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# Developing and validating survey instruments for assessing beliefs and motivation in mathematics

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*In this article, we describe the development and validation of two short questionnaires for assessing beliefs and motivation in mathematics that are (1) linked to Inquiry-Based Learning (IBL) and (2) short, concise and easy to administer quickly within the parameters of regular classroom instruction.*

*Keywords: Mathematical beliefs, motivation, IBL, scale validation.*

## Introduction

Research has shown that Inquiry-Based Learning (IBL) has the potential to improve students learning of and motivation for mathematics and science, as well as change students' beliefs about the subjects (Bruder & Prescott, 2013). However, many teachers report challenges with implementing IBL in their own classrooms (Engeln et al., 2013). The context of this study is a project called SUM, which aims to explore how IBL could be integrated in teachers' day-to-day mathematics teaching. One of the key research tasks of the SUM-project is to document and explain students' long-term development in terms of motivation for and beliefs about mathematics. In order to achieve this, the project needs survey instruments that can be quickly and easily administered to a large number of diverse-aged students in a classroom context. Several questionnaires have previously been developed to explore students' beliefs and motivation, but a review of the literature disclosed multiple reasons why there is a need for new instruments, particularly with respect to beliefs. First, we identified no instruments that have been conceptually linked to IBL. Second, most existing survey instruments were long with many items. Brevity is important when instruments are administered in classroom contexts, as the longer the inventory, the less students may take care completing it (Entwistle & McCune, 2004). Finally, earlier scales used to measure beliefs have mostly not been systematically validated, and according to Diego-Mantecon et al. (2019), there is still a need for a more thorough methodological analysis of these variables, particularly in the case of beliefs, using advanced psychometric tools.

In this paper, we describe how the aforementioned issues have been addressed through developing and validating a questionnaire assessing students' beliefs about mathematics, and adapting and validating existing scales measuring motivation. This resulted in the *Mathematical Beliefs Questionnaire* (MBQ) and the *Mathematical Motivation Questionnaire* (MMQ). Our main research questions are: a) What is the factor structure of the MBQ and the MMQ, and how can we achieve robust scales measuring the underlying constructs with as few items as possible? and b) Do the six proposed scales in the MBQ and MMQ have acceptable internal reliability and discriminant validity?

## **Theoretical foundation and initial development of instruments**

### **Mathematical beliefs – the MBQ**

The affective domain is characterized by many overlapping and related terms such as beliefs, attitudes, values and emotions – that are not used in a uniform way (Philipp, 2007). As such, research on beliefs is complicated by numerous factors, including both methodological and conceptual issues. However, empirical work on an unclear concept can invite dialogue and serve to clarify it (Skott, 2015). Furthermore, although beliefs are not easily defined, reviews of the research literature have identified certain consistent features. In general, beliefs can be described as subjective knowledge (Philipp, 2007), and according to Skott (2015), beliefs are usually value-laden mental constructs that are relatively stable results of prior experiences. Beliefs are also organized in complex and quasi-logically connected clusters (Philipp, 2007). Although it is possible to group these clusters in a variety of manners, beliefs about mathematics have commonly been grouped according to beliefs about the nature of mathematics, beliefs about mathematics learning, and beliefs about mathematics teaching (Ernest, 1989; Beswick, 2012). Each of the three belief clusters can be mapped on a continuum that ranges from formal beliefs at one end to informal beliefs at the other (Collier, 1972). Formal beliefs about the nature of mathematics views mathematics as a hierarchically structured subject of procedures, rules, algorithms and formulas. Knowing mathematics is equivalent to recall and application of these procedures, and teaching mathematics is envisioned as a teacher-centered activity in which the teacher presents and demonstrates the use of said procedures. Informal beliefs about the nature of mathematics identifies it as a creative subject of investigative processes tied to problem solving, proof, reasoning, communication, connections and making sense of the world around us. Knowing mathematics is displayed through active and successful engagement in these processes, and teaching mathematics is envisioned as a student-centered activity in which the teacher facilitates students' active knowledge construction through exploratory and open-ended processes.

In the development of the MBQ, we sought to relate each of the aforementioned three belief clusters to key characteristics of IBL. First, IBL is premised on an epistemological belief of mathematics as a dynamic and creative endeavor, where the focus is primarily on the process of mathematics and not necessarily on the product (Ernest, 1989; Artigue & Blomhøj, 2013). A belief cluster we may call Mathematics as a creative subject (Creative) points to the view of mathematics as a creative and human endeavor, where the purpose is to solve interesting problems and understand the world around us using creative and original thought. Second, a belief cluster we may call Mathematics instruction should be inquiry based (Inquiry) points to the view that teaching should be based on inquiry related activities, where students are provided with opportunities to explore and try out their own ideas (Artigue & Blomhøj, 2013). Third, a cluster we may call Mathematics is not an innate ability (Adventitious) rests on the premise that education should be for all, and, more importantly, that all students have the capacity for learning and becoming proficient in mathematics through effort and dedication (Kloosterman, 2002).

A literature review confirmed that there were no existing previously validated instruments for measuring students' epistemological beliefs about mathematics along these three domains. Therefore, a panel of researchers in the SUM-project generated a pool of items that captured aspects of each of

the three aforementioned belief clusters. This item pool was generated based on an approach consisting of: a) identifying key characteristics of the theoretical definitions, and b) selecting and translating the most relevant items from existing literature (particularly Tatto, 2013; Collier, 1972) as well as supplementing this by formulating some new items. This process resulted in 17 items – significantly longer than the target length.

### **Motivation – the MMQ**

Like beliefs, motivation has long been the focus of educational research, and there exists a number of theoretical approaches and associated concepts – many of them appearing related or partially overlapping. In the context of mathematics education, Schukajlow, Rakoczy and Pekrun (2017) identified expectancy-value theories and self-determination theory as especially important theoretical approaches, and the conceptualization of motivation in the MMQ encompass features of both.

The choice of motivation constructs in the MMQ was mainly influenced by the expectancy-value theory of Eccles et al. (Eccles & Wigfield, 2002), whereby students' motivation for engaging in an activity is thought to depend on both their expectations for success and how they value the activity. Specifically, our review of the motivation literature led to the conceptualization of students' motivation for mathematics through three constructs: Perceived competence, which relates to their perception of their mathematical ability and expectation for success, Intrinsic value, which can be linked to intrinsic motivation in self-determination theory (Ryan & Deci, 2000) and relates to the enjoyment they may experience when engaging in mathematics or the subjective interest the students' have in mathematics, and Utility value, which relates to how engaging in mathematics may be useful to the student, for instance with respect to pursuing future career goals (Eccles & Wigfield, 2002). These constructs are not only widely supported in the motivation literature, they can also be related to key characteristics of IBL. For instance, Fielding-wells et al. (2017) demonstrates the use of expectancy-value theory for developing insights into how engaging with IBL affects student's competence-related beliefs, sense of autonomy, and intrinsic valuing of mathematics, and calls for further research on how IBL may foster a perception of the utility of mathematics.

A number of scales for measuring aspects of student motivation exists, and the MMQ combines translated and adapted versions of items from the IMI - Intrinsic Motivation Inventory (Center for Self-Determination Theory) in order to address Perceived competence and Intrinsic value, as well as adapted items from the Usefulness scale in the Conceptions of mathematics inventory (CMI, retrieved from (Star & Hoffmann, 2005)) to address Utility value. This resulted in 20 items – again significantly longer than the target length.

### **Method**

Many of the items in the item pool originated from well-known instruments, but in the MBQ and MMQ they are combined and adapted to our specific situation (IBL, mathematics, students aged 10-19). Furthermore, our aim is to achieve robust scales measuring the underlying constructs with as few items as possible. Therefore, we have conducted a validation study in order to explore the factor structure of these items and establish internal reliability and discriminant validity.

## **Participants and procedure**

Data for the validation study was collected in the fall of 2019, and the participants were 377 students in the age range 10-19 (48% male) from 10 different schools in the SUM-project. Note that this constitutes a convenience sample where the students are nested in classes, and as one might expect a greater similarity between students in the same class, this is a limitation of the present study.

The participants responded to an electronic questionnaire consisting of 49 items, all answered along a 5-point Likert scale, as well as five background questions. The 49 items were separated into three sections: 1) 17 items from the MBQ, 2) 20 items from the MMQ, and 3) 12 items assessing students' past experiences in mathematics classroom. Before analyzing the data, missing values were handled by interpreting them as a "neither agree or disagree"-response and imputing the Likert-scale midpoint value 3. To assess the theorized scales in both the MBQ and the MMQ, the sample was randomly split in half and an EFA (exploratory factor analysis) was conducted on the first half (196 students) and a CFA (confirmatory factor analysis) was conducted on the second half (181 students). Small sample sizes can result in unstable factors, but ca. 200 is considered fair – especially if communalities and factor loadings are high.

## **Results**

### **Exploratory factor analysis (EFA)**

In line with Carpenter's (2018) summary of recommendations for non-normally distributed variables and possibly correlated factors, we conducted separate exploratory factor analyses using Principal Axis Factoring and oblique rotation to identify the latent factor structure of the MBQ and MMQ. In both cases the factorability of the items was deemed satisfactory, as the Kaiser-Meyer-Olkin measures were .74 for the MBQ and .93 for the MMQ, and the Bartlett's Tests of Sphericity were significant.

For the MBQ, five items were excluded to improve the interpretability due to low communalities and high cross-loading. The number of factors retained was based on several criteria. First, the Kaiser "eigenvalue greater than one" rule led to an initial extraction of three factors and the scree plot indicated the extraction of three or four factors. Second, a parallel analysis (100 datasets; CI 95%) indicated the extraction of three factors. Finally, the following criteria were also taken into consideration: the factors should be homogenous and theoretically meaningful, Cronbach alpha has to be sufficiently high ( $>.65$ ), each factor should contain enough items ( $>2$ ) with high loadings ( $>.50$ ) and the total amount of variance explained must be sufficient (approx. 50%) (Carpenter, 2018).

A three factor-solution was preferred in the end because of its simple theoretical interpretation, sufficient number of items per factor, and general parsimony, along with sufficient total variance explained. Factor one (eigenvalue = 3.63, 24.70% of variance) was Mathematics as a creative subject (Creative). The three items retained all reflect seeing mathematics as a subject that is creative, humanistic and related to the world we live in (e.g.: C3 - "Mathematics is first and foremost about understanding the world around us"). Factor two (eigenvalue = 2.10, 14.13% of variance) was Mathematics instruction should be inquiry based (Inquiry). The three items reflect a belief that mathematics teaching and instruction should allow students to explore and understand mathematics in settings similar to how mathematicians work (e.g., I2 - "In mathematics class we should first and

foremost experiment and try out our own ideas”). Factor three (eigenvalue = 1.95, 12.18% of variance) was Mathematics is not an innate ability (Adventitious). The three items retained in this scale reflect the belief that mathematical ability is the result of hard work and not innate (e.g.: A3 – “Everyone can become proficient in mathematics”). The three factors explained 51.01% of the total variance, and the factor loadings are summarized in Table 1.

**Table 1: Factors and loadings for the MBQ and MMQ. Note: Factor loadings < .5 are suppressed**

MBQ	Item Factor	C1	C2	C3	I1	I2	I3	A1	A2	A3				
	1	.65	.62	.59										
2				.73	.64	.53								
3							.74	.68	.63					
MMQ	Item Factor	IV1	IV2	IV3	IV4	IV5	UT2	UT6	UT7	UT8	PC1	PC4	PC5	PC6
	1	.84	.74	.84	.72	.95								
	2						.78	.76	.64	.63				
	3										.80	.84	.81	.87

For the MMQ, inspection of the communalities and the rotated pattern matrix led to the identification of 7 items with either low communalities (<.4) or relatively high cross-loadings. After a closer inspection of the wording of these items, we decided that it would be theoretically meaningful to remove them from the analysis. Most of the 7 removed items belonged to the proposed Utility scale, and referenced mathematics being useful to the student in the present (e.g. UT1 – “Mathematics is a useful subject in school”).

Repeating the EFA with the 13 remaining items indicated the extraction of three factors (see Table 1), together explaining 73.63 % of the total variance. Factor one (eigenvalue = 6.51, 50.07 % of variance) was Intrinsic value. The five items all reflect the notion that mathematics is a fun, enjoyable and interesting activity (for example, IV1 “I enjoy doing mathematics”). Factor two (eigenvalue = 1.87, 14.36 % of variance) was called Utility for future life. This is a slight narrowing of the original theorized scale, as the four items retained reflect the view that learning mathematics in school will be advantageous for the student in the future (e.g. UT6 “Mastering mathematics will help me get a job later in life”). Finally, the third factor (eigenvalue = 1.20, 9.20 % of variance) was called Perceived competence. The four items belonging to this factor all reflect a perception of oneself as being competent in the field of mathematics (for example PC4 “I have good mathematical skills”).

### Confirmatory factor analysis

To assess the stability of the three-factor solutions, we conducted confirmatory factor analyses using the AMOS software. In both cases, the chi-square value in relation to the degrees of freedom gave p-values larger than .05 (MBQ:  $\chi^2(24) = 28.61$  and  $p = .24$ , MMQ:  $\chi^2(62) = 28.61$  and  $p = .11$ ), indicating that the hypothesized models do not significantly deviate from the observed data. Model fit was further assessed using the goodness-of-fit index (GFI); the comparative fit index (CFI); the root mean square error of approximation (RMSEA); and the standardized root mean square residual (SRMR), with acceptable values near .95, for the two former and  $\leq .05$  for the two latter (Byrne, 2010). The analyses indicated a good model fit in both cases, with fit parameter values GFI = .97, CFI = .98, RMSEA = .03, and SRMR = .05 for the MBQ, and GFI = .94, CFI = .99, RMSEA = .035, and SRMR = .035 for the MMQ.

### Internal reliability and discriminant validity

We assessed internal reliability using Cronbach's alpha and the validation dataset. For the MMQ, the alpha values for Intrinsic value, Perceived competence and Utility for future life were .93, .92 and .76 respectively, indicating that the motivation scales were remarkably robust to our translation, adaptation and shortening. For the MBQ, the alpha values for Adventitious, Inquiry and Creative were .76, .68 and .70 respectively. Although alpha values greater than .70 is often referred to as a rule of thumb, this appears to be a rather arbitrary cutoff and lower values may be considered satisfactory for scales with few items. As such, we conclude that all six subconstructs have acceptable internal reliability.

**Table 2: Scale intercorrelations. Coefficients marked with \*\* are significant at the 0.01 level.**

	1	2	3	4	5	6
1 Creative						
2 Inquiry	.34**					
3 Adventitious	.20**	.08				
4 Intrinsic value	.56**	.04	.47**			
5 Perceived competence	.36**	.08	.37**	.66**		
6 Utility for future life	.37**	.13	.45**	.53**	.32**	

Table 2 shows the bivariate Pearson correlation coefficients between all the subscales. Significant correlations are observed between all scales except for Inquiry, and this is to a large extent expected as the subscales Creative, Adventitious, Intrinsic value, Perceived Competence, and Utility for future life all reflect optimistic beliefs and motivations about mathematics and mathematics learning, as well as mathematics being important for one's own future life.

Given the rather high correlation between some of the subscales, we assessed the discriminant validity using the Fornell and Larcker criterium (1981). The Average Variance Extracted (AVE) was calculated for each of the six subscales, and compared to the highest squared correlation with any other subscale within MBQ and MMQ respectively. For all the subscales, the AVE exceeded the highest squared intercorrelation, indicating that the subscales may be reliably distinguished.

## Concluding remarks

Our analyses show that we can achieve robust measures of subscales pertaining to mathematical beliefs (Mathematics as a creative subject, Mathematics instruction should be inquiry based, Mathematics is not an innate ability) and motivation for mathematics (Intrinsic value, Perceived competence, Utility for future life) with relatively few items. Furthermore, these six proposed scales all have acceptable internal reliability and discriminant validity.

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