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Persuasive moments, and interaction between authoritative discourse and internally persuasive discourse when using GeoGebra

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The purpose of this paper is to explore students' discourse when working in small groups on mathematical problems using GeoGebra. Specifically, the interest lies in to what extent the discourses are internally persuasive and to what extent they are alien to the students themselves. The data analysed are drawn from an upper-secondary class of students with histories of low attainment, focusing on functions and the Cartesian connection between algebra and geometry. Discourses of visual appearance were more present than academic mathematical discourse, and discourses of school as compliance counteracted students' appropriation of mathematical discourses into their own internally persuasive discourses.

Keywords: Classroom communication, discourse analysis, inquiry-based learning, dialogism, dynamic geometry software.

Introduction

Mathematics teachers and educators want students to integrate mathematical ways of thinking and communicating into their own thinking and communicating. According to sociocultural theories, the main goals of learning consist in expanding the students' action- and meaning potentials (e. g. Wells, 1999, p. 48), and other researchers have also argued for the motivating satisfaction that people derive from being able to "do something one could not do before" (Papert, 1980, p. 74). Unfortunately, for many students, mathematical ways of thinking and communicating stay alien to a large extent. Mathematics is only ever thought of as the words of authority, to be imitated in order to satisfy the requirements of the teacher, and ultimately, the school system. Many students thus rarely use mathematics studied in school in order to think or communicate about anything except school tasks.

The affordances offered by dynamic geometry software to visually perceive representations of mathematical objects, including the covariation of variables, have the potential to facilitate student's experiences of being able to do something one could not do before. Yet, there is a lot to learn about how students interact with such software in the classroom and the different types of learning made possible by different didactical designs. I therefore explore dynamic geometry problem solving discourses in an upper-secondary mathematics classroom of students with histories of low attainment. Here *discourses* refer to sequences of utterances.

Learning and authoritative and internally persuasive discourse

When we communicate, we are always responding to, and making use of context, which includes social and physical settings. What we assume to be our common ground, our shared assumptions about the world and the situation we are in and what it is we are trying to achieve, is crucial. When a group of people frequently interact in some sphere of shared activity, they develop patterns and types of utterances that are relatively stable. Bakhtin refers to these types as *speech genres* (1986, p. 60).

The goal of learning mathematics, from a dialogical perspective, is that students expand their discursive repertoires to include *mathematical discourse*—the historically established ways of communicating that competent users of mathematics employ (e.g., Sfard 2008). This constitutes the speech genre of mathematical discourse. But not all talk in mathematics class is directly related to mathematics. An important explicit theme as well as background assumption of communication in classrooms are the demands that the school and the teacher make on students. This I refer to as *schoolwork discourse* – utterances that refer to, or seem interpretable only in the context of the school as an institution that makes demands on students to finish certain work to some standard.

I relate learning to Bakhtin’s concepts of *authoritative discourse* and *internally persuasive discourse*. The former is a type of discourse that demands acceptance, and derives its power from social authority, “independent of any power it might have to persuade us internally” (Bakhtin, 1981, pp. 110–111). For example, when mathematics functions as prescriptive rules to be followed, without justifications that are convincing to students, it is an authoritative discourse. On the other hand, a discourse is internally persuasive when it becomes tightly interwoven with “one’s own word” (Bakhtin, 1981, p. 345). It is discourse that enters into an interaction and struggle with other internally persuasive discourses, which are all the other “available verbal and ideological points of view, approaches, directions and values” (Bakhtin, 1981, p. 346). It connects with and has an effect on our own discourses, being partly assimilated, partly modified, and always subject to our own creative intentions. In other words, it expands our discursive repertoires.

While a discourse can be internally persuasive without any observable indicators, some types of behaviour would imply that discourse is internally persuasive: when students explicitly make, test and modify mathematical conjectures themselves, they show that the mathematical discourse is interwoven with their own discourse and that mathematics interacts with their other internally persuasive discourses. In contrast, when students apply rules for calculation, without making sense of the rules themselves, after which they ask the teacher “is this right?”, the teacher’s (or the textbook) discourse has not become internally persuasive, it is only authoritative and remains alien to the students. The research question guiding this study is: How do everyday, mathematical and schoolwork discourses intertwine and interact in the problem solving discourses and to what extent do mathematical discourses become internally persuasive for students?

Method

This paper builds on my longitudinal case study of the classroom. In prior papers I described whole class discussions on contextual (paper-based) tasks (Gíslason, 2019), and a dynamic geometry task (Gíslason, 2021), while here the focus is on students in interaction with peers, while working on tasks.

The setting of the study is an upper-secondary school classroom in which most of the students have a history of low achievement in mathematics. The teacher of the class did not rely on textbooks available on the market, but rather found, translated, and adapted tasks from various sources, and made working with dynamic geometric software (GeoGebra) a centrepiece of classroom work.

The data analysed in this paper is drawn from two lessons, all involving the solving of tasks that are intended to make students aware of the Cartesian connection between algebra and geometry, more precisely on concepts of functional dependency and the interpretations of graphs. I recorded pairs of

students selected at random, with a hand-held video camera that followed my focus of attention. My role was generally passive, but I was not invisible and the students sometimes addressed me, both to chat and to ask about mathematical tasks. All verbal utterances were transcribed verbatim in Icelandic and finally excerpts chosen to be presented in this paper were translated to English.

In analysing the data, I followed a dialogical approach. This means that I interpreted communicative actions (such as an utterance, gesture, or an input to a computer program) in context, as responses to what was said and what happened before and as initiations to further responses. The interpretations are also informed by my experience as a mathematics teacher and researcher and I began the exploration with an a priori theoretical distinction in mind, that of discourse being either closer to *everyday discourse* or closer to *mathematical discourse*. In the iterative process of analysis, it became clear that these often were intertwined, and the third major category became apparent, that of *schoolwork discourse*. Finally, I noticed that sometimes students seemed to be convinced by themselves that they had found an answer to a question, while at other times they sought external confirmations from an authority. I took this difference to correspond well to Bakhtin's notions of internally persuasive discourse for the former and authoritative discourse for the latter.

Analyses of episodes

In the following I present two sequences of dialogue, selected to illustrate the three main types of discourse, how they intertwine, and function as internally persuasive and authoritative discourse. The students have all been given unique pseudonyms.

Whether to believe one's eyes: everyday speech and alienated mathematics (episode 1)

Ragna and Drífa have created a straight line using the line-tool in addition to the line that is modelled with the parameters m and b (which may be changed by moving sliders). The intention of the teacher was that students would use the sliders to change the line in accordance with task-directions. The first question asked the students to make it horizontal and go through a fixed point (that they had chosen freely at (4,2)). In this sequence the students address me, as I am recording them, treating me as an authority on the mathematics. Underlining indicates vocal emphasis.

- 1 Ragna: So you want this to go [Addressing me, the researcher.]
- 2 Drífa: through? [Ragna moves the slider for m back and forth, the line changes slope.]
- 3 Researcher: We want this line to go through the point, and be ... and be horizontal.
- 4 Drífa: Okay. [The line now approximately going trough P but it is not horizontal yet.]
- 5 Ragna: It isn't horizontal.
- 6 Researcher: Then you must change it so that it will become horizontal.
- 7 Drífa: You have to move this one. [Drífa points on the slider for b (y-intercept).]
- 8 Ragna: Oh, okaay. [Ragna moves the slider for m so that the line goes trough P, and then she moves the slider for b which moves the line in parallel off the point. This way does not work.]
- 9 Ragna: "No can do". [In English.] [Ragna now adjusts the slope, until $m=0$, and the line is horizontal, but does not go through P.]
- 10 Drífa: Yes! Yes like this. And then you move, no. [Ragna increases and decreases the slope (m), back and forth. Drífa points her finger to the point P.]

- 11 Drífa: I think you should do ...
- 12 Ragna: Aargh. Difficult to own a mac. [Tinkers with the slope until the line is horizontal.]
- 13 Drífa: Woah. [Swiftly moves the line, so that it goes through the point P.]
- 14 Ragna: What is up!?! [A phrase used to express surprise or joy.] [They open the note where they have copied the task-questions.]
- 15 Drífa: Tada! ... And then m gives zero and b gives two. Doesn't it?
- 16 Ragna: Yeah [They start typing in the note: $m=0$] Okay
- 17 Drífa: Okay. Isn't that right?
- 18 Ragna: Is it correct? Please tell.
- 19 Researcher: Is this a horizontal line through the point P?
- 20 Drífa: Yes.
- 21 Ragna: It lies!
- 22 Researcher: Yes.
- 23 Ragna: Totally horizontal.
- 24 Researcher: So why are you asking me?

At the beginning of episode 1, the students show that they want to comply with what the authority wants. In turn 1, they seem to assume that I, the researcher, is in the same authoritative position as the teacher and that they are responding to the perceived demand of the schooling situation. I respond with the “we” pronoun, as is common for teachers, possibly to try to frame the task as shared, something to be achieved together. I take this to be an example of schoolwork discourse, as the goal of “finishing the job” is an ubiquitous assumption of schoolwork.

In episode 1, students expressed joy when they managed to adjust the sliders to get a horizontal line (turns 13–15). Their first attempt was to move the slider for the variable b (constant term), until the line coincided with the fixed point, and then change the value of m (slope) to make the line horizontal (turns 4–9). They claim that this is impossible, perhaps perceiving that the “center of rotation” of the line is not in the fixed point. They then try the other way around; change m to make the line horizontal and then change the constant term to translate it (turns 10–14). In turn 15 Drífa reads the parameter values, $m = 0$ and $b = 2$, from the algebra window. By doing so, and noting “the answer”, they show awareness that the mathematical object has both an algebraic representation and a graphical representation. They both express joy that they have found the solution (turns 16–17). However, as far as can be discerned in their talk and interaction with GeoGebra, they relied solely on slider manipulation and visual appearance, without any use of meaningful links between the algebraic and the graphical representations. For example, they did not mention that the slope should be zero to get a line parallel to the x -axis, nor did they express indications of recognising this after the fact. They achieved their goal using mainly everyday speech, more or less bypassing mathematical vocabulary and reasoning based on properties. Having found an answer, they were not fully persuaded by the visual appearance, and rather than linking their work to mathematical concepts themselves, they asked me, in turn 18, as an authority, to confirm the answer. In turn 19 and 24 I strongly imply that they *should trust* their own eyes, which can be interpreted as validating their visual, trial-and-error approach, rather than challenging them to explain their reasoning. It is possible that their experience with mathematics tells them *not to trust* their senses, and it is indeed important to reason on the basis

of properties and not only from appearance. In summary, Drífa and Ragna did not make much use of mathematical discourse and it was not present in their internally persuasive discourses. Their everyday discourses were up to the task, and they produced a solution. At the same time they did not *fully* trust that they had an answer that would satisfy the teacher. Perhaps they had some awareness that appearance can be misleading, and therefore they sought additional confirmation for themselves through an authority.

Internally persuasive mathematical discourse (episode 2)

One main goal of the class was to get students to appropriate the language of variables. An experience of covariation can be made possible by creating a variable (represented by a slider) and linking that variable to a screen object, functionally dependent on the variable. The task text was as follows:

Draw the following in GeoGebra:

- a) Make a square that can be enlarged and shrunk with a slider.
- b) Add a new slider that moves the square horizontally.
- c) Add another slider that moves the square vertically.

The link between a variable and a screen object is not given in the above task, unlike the task in episode 1. It is expected to be created by the student. The variable will create an interactive visual effect, closely linked to mathematical properties.

The teacher assisted students in constructing a dynamic square with vertices (a, a) , $(a, -a)$, $(-a, a)$ and $(-a, -a)$. In the following episode, two girls, Lilja and Anna, talk to each other and with the teacher, working on the second question, trying to create a slider that can move the square horizontally on the screen. In the first utterance Lilja suggests a modification, making it possible to move the vertex of the square via a slider determining a variable called b .

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|----|----------|--|
| 1 | Lilja: | Plus x times b. [Might mean “add one unit, b times” to the x-coordinate, although a more streamlined way is to say “add b to the x coordinate”.] |
| 2 | Teacher: | Plus x times b? [Neither affirming nor rejecting, opening for further elaboration. He either does not follow or does not want to make the interpretation for the student.] |
| 3 | Lilja: | No [Shakes head, looks at the teacher]. |
| 4 | Anna: | No... oh ... I can't remember which is x and which is y. |
| 5 | Teacher: | Okay the first number is always x and the second number is y. |
| 6 | Anna: | Okay should I then do aaah [Frustration, both hands waving]. |
| 7 | Lilja: | We just want the x you know. [Referring to her knowledge that horizontal movement is described by a change in the x-coordinate.] |
| 8 | Anna: | Yeah the x is here, I am at the x you know. [Referring to the first coordinate as “x”.] |
| 9 | Teacher: | Yes. |
| 10 | Anna: | Okay, what should I do just ... plus? |
| 11 | Teacher: | Yes, yes what. |
| 12 | Lilja: | After the brackets. [Points toward the screen of Anna.] |
| 13 | Anna: | After the brackets? |

- 14 Teacher: Na then you add both to the x coordinate and the y coordinate if you do that. [The teacher knows that $(a, a) + b$ in GeoGebra results in $(a + b, a + b)$. It's unclear that this has an impact on the following turn.]
- 15 Lilja: Then not, you should do before the second number ... hooo [Breaths in, throws head back, opens arms, visibly excited.] Before the second number do plus b! [Smiles, increased voice volume and much higher pitch.]
- 16 Teacher: Okay that's y, then you move it to the y. [The teacher seems to interpret the suggestion as to write something equivalent to $(a, b + a)$.]
- 17 Lilja: No, that, before the first number. [She seems to sense what the teacher meant and the need to make it clear that she means $(b + a, a)$ or equivalently, as she tried to express in turn 15, $(a + b, a)$.]
- 18 Teacher: Okay by the first number.
- 19 Anna: But why plus b?
- 20 Lilja: Because, because when.
- 21 Anna: But there is an a there you know.
- 22 Teacher: Yes, but yes but it...
- 23 Lilja: I got it, I got it! [Visibly excited and joyful.]
- 24 Teacher: Okay, show me.
- 25 Lilja: Wait, wait.
- 26 Teacher: You did a plus b.
- 27 Anna: I did something wrong ... I first want to see that she can do it right, then I'll trust you ... first learn to do this plus.
- 28 Lilja: Gurrll gurrll, gurrll, gurrll, gurrll. Look ... oh ... gurrll, gurrll! [Outburst of joy, smiling and using a higher pitch and volume. Lilja now modifies her coordinates, her suggestion finally implemented as she has meant it, in the software.]
- 29 Anna: Okay, uhm, how did you do it?
- 30 Lilja: Look, gets bigger and smaller. Just do plus b after the second number.
- 31 Anna: Plus b?
- 32 Lilja: You know.
- 33 Anna: Yes a plus b.
- 34 Lilja: Just a plus, ... there ... minus you do also plus a, no, plus b. That's always the first number ... plus b. Now you won't flunk this class! ! [This is accompanied by pointing by her finger on the screen of Anna. She is directing Anna to type, first $a + b$ in the first coordinate place and then $-a + b$ in the first coordinate place of the next vertex. Then she gives a generalisation: always the first number (implied: of the first coordinate-place) plus b.]

Lilja now has a square that can be moved via the slider for variable b . Her square consist of the vertices $(a + b, a)$, $(-a + b, a)$, $(-a + b, -a)$ and $(a + b, -a)$. In episode 2 the students do not immediately solve the problem, which indicates that the mathematical symbolic language of variables and coordinates (as represented in the software) was not initially a part of their internally persuasive discourses. In turn 15 Lilja expresses her excitement in having grasped the nature of the connection between the symbolic slider-controlled variable and the visual behaviour of the screen object. She has not yet implemented her idea, which only happens in turn 28 when she modifies her coordinates

and sees the results, verifying her solution. It is as if (this aspect of) symbolic algebraic mathematical discourse suddenly makes sense to her, using the Cartesian link to her own intent, incorporating it into her internally persuasive discourse. At first the teacher does not follow her “before the second number, do plus b” (turn 15), an everyday type of utterance, describing spatial arrangement. Lilja wants to replace (a, a) with $(a + b, a)$ using the variable b to control a horizontal movement of the point. She uses everyday discourse to orient to the positions of symbols on the screen, “after the brackets”, “before the first number”, and the screen object “gets bigger and smaller”, yet mathematical discourse is internally persuasive for her and is evident in her input to GeoGebra.

Lilja’s partner, Anna, seems not to understand Lilja’s description, and she seems to be more or less stuck at trying to imitate (Lilja’s) authoritative discourse, grasping for step by step instructions. In turns 6, 8, 10, and 13 she seems to be trying to follow instructions (from Lilja and the teacher) as to what she should type into the software, without consideration of meaning. The mathematical symbolic system seems alien to her, it is only authoritative discourse that does not touch her own internally persuasive discourse. In turn 19, she asks “why plus b”, which I see as her attempt to bring the authoritative discourse into contact with her own internally persuasive discourse. She wants the words that she has used to give commands to the computer to have meaning for her. Because what she has typed doesn’t work, she expresses doubts as to whether Lilja has really “got it” (turn 27). Lilja describes her solution to Anna only on a syntactical level in everyday language (what symbols to type in and where) but never addresses her why-questions. Instead she tries to encourage Anna that she will not “flunk this class” (turn 34), reminding us that we are in school, talking in a voice from the schoolwork genre. Both Lilja and Anna are concerned to pass the course, but in this episode the mathematical content became internally persuasive only to Lilja.

In contrast with Drífa and Ragna in episode 1, Lilja does not need a confirmation from an authority. She is convinced that she has grasped the symbolic language and the link between that language and the visual representation. She communicates to the computer through the input text: “ $P = (a + b, a)$ ” and experiences directly an expansion of her action potential. Her outburst of joy *preceded* her actual typing in of the command – it was not a response to seeing it work out (perhaps luckily, through trial and error, as was the case in episode 1). Afterwards, she was quick to also add a functional variable for vertical translation, generalising the method for translation of points in the coordinate system.

Conclusion

In the first episode the students were occupied with the task as a piece of schoolwork to be finished. While they expressed pleasure of having a result (achieved by trial and error), they made no explicit connection between the visual result and mathematical concepts, and also requested confirmation of their answer from an authority. Their satisfaction was due to having finished a job, not with having made mathematical discourse their own. Thus, the schoolwork discourse can be interpreted as being in this case not conducive to learning, or worse, actively working against learning.

In the second episode one student, Lilja, suddenly grasped the relationship between the mathematical symbolism and the visual representation. While she confirmed her answer visually, she was convinced that she knew what needed to be done *before* she gave any input to the software. I interpret her satisfaction as stemming from having made mathematical discourse internally persuasive. Her

partner, Anna, did not show any indication of having made the discourse internally persuasive. The schoolwork discourse's assumption that students should finish the tasks set by the teacher frustrates Anna as she was concerned that she might fail the course. Lilja tried to help Anna finishing the task, but described only a step-by-step recipe, without reference to meaning. Lilja, therefore, was not hurt by the schoolwork discourse in this case, while Anna's learning suffered.

The two tasks worked on provide different opportunities to use mathematical discourse to achieve goals. In episode 1, the students manipulated ready made sliders to observe covariation of parameters and a visual representation. This did not make the *mathematical* relationship the center of attention. In episode 2 the students were expected to create sliders for variables and then define the mathematical objects themselves, using the variables. This required students to use mathematical discourse as a semantic tool, which means incorporating it into internally persuasive discourse. One of the students did so, while the other did not.

In their problem solving, students drew on everyday discourse to describe visual elements, both the visual representations of geometric objects and strings of symbols. They also assumed the everyday practice that to be persuaded of something, it is both necessary and sufficient to empirically check its appearance. Mathematical discourse was present in their discourses to a much lesser extent, and in a way that focused more on the surface (the syntax), rather than the conceptual meaning. Schoolwork discourse was always in the background if not explicitly apparent in talk about failing the course. Schoolwork discourse seems push students to imitate authoritative discourse, that is, using mathematical discourse without having made it their own. In other words, schoolwork discourse does not bring authoritative mathematical discourse into contact with internally persuasive discourse. Rather, it functions to keep mathematics only authoritative, and alien to the students themselves.

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