-digit numbers, Representation of fractions) and forty-two themes (for instance, addition with fractions of the different denominator), respectively, based on the table of the program. Table 1 shows in which grade(s) a given theme is to be taught.

We finally show a more detailed model of one particular theme: Addition of 1-digit numbers (Table 2), with types of tasks (T) and technique ( $\tau$ ), along with examples of tasks from the textbooks. This is the kind of model presented and used (more extensively) by Wijayanti and Winsløw (2017).

**Table 2: A praxeological reference model of one theme: Addition of 1-digit numbers**

Type of tasks	Technique	Example of task
T <sub>1</sub> : The addition of 1-digit numbers (cardinal numbers, cases of combination)	$\tau_1$ : Put the two sets or quantities together and count the result	<p>t<sub>1</sub>: Write the equation. We put 2 pencils in the box. How many pencils do we have in total?</p> 
T <sub>2</sub> : The addition of 1-digit numbers (cardinal numbers, cases of increase)	$\tau_2$ : Counting up from an augend by the addend	<p>t<sub>2</sub>: Write the equation. There are 4 flowers in the vase. The girl puts 3 more flowers in there. How many flowers are there in total?</p> 
T <sub>3</sub> : The addition of 1-digit numbers (ordinal numbers).	$\tau_3$ : Converting a given ordinal number into cardinal number based on a figure, then use $\tau_2$ .	<p>t<sub>3</sub>: Sora is 6th from the front. Behind Sora, there are 4 people. How many people are there in total?</p> 

## Conclusion

In this paper, we have shown how ATD can be used to answer a methodological question: how to elaborate a large-scale model of a curriculum. As a concrete case we have shown different scale



models for the domain of arithmetic in Japanese primary school. Such a model can be used, for instance, to precisely situate observations from a lesson in a larger context. More generally, the levels of codetermination (subject, theme, sector) and praxeologies allow us to elaborate different scale models and relate them to each other. We acknowledge that the form and characteristics of study programmes and textbooks may vary from country to country. For instance, our next step will be to develop similar models for the domain of arithmetic in primary school in Denmark. However, the Danish curriculum is (roughly speaking) less explicitly structured than the Japanese one; therefore, the precision of the model will be different. We will later use the model to address the main questions of my PhD project: how the domain of Japanese arithmetic, defined by MEXT, transposes to the Japanese supplementary school in Denmark (held on Saturdays), and how the students there experience it, as they attend Danish school on working days.

## References

- Artigue, M., & Winsløw, C. (2010). International comparative studies on mathematics education: A viewpoint from the anthropological theory of didactics. *Recherches en didactique des mathématiques*, 30(1), 47-82.
- Bosch, M., & Gascón, J. (2006). Twenty-five years of the didactic transposition. *ICMI bulletin*, 58(58), 51-65.
- Fujii, T., & Majima, H. (2021). *Atarashi sansu 1 [New mathematics for elementary school for grade 1]*. Tokyo: Tokyo Syoseki Shuppan.
- Fujii, T., & Majima, H. (2021). *Atarashi sansu 2 [New mathematics for elementary school for grade 2]*. Tokyo: Tokyo Syoseki Shuppan.
- Fujii, T., & Majima, H. (2021). *Atarashi sansu 3 [New mathematics for elementary school for grade 3]*. Tokyo: Tokyo Syoseki Shuppan.
- Fujii, T., & Majima, H. (2021). *Atarashi sansu 4 [New mathematics for elementary school for grade 4]*. Tokyo: Tokyo Syoseki Shuppan.
- Fujii, T., & Majima, H. (2021). *Atarashi sansu 5 [New mathematics for elementary school for grade 5]*. Tokyo: Tokyo Syoseki Shuppan.
- Fujii, T., & Majima, H. (2021). *Atarashi sansu 6 [New mathematics for elementary school for grade 6]*. Tokyo: Tokyo Syoseki Shuppan.
- Ministry of Education, Culture, Sports, Science and Technology. (2017). *Syougakko gakusyu shido yoryo kaisetsu, sansu-hen [Primary school teaching guide for the Japanese course of study: Mathematics]*. Tokyo: Nihon Bunkyo Shuppan.
- Wijayanti, D., & Winslow, C. (2017). Mathematical practice in textbooks analysis: Praxeological reference models, the case of proportion. *REDIMAT*, 6(3), 307-330. doi: 10.17583/redimat.2017.2078