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# What makes doing mathematics in PD productive: Lifting and adapting a framework for insights on leader PD and facilitation

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*Well-prepared facilitators of mathematics PD are essential, yet few studies offer nuanced insights and content-specific practices for how facilitators support teachers' productive mathematical engagement. This study utilizes a framework of productive disciplinary engagement (PDE) to investigate the social and mathematical practices of 48 aspiring facilitators and two expert PD leaders across a two-phase design-research study. We found that the PDE framework allowed us to identify participant and facilitation practices that afforded and constrained productive engagement with disciplinary content and practices. We discuss the use of PDE as an analytic and design framework for facilitator and teacher PD. This study contributes insights on content-specific PD practices applicable to a wide range of mathematics PD models.*

*Keywords: Facilitators, professional development, mathematics education, facilitator practices.*

## Introduction

Decades of U.S. policy and international research advance that effective professional development (PD) must actively engage participants in disciplinary work. In mathematics PD, this means offering opportunities for teachers to engage in the mathematics relevant to their classrooms. Previously, we have argued that goals for doing mathematics are underspecified, advocating for PD to attend to teachers' specialized content knowledge (SCK) (Ball et al., 2008; Elliott et al., 2009). Well-prepared facilitators of mathematics PD are essential to realizing these goals and advancing PD research (Karsenty, 2021; Lesseig et al., 2017). Over the past decade, attention has been paid to the growth of PD facilitators (Krainer et al., 2021); however, few studies offer nuanced insights and content-specific practices for *how facilitators support* productive mathematical engagement when working with teachers in PD (Borko et al., 2014). We take up this aim by examining a nested context of facilitator PD focused on teacher PD, in which two expert leaders (ELs) worked with aspiring facilitators. Knowing how and what moves to support teachers' productive engagement is vital to high-quality PD (Tekkumru-Kisa & Stein, 2017).

This study is situated in a two-phased design research project, *Researching Mathematics Leader Learning* (RMLL). The paper examines participants' collective work on one mathematics task across four small groups and two whole group discussions using a framework of *productive disciplinary engagement* (PDE; Engle & Conant, 2002). We use this framework to understand what makes "doing mathematics" in PD effective and ways facilitators might guide collective mathematical activity toward particular goals for teacher learning.

## Theoretical perspective

Elsewhere we have argued that teacher learning of mathematics differs from students' learning in at least two specific ways (Elliott et al., 2009). Teachers have experienced the content at least once as

students and often have revisited the mathematical ideas as teachers or PD participants. Second, in PD, teachers are working with peers and are often colleagues of the facilitator, resulting in different authority arrangements than those among teachers and students. These differences, coupled with the understanding that teachers come into PD with varied orientations toward mathematics, mathematics teaching, and learning, can create tensions as social positioning is negotiated among colleagues when doing mathematics (Holland & Lave, 2009). As a result, we have argued that a goal for teachers' mathematical learning in PD is to develop SCK (Ball et al., 2008). With this goal, facilitators must anticipate and navigate the range of ways teachers may leverage mathematical concepts while simultaneously holding teachers accountable to the practices of the discipline and supporting their authority by positioning them as competent. Facilitators must also negotiate what constitutes a mathematical explanation, how errors and uncertainty are valued, and how social relations and positioning are mediated. This study utilizes the PDE framework as a lens to consider the dynamic interplay among social and mathematical aspects of the work accomplished by aspiring facilitators (teacher leaders) in a PD setting. Our analysis provides an empirical argument for the utility of the PDE framework addressing how productive disciplinary engagement is afforded or constrained in collective mathematical work in facilitator PD.

Adult learning involves cognitive, social, and cultural dynamics inseparable from contexts and content. Consistent with this situative perspective, we use PDE to examine the activity system - actors engaged with one another, using practices and resources to enact activity - as our unit of analysis (Holland & Lave, 2009). A vital feature of the study context of facilitator PD is that these actors belonged to at least three communities of practice central to doing mathematics - aspiring facilitators, previous participants in mathematics PD, and mathematics teachers. The complexity of these overlapping communities frames the collective work of doing mathematics in this setting and the social positions occupied by participants. Awareness of the practices nested within these multiple communities is vital to evaluate the productivity of PD engagement.

## **Productive Disciplinary Engagement**

Engle and Conant (2002) describe *productive engagement* as the intensity and extent to which learners progress on a problem over time. *Disciplinary engagement* involves working toward goals by individuals and groups using the concepts and practices of that discipline. PDE frames how we make sense of ideas, practices, and discourse when a group of aspiring facilitators solve and share solutions to mathematics tasks meant to support them in developing mathematical knowledge for teaching and leading. Engle and Conant (2002) operationalize PDE as a dynamic system of four coordinated tenets: problematizing–resources and authority–accountability, suggesting that when any tenet is more dominant or missing, PDE is in jeopardy.

*Problematizing* refers to efforts where disciplinary uncertainties arise. Uncertainty involves questions or indecision regarding what to do, conclude, or justify. Productivity increases as learners reflect on uncertainty and employ *resources* to alleviate it. Resources might be tools, artifacts, or practices needed to do particular kinds of disciplinary work. Problematizing is advanced when learners have sufficient resources and are encouraged to question and make proposals or challenges. As learners assert *authority*, they express their agency to define, address and resolve disciplinary problems. Authority is expanded as learners take control over problematizing the disciplinary content. Learners'

authority needs to be balanced by *accountability* to how one's ideas make sense and are relevant to the work of one's peers, others, and to the discipline. PDE was conceived as a set of design principles to understand the quality of learning settings (Engle & Conant, 2002). We extend PDE's application to focus on the supporting conditions of facilitation in the context of PD. We consider the framework's utility as an analytic and design tool across facilitator and teacher PD.

## Context

RMLL's five-year design-research project prepared facilitators to lead mathematical tasks in PD using a series of PD videocases to support productive mathematical discussions (Elliott et al., 2009). In six seminars in Phase I, facilitators solved mathematics tasks, discussed solutions, and analyzed facilitation features within PD videocases (c.f., Lesseig, et al., 2017). Math tasks were identical to those in student textbooks or PD curriculum. In eight seminars in Phase II, facilitators again solved mathematics tasks, discussed solutions, and analyzed videocases, but there was an increased emphasis on teacher learning goals to develop SCK. In Phase II, we redesigned mathematical tasks to foster and focus mathematical conversations on concepts and practices critical for teaching (Lesseig et al., 2017). With these "reframed-tasks," we aimed to invoke uncertainty by foregrounding mathematical structure while simultaneously providing facilitators with resources to coordinate representations as they explored mathematical patterns and offered conjectures and justifications.

## Methods

We analyzed two small groups in Phase I (three and four members), two small groups in Phase II (four members each), and two whole group discussions (Phase I,  $n = 11$ , Phase II,  $n = 37$ ) as facilitators worked on the *Staircase Task* (Noyce Foundation, 2005). This task is a visual patterning task in which facilitators were asked to determine the number of cubes needed to build the  $n^{\text{th}}$  staircase (i.e., the sum of consecutive integers). In Phase II, after facilitators worked on the Staircase task, they were given the *reframed* task, which pressed them to coordinate representations (visual models, expressions, and tables) to illustrate solutions to the quadratic relationship that can be generalized as  $[n(n+1)]/2$ . We were not looking for evidence of individuals' cognitive resources; instead, we examined patterns of practices and how PDE was afforded or constrained.

First, we viewed the video records to develop a consistent idea unit of leading speaker and focus. Our coding scheme captured the mathematical and social interactions in each idea unit. Disciplinary codes included claims, justifications, and representations. Social codes noted whose ideas were made public and traced how the groups took up these ideas. We also noted how individuals positioned themselves or were positioned by others. Based on this coding, we wrote summative memos of small and whole groups examining how work was accomplished in light of the four PDE tenets to illuminate similarities and differences across groups (Miles & Huberman, 1994). We report on critical ideas across phases to illustrate the interaction of facilitators' mathematical and social work (given pseudonyms initials) and the two ELs (noted as L in the transcript).

## Results

Across both phases of small groups, most facilitators revealed their uncertainty with the Staircase task identifying a recursive relationship in their table (figure number and total cubes). Patterns were described as adding the figure number to the previous staircase, the sum of consecutive addends, or

a generalized sequence. Facilitators' discussion revolved around various explanations for the recursive pattern and their attempts to find a formula that directly related the figure number to the number of cubes in the staircase. Most facilitators could not coordinate resources within their group to find the summation or move to a closed-form solution. A Phase I small group narrated a part of their uncertainty in the following interchange.

- CK: This might be too long and cumbersome, but I'm thinking  $n+n-1+n-2$  ((GT saying it at the same time))  
GT: I wonder if there's another way to state that?  
CK: I don't have any idea.  
LN: But that's what it is  
GT: Yeah, but isn't there an easier way to put it?  
CK: There's gotta be.  
LN: But how do you show a sequence?  
CK: Well, I started writing out the actual numeric equation, so for 3, it equals  $3+2+1$ , and so for 4, it equals  $4+$ and the quantity  $3+2+1$ . Do you see what I'm saying?..  
LN: I don't know how to show sequencing in algebraic thinking except for parentheses  
GT: Yeah, I don't know either. I can't remember.

### **Mathematical uncertainty made public**

As illustrated above, facilitators willingly made their uncertainty public, pushing papers into the center of the table to show their reasoning and sharing different patterns they noticed. Additional resources were needed to move forward within groups where uncertainty coalesced toward the same strategy (e.g., remembering how to sum a sequence). The compressed knowledge of previous mathematics and the various ways that different mathematical concepts were coordinated perpetuated uncertainty unless further resources were recruited into facilitators' discussions.

When uncertainty was public, facilitators often positioned themselves in terms of mathematics. When attempting to generalize patterns, we heard facilitators suggest that they were "visual thinkers," not "formula queens," and label peers as "the Algebra teacher," which conferred higher status. The positioning of facilitators often provided insights into the uncertainty they faced. We also noticed that this social positioning could result in facilitators *deflecting their authority and hampering the accountability* of the facilitator's ideas to the discipline. This showed up in various ways, such as presenting strategies but not knowing how to continue because of faulty understandings ("remembering") or in compressed solutions that were not pressed on to examine the mathematical claims and unpack justifications.

A counter-narrative to the deflecting of authority and accountability to the discipline emerged when facilitators asked how a strategy works, whether it always works, and how a variable was defined. In Phase I we noted that these questions were potentially powerful, yet justification wasn't normative, and incorrect mathematical responses were not probed. However, we saw the expert leader press for accountability in the whole group by prompting facilitators to define variables and coordinate representations (visual model with expressions) to justify claims. Facilitators, in turn, challenged mistaken ideas pressed for further elaborations of mathematical concepts, and the expert leader continued to press for accountability to the discipline. This created opportunities to slow down conversations and connect resources to the uncertainty that had emerged in small groups.

The following are illustrative interchanges from the Phase I whole group.

NC: ...So first, if you start with a 3 by 3, so when I looked at that, you could just divide this. So, this would be  $x$ -squared.  
 L: And why is it  $x$ -squared?  
 NC: If you complete the whole square...(pointing to the additional squares added to figure 3 staircase).  
 L: So, what's  $x$ ?  
 NC:  $X$  would be one dimension, one side. In this case,  $x$  would equal 3.

Moments later, in NC's solution discussion

L: So, I'm curious as to why you subtract the [pointing to quantity  $\frac{1}{2}x^2 - \frac{1}{2}x$ ]...  
 SL: Okay, so I don't get, okay, here I am again. I don't get why that's  $\frac{1}{2}x$ , those three little chunks.  
 NC: Well,  $x$  is 3 squares, right?  
 SL: Ok. Yes, yes.  
 NC: And so,  $\frac{1}{2}x$  would be one square and a half square so if this (points to the squares on diagonal that have been cut in half) equals  $x$ , then this (points to the portion of squares on diagonal that needs to be included to create staircase) equals  $\frac{1}{2}x$ .

In Phase II, pressing questions in small groups like those in Phase I resulted in accountability to one another. Yet, we also found that the reframed task served as a resource to push facilitators toward greater accountability to the discipline. As the excerpt below illustrates, we heard facilitators share solutions, solicit support to co-create solutions and revoice each other's answers until they felt mathematically confident with the ideas.

SJ: Okay, I've got another one,  $\frac{1}{2}$  of  $n^2 + \frac{1}{2}$  of  $n$ . So which one of these models shows half of the square plus half of the number? How can we show that? I know it works. You guys have to help me!  
 ED: That's different; let me think that through.  
 BK: Yea, let me write it down, say it one more time.  
 SJ: Half of  $n^2 +$  half of  $n$   
 ED: But that just goes back to that is the same formula (pointing out that if you simplify  $[n(n+1)]/2$ , the expressions are equivalent).  
 SJ: But if we look at the square (SJ picks up blocks to recreate the 3 x 3 square). Here's the square, right? So, half of that would take it to that.  
 ED: Plus...  
 SJ: Plus half of the 3 (continues rearranging the cubes to add back on and recreate 3<sup>rd</sup> staircase).  
 ED: So, half of the square is that much, and then the half of  $n$  is that much (pointing to subset of the blocks)  
 SJ: Is that what  $b$  is showing? (pointing to the square model in the reframed task)  
 ED: Yea, I think that is what that is showing because that is that half (pointing to the diagonal in figure b).

Here we see how the additional resources of the reframed task supported accountability to the discipline – enabling facilitators to coordinate symbolic and visual representations of the quadratic relationship and move beyond verifying that an expression works to justifying and making sense of expressions coordinated across representations. Small *and* whole group discussions were opportunities to coordinate uncertainty and resources and hold themselves accountable to one another and the discipline. Our analysis documents facilitators unpacking mathematical concepts and reasoning across representations to challenge ideas.

### **ELs played pivotal roles in both phases**

In Phases I and II, ELs drew upon two facilitation practices to support reconciling facilitators' uncertainty to catalyze resources. One method was to *confer authority on the facilitators to address*

*their uncertainty* by inviting facilitators to ask questions, press for justification, and compare reasoning across groups. In the exchange from Phase I, below, the expert leader acknowledged the viability of facilitators' reasoning (a recursive solution) and encouraged them to seek additional resources to address their articulated uncertainty.

GT: Yeah. We're all stuck here.

L: Well, what you might want to do is literally go in pairs and eavesdrop on the other two tables because they've actually got two different approaches. So what I would suggest is going and just listening for a couple of minutes and then...

GT: We have an approach

L: You do! And I think that you might hear some things that would help you build on your approach.

A second practice the ELs utilized to support facilitators was to *reposition resources generated via facilitators' agency but not fully explored*. This practice allowed facilitators to address uncertainty and hold themselves accountable to the discipline by coordinating the closed-form expression, the pattern in their table, and a visual representation of the algebraic ideas of multiplying the dimension of an  $n$  by  $n+1$  rectangle and dividing the figure in half.

In Phase II, the second set of coordinated leader practices emerged, *extending the facilitator's agency* by inviting facilitators to connect representations in the reframed task and *fostering facilitators' rehearsal of partial or incomplete understandings*. For example, when facilitators noticed connections between the visual rectangle model and a table showing Gauss' counting method, the expert leader invited them to explore (extend their agency) and rehearse using the fourth Staircase to articulate the correspondences across the representations.

Although some might suggest that these leader practices are not unique to PD (i.e., similar to classroom practices), we contend that because the goals for learning are geared toward SCK, they are unique from goals for students and, as a result, demand particular sensibilities from ELs that are equally different. We expand upon those differences in the discussion that follows.

## **Discussion**

Our analysis revealed critical insights on facilitators' practices and ELs' practices. One of the necessary conditions for PDE was fostering facilitators' accountability to one another and the discipline when coordinating crucial resources. Persistent accountability amongst facilitators wasn't enough to foster PDE. Facilitators needed access to resources to hold themselves accountable to the discipline. In Phase I, ELs conferred authority and strategically repositioned resources to balance facilitators' uncertainty with resources. With access to these resources, we saw facilitators use a range of socially and mathematically effective practices such as pressing for clarification, mapping correspondences amongst representations, and creating the shared meaning of each other's methods, critical markers of PDE. Similar to Borko et al. (2014), when facilitators took up this kind of social and mathematics work, we saw groups make progress on their uncertainty and productively engage with key concepts and practices in the task.

Our results led us to claim that collective agency to press on uncertainty is a supportive condition to foster PDE. When facilitators collectively asserted their authority to share, add-on, and connect ideas, they could access resources to address their uncertainty. Further, when ELs offered opportunities to slow down conversations via practices of repositioning resources, extending agency, and rehearsing

partial understanding, we saw facilitators reveal their errors, refine reasoning, and revoice solutions until they were confident with solutions. These were also spaces for peers to support accountability by asking pressing questions, pointing out insights provided thus far in an explanation, and connecting methods (Tekkumru-Kisa & Stein, 2017).

ELs' practices of conferring authority and repositioning resources were strategically deployed in ways that either re-centered facilitator authority to disrupt status or leveraged facilitator authority in invitations to examine additional resources and share reasoning. These practices pressed facilitators for accountability to one another, and the discipline, and thus were vital to advancing productivity beyond engagement (Engle & Conant, 2002). To consider PDE as a framework for teacher and facilitator PD, we must understand the ways that teachers' and facilitators' social and professional positioning frames how authority and disciplinary uncertainty afford and constrain productive engagement.

The PDE framework allowed us to understand the resources needed in PD, the role of facilitator agency and accountability, and how ELs can foster PDE. Facilitators' engagement with the reframed task highlighted the need for "just enough" uncertainty to create productive struggle alongside resources to accelerate and focus discussions on critical mathematical ideas for teaching. Strategic leader practices of extending agency and fostering rehearsals of understanding were foundational to facilitators expanding their productive engagement with SCK.

Essential to a PDE lens for PD is to consider how mathematics is framed by specialized content central to mathematics teaching and learning (Ball et al., 2008). While related to what Engle and Conant (2012) call the critical content and practice of the discipline, PDE accountability in PD would mean something slightly different. For facilitators to be accountable to the discipline, they need to leverage and connect multiple mathematical structures, examine correspondences across representations, and coordinate various solutions. The EL practices we identified in our analysis fostered this type of facilitator activity and thus advanced these goals.

## **Conclusion**

We used the PDE construct (Engle & Conant, 2002) to examine the dynamic interactions among social and mathematical aspects of doing mathematics in facilitator PD. The study offered empirical evidence for the framework as an analytic tool for PD research. The PDE framework revealed the nuanced ways that aspiring facilitators' authority and accountability were at play as they accessed resources to address uncertainty. Moreover, we identified how ELs supported PDE via a set of strategic practices related to the four tenets of PDE. We offer insights on strategic facilitation practices of doing mathematics that can inform future studies of facilitation (Krainer et al., 2021).

While we did not use PDE as a design framework in RMLL, our emerging research suggests that "lifting" the tenets of PDE to inform PD designs is a viable lens to guide task design and practices. Our analysis of ELs' practices revealed how their support of facilitators taking up resources in response to their uncertainty and fostering authority in balance with accountability was a means for cultivating SCK. Frameworks to support PD design and formative feedback are essential for advancing content-specific facilitation across PD models (Karsenty, 2021). As a potential design framework, we can imagine how the adaptation of the PDE classroom framework could orient PD designs and advance productive social and mathematical norms while doing mathematics in PD. This

study contributes insights on content-specific PD practices beneficial to a wide range of PD models where teachers engage in mathematics.

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