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# Facilitators' views on content goals, learning obstacles, and teaching resources in reference to conditional probability

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*Facilitators of teacher professional development (PD) courses mediate the course ideas conceptualized by the PD course designers. It is relevant to research what views facilitators hold in reference to the content goals, the learning obstacles, and the teaching resources that constitute the heart of a PD course. In our current study, we analyze one facilitator's deliberations in an interview conducted after a PD day on conditional probability, based on a framework of expertise on the classroom and the PD level. The results reveal which elements of the PD course were adopted wholeheartedly, and which remained more or less superficial. The analysis also suggests reasons for this distinction and how to rethink facilitator qualification and PD course material.*

*Keywords: Professional development, facilitators, expertise, conditional probability.*

## **Introduction: Challenges in teaching conditional probability**

With curriculum changes and the growing relevance of data in the modern world, stochastics (statistics and probability calculation) has moved into the spotlight of mathematics education (Batanero et al., 2011). This has led to a growing need for teacher professional development (PD) for stochastics in general, as teachers are the agents in lesson development, just as facilitators are the agents of teacher PD. Stochastics presents a challenge for numerous reasons (Burrill & Biehler, 2011): There are uncertainties to deal with, e.g. when predicting future frequencies from probabilities. And modelling must be taken seriously; the step from reality or real data to the world of mathematics involves an awareness of idealizations, and the necessary interpretation of results requires considering restrictions of the model.

In this paper, the challenges are exemplified by the content of conditional probability, which is connected to the concept of stochastic independence and Bayes' theorem. The common mistakes respectively misconceptions in this area (Bar-Hillel, 1983; Gigerenzer & Hoffrage, 1995; Kahneman & Tversky, 1973; McDowell & Jacobs, 2017) cover confusing condition and event, misinterpreting stochastic dependence as causality, and underestimating the relevance of the base rate  $P(A)$  for the calculation of the conditional probability  $P(A|B)$ . Substantial knowledge of common mistakes and misconceptions is prerequisite for choosing suitable teaching resources and supporting students in reaching the respective learning goals. Conducting a teacher PD, however, is accompanied by additional challenges. While providing necessary content knowledge or illustrating misconceptions, facilitators need to address aspects which are specific for a PD, like participants' heterogeneity or their pre-formed opinions on learning pathways. A framework for design of and research on teacher PD, the three-tetrahedron model for content-related PD research (see Prediger et al., 2019) covers these aspects comprehensively: The four corners of a tetrahedron, referring to educator, learners, content, and teaching researches respectively, are specified for three levels, the classroom level, the PD level, and the qualification level. On the PD level e.g., the facilitators are the educators of the

learners, who are the participating teachers. This model also illustrates the connections between the different levels. Classroom level issues are nested in the PD level insofar as the content goals and learning obstacles for students are, together with suitable teaching resources, the content at the PD level. Therefore, facilitators should be experts in the content goals, the learning obstacles, and the teaching resources of both the classroom and the PD level. In how far this is the case and what might be the reasons behind, is the focus here.

### **Theoretical considerations: Expertise for teaching conditional probability**

We base our considerations on a situated approach and chose a framework for teaching expertise (Prediger, 2019, adapting Bromme, 1992), which distinguishes between *jobs*, *pedagogical tools*, *categories*, and *orientations*. These concepts allow to describe and explore what teachers or facilitators focus on doing in a specific situation (*jobs*), which thinking *categories* they activate, what they utilize in order to reach their goals (*pedagogical tools*), and which *orientations* influence their choices. The thinking categories, in particular, cover specificities of the content, e.g. the procedural and conceptual learning goals, the possible learning pathways, and the learning obstacles. The framework is tuned towards the actual teaching / learning situation, with its carefully orchestrated resources and its *ad hoc* reactions and decisions.

For this paper, we focus on content goals and learning obstacles (which are parts of the thinking *categories*), and on the *pedagogical tools*, which are closely connected to content goals and learning obstacles – as teachers / facilitators choose their pedagogical tools (e.g. specific tasks or activities, visualizations, software applications) with the aim of supporting their students / the participants in their PD course in reaching the intended content goals, keeping possible learning obstacles in mind, i.e. finding ways to overcome them. In our PD setting, a content goal is to comprehend the relevance of the base rate  $P(A)$  for the calculation of the conditional probability  $P(A|B)$ , which is often underrated when the corresponding calculations are executed with probabilities and Bayes' rule. Using natural frequencies and an easily accessible representation, e.g. a double tree diagram, can help to overcome this learning obstacle (Gigerenzer, 2011; Wassner et al., 2007). Our PD also aimed at generally promoting the use of simulations as a teaching resource, which has been shown to “have the potential to make learning statistics easier” (Lane & Peres, 2006, p. 6). On the one hand, simulations were presented to foster the frequentist view on probability. On the other hand, we introduced simulations as an adequate tool for calculating probabilities where learners' analytic means are insufficient.

The thinking categories at classroom level cover, among other aspects, the content goals that are to be addressed and the learning obstacles that might hinder reaching these goals. In the course section focused here, the content goals can be described as knowing the definition for conditional probability, understanding the sense of this definition, calculating conditional probabilities (via the definition or by using Bayes' rule or other strategies), and being aware of the impact of different base rates. At PD level, knowing how to introduce conditional probability and stochastic independence by utilizing appropriate tasks, activities, simulations, and visualizations, as well as considering misconceptions when planning lessons, would be added to this list.

At classroom level, various learning obstacles should be considered by the teacher, e.g.

- modelling issues as a result of idealizations involved in probabilistic models
- sampling issues, sampling variation and differences between population and samples
- motivation issues that might hinder students from grasping or memorizing the content,
- misconceptions like confusing condition and event, misinterpreting stochastic dependence as causality or underestimating the relevance of the base rate, and
- a high level of abstraction that could present a learning obstacle in itself.

At PD level, additional learning obstacles comprise the heterogeneity of the group of participating teachers (e.g. referring to their individual knowledge or their respective professional learning groups), pre-formed opinions on certain teaching approaches, or previous (positive or negative) experiences when teaching the same or a similar content. For example, teachers might infer from their own learning history that the use of digital technology is not worth the time needed to come to grips with it. Or they might not consider modelling issues as relevant enough to discuss explicitly.

The pedagogical tools comprise teaching and learning resources, which can have a close connection to the content goals. For the content of conditional probability at classroom level, these are: the reflected use of absolute (natural) or relative frequencies, traditional tree diagrams with probabilities, (double) tree diagrams with absolute frequencies, 2x2 tables, linguistic scaffolding, hands-on experiments, digital simulations, data from digital simulations, ideal simulation (gaining natural frequencies by using artificial population sizes), authentic problems, and problems with artificial stories. The resources listed here, taken from the PD course at hand, share the characteristic that they address conceptual understanding, rather than procedural skills (see Binder et al., 2020 for an explanation of the different tools for visualization).

At PD level, these pedagogical tools can also be utilized, in a reconfigured form with a perspective on the PD situation: For example, the content and its associated pedagogical tools from the classroom level can be arranged in possible sequences, to present a range of teaching options. Then, the PD participants can be asked to work on the tasks and materials belonging to the different options, from a student perspective. This, in turn, can be followed by group discussions to reflect upon the teaching options, led by the facilitator who can incorporate his own experience with the material into the discussion, integrating participants' concerns and misgivings.

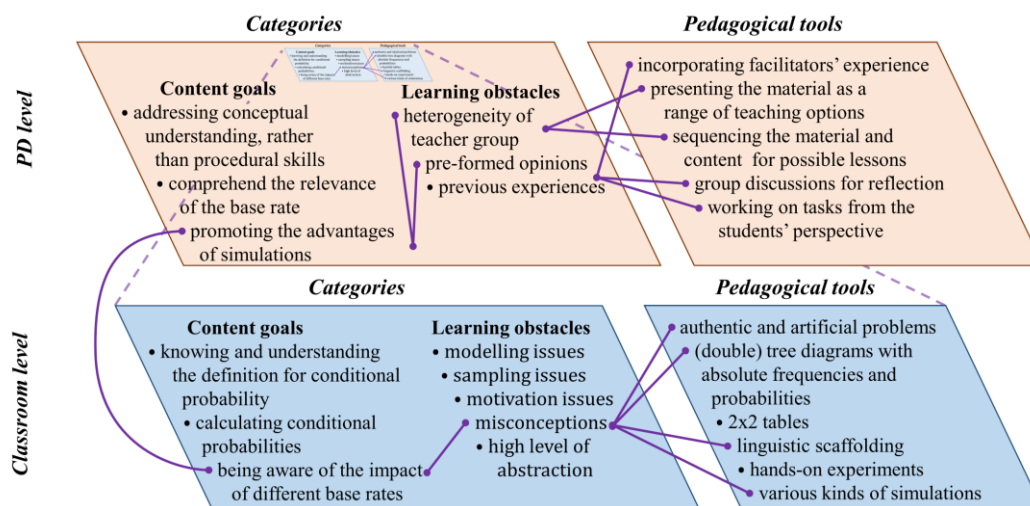
There are various possible connections between the content goals, the learning obstacles, and the teaching resources; and teachers' or facilitators' more general (and less content-specific) orientations can reveal the underlying reasons for their interpretation and the performance of their jobs: For example, an orientation to actively address misconceptions influences the choice of an activity or a teaching resource; an awareness of modelling issues implies integrating validation considerations; the belief that language matters encourages offering content-specific language learning opportunities. The exemplary connections in Figure 1 also illustrates the interconnection of the classroom and PD level. Using (ideal) simulations in class is an adequate tool to create an awareness for the impact of base rates. Promoting the use of simulations in the classroom implies addressing their advantages in the PD by pointing out their usefulness in e.g., group discussions.

Our research goal is to learn more about facilitators' views connected to the content goals, learning obstacles, and teaching resources – so we aim to answer these questions:

RQ1: Which content goals, learning obstacles, and teaching resources are mentioned by the facilitator and how are they accentuated?

RQ2: Which connections between the content goals, learning obstacles, and teaching resources are mentioned by the facilitator and how are they elaborated upon?

As a perspective, we are interested in exploring in how far the facilitator's notions coincide with the conceptual ideas of the original PD designers. Therefore, our research interest is in knowing which orientations can be inferred from the above, in particular in reference to the facilitator's adaption of the teaching concept. This also includes exploring if and when the facilitator focuses on student learning and / or on teacher PD.



**Figure 1: Nested facilitator expertise (categories, pedagogical tools) for teaching conditional probability, with exemplary connections**

## Context of the PD course

The PD course is part of a five-day PD program on stochastics for upper secondary level, developed at Paderborn University, Germany (Barzel & Biehler, 2017). The PD program envisions a teaching approach based on the principle of consistently promoting concept formation, e.g. via the use of simulations, digital tools, authentic examples and real applications. During the PD course on conditional probability, stochastic (in)dependence, and Bayes' theorem, the teaching recommendations focus on the use of natural frequencies, e.g. gained in simulations, and their use in double tree diagrams, which are regarded as an innovation in the German school context. A more traditional form of representation, 2x2 tables, is mentioned along the way.

The whole PD program was discussed at length and re-designed with four experienced facilitators over a period of three years, in cooperation with a regional education administration. Afterwards, the facilitators moderated the program more than once in teams of two.

## Methodology

Directly after each PD day, guided interviews with the facilitators were conducted, audio-recorded and later transcribed. Among other aspects, the interviews covered the PD learning goals (both from the facilitator's and from the course designers' view), possible learning obstacles and how to

overcome them, and the teaching resources offered by the PD course for the classroom level. In the course of the interview, facilitators were asked to elaborate on a printed list of PD goals.

In this paper, we concentrate on one facilitator, who we call “Mike”, who was involved in the re-design of the PD program, and on the part (on day 2 of the program) on conditional probability, stochastic independence, and Bayes’ theorem. Mike is male and has 16 years of experience as teacher, and 13 years as facilitator (mostly for other content than conditional probability). The interview with Mike lasted 70 minutes and has 178 turns; the interviewee’s turns ranging in length between short comments of very few words and extensive elaborations of over 450 words.

The transcribed interview was analyzed in three steps: First, the passages relevant for content goals, learning obstacles, and teaching resources were identified, respectively, by the first and second author separately. Second, a consensus was reached between them about which interview passages belonged to which aspect. Third, a qualitative analysis was conducted (Kvale, 2009) in order to dissect the relevant text passages and phrase answers to the research questions.

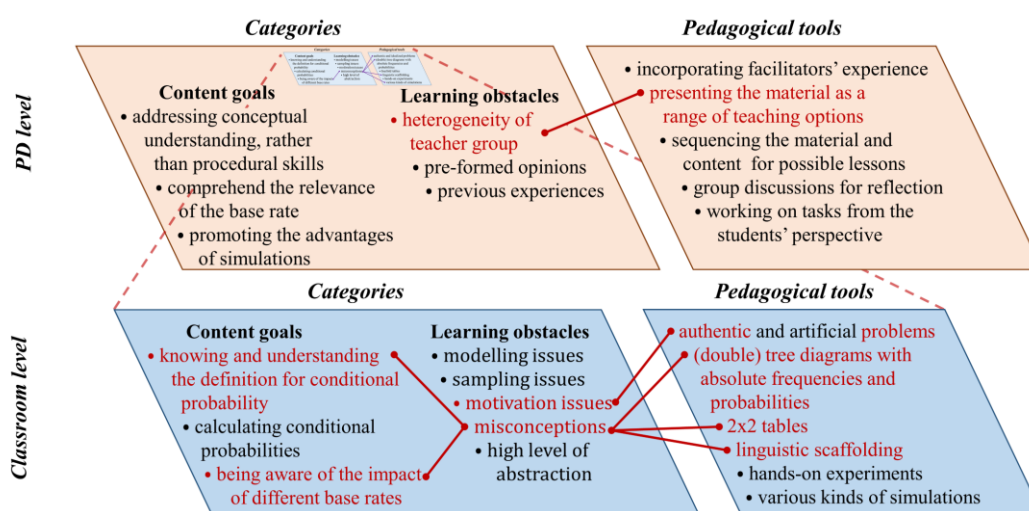
## Results

The research questions (RQ1: Which content goals, learning obstacles, and teaching resources are mentioned by the facilitator and how are they accentuated? RQ2: Which connections between the content goals, learning obstacles, and teaching resources are mentioned by the facilitator and how are they elaborated upon?) can be answered as follows (see Figure 2 for an overview):

Mike mentions the contents conditional probabilities and Bayes’ theorem as the most important goals for the PD day, without specifying what exactly is relevant for these topics (turn M\_002). He emphasizes that the PD concept is to promote students’ understanding and argumentation skills (M\_004, M\_010, M\_014, M\_018, M\_064) and sees this aspect as an indication for better teaching (M\_010, M\_012, M\_022). There is no mention of procedural skills. Mike connects the advancement of understanding with an awareness of common misconceptions (M\_010, M\_018, M\_111, M\_121, M\_127, M\_165, M\_171, M\_175) and finds that the most relevant general problem is that “students show very many misconceptions, even with everyday relevance” (M\_010), where at the same time he assesses everyday applications as beneficial for students’ motivation (M\_010). Mike has noticed that PD course participants often hold misconceptions themselves (M\_107, M\_167), so addressing misunderstandings is an issue both at the classroom and at the PD level (for the connections between the different aspects of expertise, see Figure 2). Mike does not mention simulations, an adequate tool for fostering students’ understanding.

The teaching resources Mike specifies can all be located on the classroom level and mostly refer to specific tasks (M\_010, M\_012, M\_127, M\_128, M\_145, M\_157, M\_173) that have an authentic background and touch upon the common mistakes. Other pedagogical tools that Mike mentions are double tree diagrams (M\_022, M\_149, M\_151) and 2x2 tables (M\_121, M\_149, M\_151), again with a perspective on classroom teaching, not on teacher PD. He explains at length that he prefers 2x2 tables (M\_149, M\_151) and gives as reasons that they help students to connect absolute and relative frequencies, and to bridge the transition from a tree diagram to the reversed tree diagram (M\_149), therefore connecting a teaching resource to a content goal. Although double tree diagrams, as presented in the PD course, comprise didactic advantages, Mike states he would use this resource

only subsequently (M\_151). In addition, Mike sees the advantages of using absolute over relative frequencies (M\_129, M\_131) for the content goal of promoting understanding, but comments on this teaching / learning resource only when hinted by the interview material. He stresses the fact that, in a PD course, the aim is not to offer an ideal teaching approach that works perfectly in every setting (M\_127), but to present a range of teaching options (often in the form of tasks, M\_052) teachers can choose from. Therefore, a lesson plan is not regarded as an appropriate teaching resource at the PD level, but a collection of tasks and activities is. Mike outlines that he would conceptualize his own lessons following the principle to orchestrate an easy access, stressing connections to previous knowledge elements, and introducing more complex considerations only when students feel secure on the new ground (M\_141). The scenario he refers to particularly attends to weaker students and examination situations (M\_121, M\_145, M\_147).



**Figure 2: Aspects of expertise and their connections mentioned by facilitator Mike (highlighted)**

The use of hands-on experiments or digital simulations triggers questions for Mike, as to when (or if, the German language does not distinguish this) these are helpful (M\_109, M\_143), and he finds that the result matters, independent of coming from a simulation or from a calculation (M\_018). In this context, Mike is keen to refer to hands-on experiments (M\_014), thus indicating a certain reserve towards digital simulations. More importantly, Mike always connects digital simulations with the technical skill of handling Graphing Calculators (M\_024). As simulations require predetermining the number of overall experiments, Mike does not see the advantage of simulations over 2x2 tables – and he is unaware that these can indeed represent ideal simulations (M\_143-145). He would utilize simulations when the probabilistic model is unclear, though (M\_018, M\_145).

Particularly here, it becomes clear that Mike's argumentation routinely refers to the decisions he has made or would make for his own teaching (M\_097, M\_145), the PD course participants do not feature in his deliberations as active agents of their own teaching. He visualizes himself teaching, not qualifying the PD participants teaching their respective students (M\_141). This is a key point in our analysis, as it not only reveals Mike's self-concept of himself as a facilitator, but also provides a method to spark reflections on this self-concept (via visualizing the prevalent scenes in one's mind when leading a PD course), and categories for facilitator self-concepts (e.g. as teacher, as agent for the PD of the participating teachers, or even as erstwhile learner) in general.

## Conclusion

Mike, one of four facilitators, expresses his views openly in the interview. What he does and does not mention in reference to content goals, learning obstacles, and teaching resources allows insights into a facilitator's views on the specific PD course at hand and on teacher PD in general.

Mike's utterances indicate an orientation to stick to teaching strategies that yielded satisfying learning results in the past, e.g. preferring 2x2 tables (a standard form of representation) over double tree diagrams (rather uncommon in German textbooks, but suggested by didactic research and successful teaching experiments). It is remarkable that Mike mentions of his own accord only teaching resources that he either favors (e.g. 2x2 tables), or that are both innovative *and* stand the test of him introducing them into his own lessons (e.g. double tree diagrams). He comments on other resources (like digital simulations) when these are mentioned by the interviewer, but does not introduce them into the conversation himself. This shows that he has remained skeptical of using digital simulations for improving learning in the classroom, and he states that he would only take recourse to them if there is no other way of establishing a probability.

All in all, it becomes obvious that Mike favors the perspective of focusing on the classroom level (M\_026, M\_125, see Figure 2). He sees the main purpose of PD in teachers discussing and reflecting on concrete teaching situations, himself as *primus inter pares* – albeit acknowledging parallels between the PD course and a mathematics lesson. Mike switches to reflections on the PD level in the later parts of the interview (M\_127 onwards), but retains his focus on his own suggested teaching, and on teacher professional development only indirectly via the intended student learning.

Consequently, it remains doubtful how far certain aspects the PD course ideas have been conveyed successfully, in spite of intensive and prolonged cooperation between course designers and facilitators. It seems that orientations are not easily changed, in particular if they are based on previous experience, and addressing them should be planned very carefully. Introducing new teaching resources or pedagogical tools, on the other hand, might be presumed successful up to a certain level. And it we hope that these can impact on orientations in the long term.

What is more, Mike's focus on lessons and students leaves the issue unresolved if the PD courses he leads concentrate on teacher PD in the sense of advancing teacher expertise – which is more than reflections on advancing student learning. Mike emphasizes that teachers are presented with a range of tasks and activities to choose from or to adapt, but does not address the necessary skills for this selection or adaptation process. Ideally, these skills should be promoted during phases of discussion and reflection in the PD course, and the facilitator would disengage from the role of a colleague and view the PD course participants as individuals whose learning processes are also his responsibility.

It will be interesting to explore if this interpretation can be supported by Mike's actions and utterances during the PD course, which was audio-recorded. Although acting as a team, the analyses of the other three facilitators' interviews and moderation will probably reveal different aspects and thus paint a more differentiated picture of facilitators' views.

## References

- Bar-Hillel, M. (1983). The base-rate fallacy controversy. In R. W. Scholz (Ed.), *Decision making under uncertainty: Cognitive decision research social interaction development and epistemology* (pp. 39–61). Elsevier.
- Barzel, B., & Biehler, R. (2017). Design principles and domains of knowledge for the professionalization of teachers and facilitators: Two examples from the DZLM for upper secondary teachers. In S. Zehetmeier, B. Rösken-Winter, D. Potari, & M. Ribeiro (Eds.), *Proceedings of the Third ERME Topic Conference on Mathematics Teaching, Resources and Teacher Professional Development, ETC3* (pp. 16–34). Humboldt-Universität, Berlin.
- Batanero, C., Burrill, G., & Reading, C. (Eds.). (2011). *Teaching statistics in school mathematics: Challenges for teaching and teacher education, joint ICMI/IASE study: 18th ICMI study*. Springer.
- Binder, K., Krauss, S., & Wiesner, P. (2020). A New Visualization for Probabilistic Situations Containing Two Binary Events: The Frequency Net. *Frontiers in Psychology, 11*(750). <https://doi.org/10.3389/fpsyg.2020.00750>
- Bromme, R. (1992). *Der Lehrer als Experte: Zur Psychologie des professionellen Lehrerwissens*. Huber.
- Burrill, G., & Biehler, R. (2011). Fundamental statistical ideas in the school curriculum and in training teachers. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics: Challenges for teaching and teacher education, joint ICMI/IASE Study: 18th ICMI Study* (Vol. 14, pp. 57–69). Springer.
- Gigerenzer, G., & Hoffrage, U. (1995). How to improve Bayesian reasoning without instruction: Frequency formats. *Psychological Review, 102*(4), 684–704.
- Gigerenzer, G. (2011). What are natural frequencies? *BMJ (Clinical Research Ed.)*, *343*, d6386.
- Kahneman, D., & Tversky, A. (1973). On the psychology of prediction. *Psychological Review, 80*(4), 237–251.
- Kvale, S. (2009). *Interviews: An introduction to qualitative research interviewing* (2nd ed.). Sage.
- Lane, D. M., & Peres, S. C. (2006). *Interactive simulations in the teaching of statistics: Promise and pitfalls*. Proceedings of the International Conference on Teaching Statistics, ICOTS7
- McDowell, M., & Jacobs, P. (2017). Meta-analysis of the effect of natural frequencies on Bayesian reasoning. *Psychological Bulletin, 143*(12), 1273–1312.
- Prediger, S. (2019). Investigating and promoting teachers' expertise for language-responsive mathematics teaching. *Mathematics Education Research Journal, 31*(4), 367–392.
- Prediger, S., Roesken-Winter, B., & Leuders, T. (2019). Which research can support PD facilitators? Strategies for content-related PD research in the three-tetrahedron model. *Journal of Mathematics Teacher Education, 22*(4), 407-425.
- Wassner, C., Biehler, R., Martignon, L. (2007). Das Konzept der natürlichen Häufigkeiten im Stochastikunterricht. *Der Mathematikunterricht, 53*(3), 33-44