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# Handmade diagrams – learners doing math by using gestures

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*It is a well-investigated fact that human interaction implies a huge range of expressive modes. Unsurprisingly young learners although do so when doing mathematics together. In the last decade, more and more research has been concerned with this so-called multimodality in mathematical situations. Diverse modes and their interrelations were under investigation, like gestures, speech, or written expressions, sometimes actions. A systematic description of potentially mathematically used gestures by young learners as a part of this mix of modes is still missing. The paper is theoretically framed by an interactionist-semiotic approach wherein mathematics is seen as a social activity of using diagrams. The subject-specific role of gestures is to be clarified: Do gestures play a constitutive role in and for mathematical interactions, literally as a handicraft to work diagrammatically? The exemplary analysis reveals different functions of gestures in the diagrammatic work of the learners.*

*Keywords: Functions and forms of gestures, subject-specific gestures, diagrammatic reasoning.*

## Introduction

Observing young learners while doing mathematics together reveals the use of different expressions: Speaking, writing, using a tool (digital or analogue), gesturing or using facial expressions are perceivable as a kind of interwoven mix of modes. Theoretically, this mix of modes is often framed by the concept of *multimodality* (Arzarello, 2006; Radford, 2009). Radford (2009) states, “[...] that mathematical cognition is not only mediated by written symbols, but that it is also mediated, in a *genuine sense*, by actions, gestures, and other types of signs.” (p. 112). This *genuine sense* of multimodality, and also of gestures for mathematics learning is often seen as given in advance (Huth, 2018, p. 219). A theoretically supported rationale for this assumption is missing, or at least not described in existing approaches to gestures in mathematics education. The present paper opens up a theoretically grounded view on gestures as signs in mathematical interactions by dint of the theoretical concept of *diagrams* and *diagrammatic reasoning* after C. S. Peirce. *Diagrammatic reasoning* in this sense includes the *construction of a diagram*, the *observation of relations* in that diagram and the *formulation of general conclusions*. It potentially leads to mathematical insights and further developed diagrams (Peirce, 1931, CP 1.54). The use of diagrams, which can be of various forms (Schreiber, 2010), is still perceptible at a very early state in the mathematics learning process. The question arises, if the use of modes in mathematical interactions of young learners and their use of diagrams are interwoven as well. For this purpose, the paper will first consider multimodality in mathematics learning, then offer a definition of gestures in mathematical interaction, and describes gesture as a sign after Peirce. Finally, the diagrammatic theory is used to focus the subject specific-role of gestures in the analysis. It should be investigated, whether gestures *represent* or rather (co-)generate learners’ diagrammatic reasoning by the following question: *Do gestures play a constitutive role in and for mathematical learner interactions, literally as a handicraft to work diagrammatically?*

## **Multimodality: A systemic feature of language – and of mathematics learning?**

Radford (2013) claims, that thinking is deeply interwoven with bodily acting in the material world. In this sense, thinking in general, especially mathematical thinking, is *shaped and performed* by using different expressive modes. In line with these ideas, Arzarello (2006) advocates his theory of the *semiotic bundle* to describe a bunch of signs which are used by participants in mathematical interaction. This concept unites e.g., speech as a conventionalized, rule-based semiotic system, and also gestures as a more non-conventionalized mode. It intends to break down the rigid boundaries of considering exclusively semiotic systems in mathematical situations. Krause (2016), in line with Azzarello (2006), emphasizes the representativeness of gestures in mathematical interactions. She investigates mathematical occupations of upper school pupils and describes a “multimodal sign” (Krause, 2016, p. 49) of speech, gestures and inscriptions. The representative role of gestures is grounded in the view, that mathematical objects are not directly accessible because of their alleged abstractedness (p. 16) – a different perspective than it is supposed in the present paper. With regard to the concept of multimodality, Radford (2013) and Krause (2016) refer on the psychological theory of *sensuous* respectively *embodied cognition*. It emphasizes a mind-body-embeddedness in thinking, generating and experiencing the world. *Sensuous cognition* breaks down with a dualistic view of mind (internal and external). Body and mind are no longer separated, but related parts of human cognition.

In linguistic approaches, multimodality is grounded in the *system* of spoken language itself. Fricke (2012) proves a *systemic relevance of gesture* in spoken German language. In sentence constructions, gestures can be integrated in speech to fulfil syntactical positions (so-called *code-integration*, p. 75). In addition, gesture reveals features of language in their structural use, like e.g., *recursion*<sup>1</sup> (so-called *code-manifestation*). Some utterances show aspects of content exclusively coded in gesture. Sometimes, gesture is even obligatory e.g., for using some deictic expressions (p. 74). In addition, gestures can lead to conventionalized forms in interaction, using the same gestures for the same meant recurrently. *Multimodality* in this sense is seen as a kind of characteristic of the language system. This idea of a fundamental *grammar of speech and gesture* underlines the constitutive role of gestures for using language in human interaction. In psychological approaches McNeill (2005) admittedly assumes an “‘unbreakable bond’ of speech and gesture” (p. 24) and coined the idea of an integrative language system. However, he also states a language-gesture dialectic, where gestures can never be part of grammar in his view. In line with Fricke (2012), the present paper locates multimodality in the language-systemic sense, whereas gestures and spoken language construct this system.

### **Gestures: Definition, and its significance in interaction**

To meet the plenty of manifold definitions of gestures, Andr n (2010) summarizes two main perspectives: The first includes nearly every body movement to be a gesture: movements of hands, arms, head, mimic or facial expressions, gaze, action, etc. In the second perspective, the gesture definition is more narrowed (p. 11). It is leaned on Kendon (2004) who investigates how body movements are *interpreted* by interlocuters in interaction and due to the situational context (pp. 14–

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<sup>1</sup>With a finite number of elements and well-defined rules of using these elements one can generate an infinite set of language constructions (Chomsky, 1983, quoted after Fricke, 2012, p. 120).

15). He basically describes gestures as “visible actions” (p. 7) and with “features of manifest deliberate expressiveness” (p. 15). The gesturer is ascribed a “voluntary control” of his gestures which are “being done for the purpose of expression” (p. 15). Goldin-Meadow (2003) differentiates gestures from *functional acts at objects or persons* (p. 8), e.g., adjusting the glasses. In her view, such actions are not seen as gestures, even though they can be done parallelly to speech. She defines gestures based on two criteria: (1) Gestures are hand motions uttered while the *communicative act of speaking*, and (2) are *no functional act* on persons or objects (p. 8). The criteria appear to be mutually exclusive and obviously strictly separates actions from gestures. It can be questioned what is meant by *a functional act on an object or person* and if there could be rather such acts which *includes* a communicative act. A simple example is the (maybe wordless) placement of a chair to broaden an ongoing discussion group while a delayed person enters the room. Can’t we assume here a *communicative act*, although a *functional act* is performed on the chair? The placement is *readable* as a communicative act by the participants, e.g., as an invitation to participate in the discussion, the *where* and *how*. These interpretations are based on a culturally grounded commonly shared background and the actual situation in which the participants *act with* and *to* each other mutually and interactively. Thus, the *existence* of a communicative act attributed to an action or a gesture is rather the *result* of an interpretation and not independently given, or ascribed from an outer view. In mathematics education Sabena (2008) also focusses a separative definition of gestures while doing mathematic: She claims, they “*are not a significative part of any other action (i.e. writing, using a tool, ...)*.” (p. 21). As a distinguishing feature, she constates, that actions are intentional and goal-oriented (p. 35). Gestures in Sabena’s sense obviously cannot be *manifest deliberate* like Kendon (2004) claims. They are to be seen strictly differently than writing or using a tool. However, Arzarello (2006) speaks of gestures that can be *fixed in written signs* in an ongoing mathematical occupation (p. 291). Harrison (2018) emphasizes the communalities of actions and gestures. He claims, that gestures are performed according to the material world and that some actions show comparable structures with gestures in their co-expressiveness with speech. A person sitting next to another person and performs a description of a direction may draw gesturally a kind of map on his or her thigh. The thigh is used as a paper to draw gesturally the map on. It is interpreted as a *part of that gesture* which in turn depends on this thigh-paper-map. With this in mind, it seems less useful to strictly distinguish gestures from actions or body touches. Their interwovenness is also reconstructed by Vogel and Huth (2020). For mathematical learning they found an intersecting action-gesture-use in *chronology*, the *semantic attribution of meaning*, and in *function* (pp. 241–243). Billion (2021) also takes a look at gestures and their intertwining with actions in mathematical situations to reconstruct diagrammatic interpretations of learners. This paper adopts Kendon's (2004) definition of gesture and draws on Harrison's (2018) view of gestures and actions: Gestures are those movements, which are ascribed the “features of manifest deliberate expressiveness” (p. 15) by the interlocutors. Material-use or exclusive body parts are no exclusive criterion. This definition can be linked to the considered interactionist-semiotic perspective on mathematics learning, where the interpretation of the participants is of core interest.

### **Mathematics learning: Diagrammatic reasoning in interaction**

To focus the role of gestures in mathematical interactions of learners, the nature of mathematics learning has to be described in line with an interactionist-semiotic perspective. Krummheuer and

Brandt (2001) claim, that interactions in peers and with an experienced opposite constitute mathematical learning. From a semiotic view, the use of diagrams is the core of doing mathematics (Peirce, 1931). Both approaches combined, mathematical interactions provide a space for experiencing “a social practice with, on, about, and through diagrams” (Dörfler, 2006, p. 105). Accordingly, mathematics learning depends tremendously on being and interacting with others about, on and with diagrams to increase one’s own proficiency in acting mathematically. Learners gain a kind of autonomy in using mathematical signs and argumentations and develop from the role of participant to an autonomous actor (Krummheuer & Brandt, 2001, p. 20–21). In the ongoing process of negotiation, the interlocutors form together the content of interaction and commonly coordinated interpretations. These interpretations can be used as a kind of template to contrast and adopt one’s own insights. The common coordination tends to offer stabilization, and that is, what Schreiber (2010, p. 59) calls the *framing of the situation* in line with Peirce’ *ground of signs* (1932, CP 2.228). Consequently, Krummheuer and Brandt (2001) claim a social responsibility for utterances in interactions, even if one interlocutor produces its perceivable form. It emerges from the social. Utterances in multimodal designed mathematical interactions of learners can be described as *signs* of different kind with a triadic structure (Peirce, 1932): They consist of a *representamen*, an *interpretant* and an *object*. If a gesture is perceived as a *sign*, it can be described as a sign. It has a perceivable outer form – its *representamen*. The gesture sign evokes an interpretation in the mind of the sign reader – the *interpretant* – related to the probably referenced *object* of the sign (Schreiber, 2010, p. 32). The sign interpretation of the sign reader is shaped by his/her *ground* of the sign which can be named as the activated *framing* in the above-explained interactionist view. It includes theories, habits and experiences as an interpretation background. This concept of *framing* assigns a contextual, cultural and social dimension to the interpretation of signs. The interpretant of the sign is describable as the *effect of the sign* in the mind of the sign reader. It can be uttered again, and used as a new representamen. An endless process of semiosis emerges. In the Peircean semiotic, the concept of *diagrammatic reasoning* and the use of *diagrams* is central according to mathematical occupations. This approach shifts the view from supposedly abstract mathematical objects to mathematics as using signs and diagrams that are materially perceptible and manipulable by hands (Dörfler, 2006). Diagrams are rule-based fixations which show particular relations to each other and can be of different kind: written, created on a screen or even developed by material arrangements (Schreiber, 2010, p. 27). The relations and rules of the diagram are not given in advance, but are based on social negotiation (p. 41). They can be seen as a part of the ground of the sign or diagram interpretation.

### **Empirical example: Gesture use of learners while doing math**

In the following extract of analysis the main research question is: *Do gestures play a constitutive role in and for mathematical learner interactions, literally as a handicraft to work diagrammatically?* Two second graders, Maya and Dennis, ought to find all possible permutations of three elements, presented by plastic animal figures (white tiger, brown tiger, elephant). A huge amount of paper cards with the animal’s faces are available for documentation. An accompanying person B offers the mathematical problem and the material. Maya and Dennis put in total 7 rows of paper cards (one of them twice) in front of them. They are finally sure ‘*we found all of them*’. For the analysis, the situation was videotaped and transcribed: First, an interaction analysis (Krummheuer & Brandt, 2001,

p. 90) is applied. Based on this, a semiotic analysis (Schreiber, 2010) is conducted with the multimodal adopted *semiotic process-cards* (SPC) to map graphically the chronological sign process (Huth, 2018). Every utterance is pictured by two sign triads, one for speech and one for gesture respectively with R (representamen), I (interpretant) and O (object). The gesture-speech triads are connected via a common interpretant. Simultaneous utterances are mapped with a and b in one row.

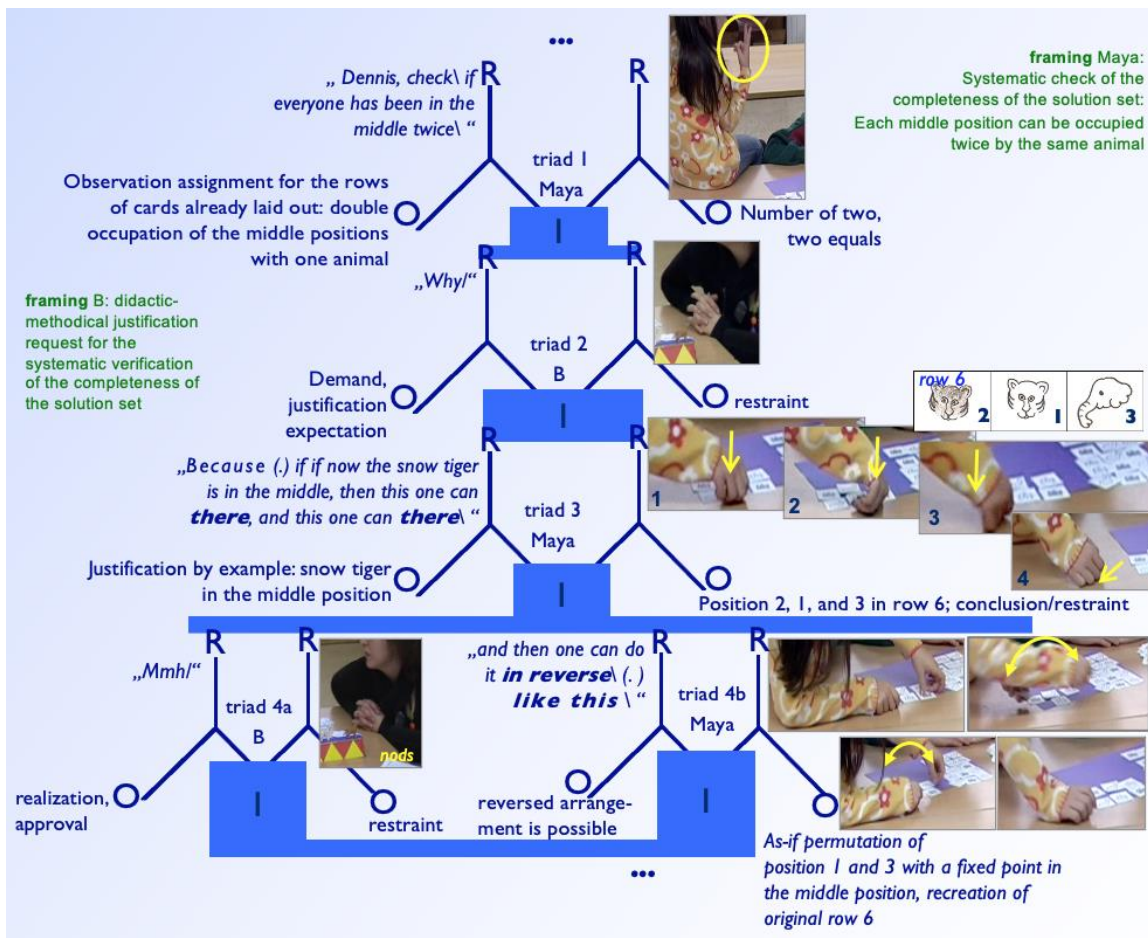
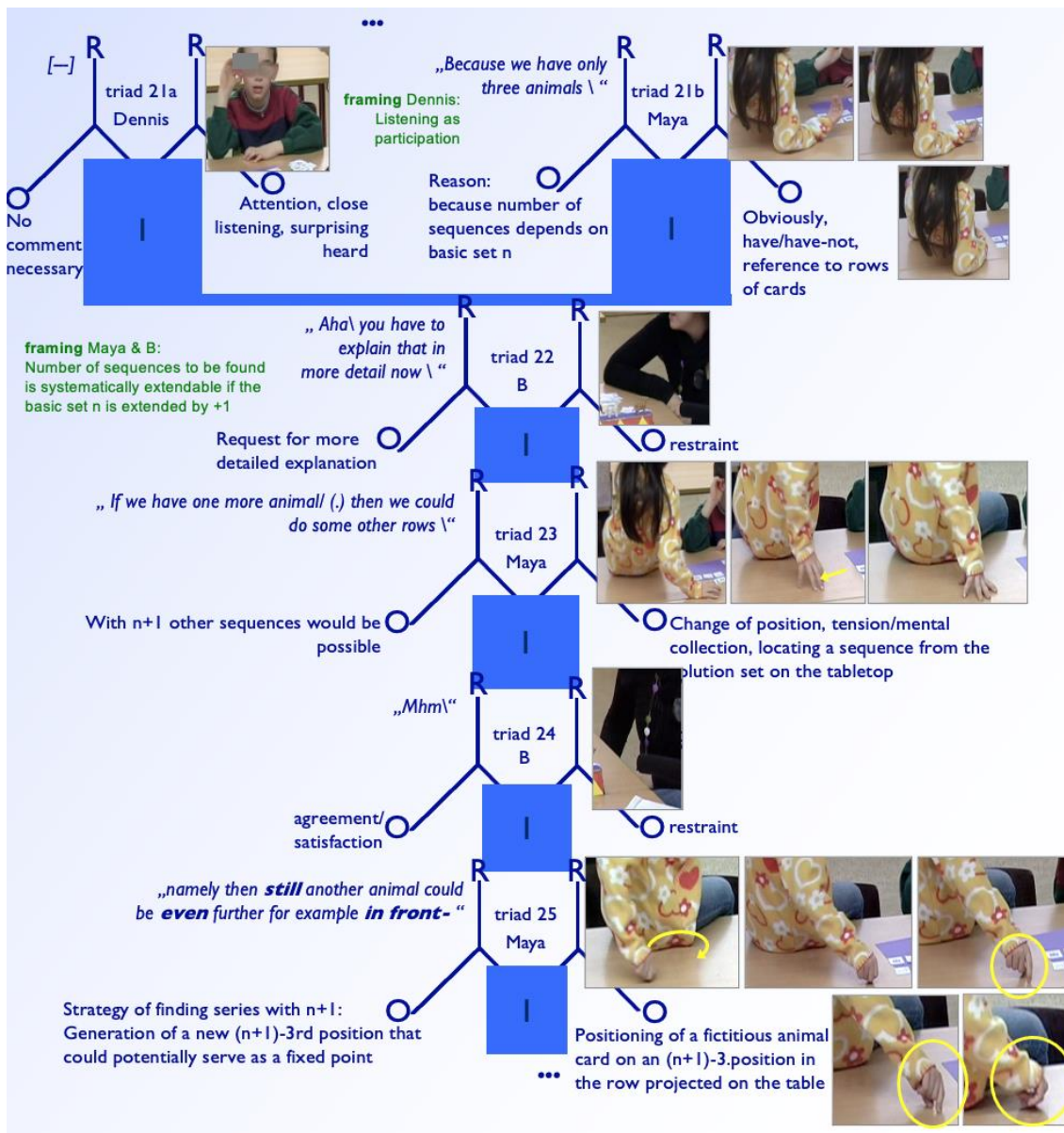


Figure 1: SPC, excerpt 1, Maya's observation instruction to Dennis

Maya starts with an observation order that refers to a first idea of a recognized generalization across all card rows: Dennis should check whether each animal has already been twice in the middle position (triad 1). Maya emphasizes gesturally the number with two similar fingers. She performs this gesture in relation to her speech in front of herself, not to the diagram of card rows, and refers to the mathematical idea of fixing points. In the SPC (Figure 1), it comes to the fore, how strongly gestures and actions are interconnected in the learners' diagrammatic reasoning: After a question of B (triad 2) the explanation of Maya is performed on the diagram of paper cards in both modes (triad 3 & 4b). The fixing point is emphasized gesturally by pressing on the middle position, and in action by leaving this position unaffected. The interchangeable positions are quickly typed or briefly swapped and returned. As activated framings we can assume a systematic review of the card rows based on a mathematical idea of fixed points. B's framing is additionally didactically overdyed. Figure 2 shows a later excerpt of the same SPC. Maya generates a new or more developed diagram mainly in her gestures (triad 23 & 25) after B asked why there are no more rows to find. Maya initially justifies her

answer in the quantity of available animals. Then she uses the generated diagram of all rows to expand  $n$  to  $n+1$ , enriched with her idea of fixed points.



**Figure 2: SPC, excerpt 2, Maya’s gestural diagram construction and manipulation, rows out of  $n+1$**

Maya extracts gesturally one row out of the card-diagram to generate an example of a row  $(n+1)$ . A new diagram next to the initial diagram is generated. In a Peircean sense, in her gesture she *creates a new diagram* and *generalizes the fixing point idea* to see, if it holds for all comparable diagrams. Maya gesturally outlines the contour of the new row  $(n+1)$ , starting with an arched sliding gesture from the last to the first position over the table. Then she establishes a new first position in the outlined row, by using a kind of pantomiming action-like gesture with which she lays down exactly an animal card on the new front position (triad 25). With her speech, she indicates how to use the initial rows with  $n$  elements to generate rows with  $n+1$  by referring to the concrete animal. Maya relates both diagrams to explain how to generate the second out of the first. Her gestures are used as mathematical signs to build a further developed diagram and to manipulate this gestural diagram again by gesture.

Dennis seems to honor Maya's ideas in a special way and marks it as important due to his ear-pinning listening gesture (triad 21a).

## Conclusions

The example shows, that gestures play a constitutive role in and for the mathematical interactions of learners. They can be literally used as a handicraft to work diagrammatically on initially generated diagrams (Dörfler, 2006; Peirce, 1931). With Fricke's (2012) theory of a systemic relevance of gestures for language, one can conclude, that gestures in the Peircean sense have a comparable systemic relevance for acting mathematically: In the learner's diagrammatic reasoning, gestures are an *integral part of mathematical constructions*, like here the idea of fixed pints (mathematical *code-integration*). In addition, perceivable in the diagrammatic reasoning, the commonly negotiated use of *gesturally generated new mathematical diagrams* is observable (mathematical *code-manifestation*). In the shown example, the performed gestures forms range from an explication of speech (triad 1), to gestures, which create links between parts of the diagram (triad 3), until gestures, that are closely related to actions and the material world (triad 3 & 4b). The central mathematical idea is that of *fixing points* in permutations, which is first explained on *one row*, and then *extended* for the gesturally constructed new diagram. In line with Vogel and Huth (2020) gestures and actions are closely interrelated: Both modes are used in a *subject-specific sequence* to expand the generalization of a mathematical idea to  $n+1$ . Thus, with regard to the used gestures' functions, one can reconstruct a subject-specificity. They (1) *structure the mathematical interaction* (triad 21a & 21b), (2) which are *used to clarify the framing* (triad 1, 3 & 4b), (3) which *are used to present a mathematical idea* (triad 1), (4) which *manipulate the diagram* (triad 3, 4b, 23 & 25), and, (5) which are *used diagrammatically* to a general idea (triad 25). Considering that gesture mode is flexible due to its functions and forms, these descriptions should not be taken exclusive. In mathematics education research the focus should be shifted on such gesture functions in interaction to apply them to a larger database e.g., in different ages of learners. From this, suitable concepts can be developed to consider and foster the gesture use in mathematics classroom as a constitutive mode for *handmade diagrams* of learners and teachers.

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