



HAL
open science

Pre-service teachers learn to analyse students' problem-solving strategies with cartoon vignettes

Marita Friesen, Alyssa Knox

► To cite this version:

Marita Friesen, Alyssa Knox. Pre-service teachers learn to analyse students' problem-solving strategies with cartoon vignettes. Twelfth Congress of the European Society for Research in Mathematics Education (CERME12), Feb 2022, Bozen-Bolzano, Italy. hal-03745715

HAL Id: hal-03745715

<https://hal.science/hal-03745715v1>

Submitted on 4 Aug 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Pre-service teachers learn to analyse students' problem-solving strategies with cartoon vignettes

Marita Friesen¹ and Alyssa Knox¹

¹University of Education Freiburg, Germany; marita.friesen@ph-freiburg.de, alyssa.knox@stud.ph-freiburg.de

The flexible use of different problem-solving strategies is key for processing mathematical problems successfully. The question of how future teachers can be supported to learn about such strategies and their use in the mathematics classroom is consequently of high relevance. In a one-semester university course, we provided a unit on problem solving focusing on the use of primary-school students' problem-solving strategies. To support the pre-service teachers' learning, we designed two types of cartoon vignettes: short cartoons, each illustrating a different problem-solving strategy (e.g., work backwards) and more complex cartoons providing the opportunity to analyse how students make use of different strategies and/or struggle while solving a non-routine problem. Our evaluation showed that the participants perceived the cartoons as valuable learning opportunities. Learning to analyse students' problem-solving strategies could be supported but remained a challenging task.

Keywords: Vignettes, cartoons, problem-solving strategies, teachers' competence of analysing.

Introduction

Mathematical problem solving is considered a core mathematical activity and it has been incorporated as a key component in mathematics curricula and national standards worldwide (Liljedahl et al., 2016). Related studies have provided evidence that the flexible use of so-called heuristics or problem-solving strategies can enhance problem solving (e.g., Carlson & Bloom, 2005). Various mathematical problem-solving strategies, such as *make a table*, *draw a picture* or *look for a pattern*, can already be introduced for primary school children (Charles et al., 1992) and corresponding instruction achieved a positive impact on students' use of strategies and problem-solving abilities (Verschaffel et al., 1999). However, there are also studies showing that many students do not spontaneously use problem-solving strategies or struggle when applying them to non-routine problems (e.g., Schoenfeld, 1992). It can therefore be assumed that mathematics teachers do not only need professional knowledge about students' problem-solving strategies but must also be able to analyse their students' strategy use in order to provide them with support. With this study, we build on prior research into mathematics teachers' analysing of classroom situations, described as the competence to link classroom observations with relevant professional knowledge in order to make sense of what has been observed (e.g., Friesen & Kuntze, 2020). In the present study, we were particularly interested in the question of how future primary-school teachers can be supported in their professional learning to analyse students' problem-solving strategies. The study was conducted in the framework of the ERASMUS+ project coReflect@maths, aiming at designing and evaluating vignette-based learning material in mathematics teacher education courses with a particular focus on cartoon vignettes (cf. Ivars et al., 2020). Correspondingly, we explored in this study the potential of different types of cartoon-based vignettes for (1) developing pre-service teachers' professional knowledge about various problem-solving strategies and (2) their competence to analyse primary-school students' use of such strategies

in classroom situations. The following sections outline the theoretical background of the study, in particular with respect to mathematical problem-solving strategies and teachers' corresponding competence to analyse classroom situations. We then introduce our vignette-based approach and describe the unit about problem solving in which the pre-service teachers of this study participated.

Mathematical problems and the use of problem-solving strategies

We follow Schoenfeld's (1983) definition of a mathematical problem as an *unfamiliar situation for which an individual does not know how to carry out its solution* since no routine or familiar procedures are (yet) available to him or her. Such problems are often described as non-routine problems in contrast to routine problems, where students can apply familiar or routine solution methods to solve them (Elia et al., 2009). Solving non-routine mathematical problems, in contrast, requires the use of so-called heuristics or problem-solving strategies (Liljedahl et al., 2016). Various problem-solving strategies can already be introduced to primary-school children, such as guess–check–revise, draw a picture, act out the problem, use objects, solve a simpler problem, make a table, look for a pattern, make an organized list, write an equation or work backwards (cf. Charles et al., 1992). A study with primary-school children by Elia et al. (2009) showed that the flexible use of strategies (adapting the strategy to the given task or changing in case the chosen strategy was not successful) led to higher performance in problem solving. Knowing various problem-solving strategies and being able to change or modify a chosen strategy is therefore an important prerequisite for the successful solving of problems. However, the same study also showed that many of the primary-school children struggled with applying different strategies or could not reach a correct answer since they did not have a sufficient understanding of the problem (Elia et al., 2009). These findings highlight in particular the important role of the teacher in supporting the students in understanding the given problem as well as in choosing, applying and reflecting on the use of different problem-solving strategies.

Analysing classroom situations regarding the use of problem-solving strategies

In order to support students in using different problem-solving strategies in a flexible manner, a mathematics teacher must know about different strategies, how students can apply them to problem-solving tasks and where they might struggle. Taken together with the definition of problem solving which is characterised by the fact that the problem-solver cannot simply apply a routine or familiar solution method (e.g., Schoenfeld, 1983), a mathematics teacher's knowledge about *content and students* (KCS, cf. Ball et al., 2008) can be assumed to form an important prerequisite in this context. However, to provide learners with appropriate support, teachers must also be able to *apply their knowledge* in the course of instruction, meaning that they have to link relevant observations in the classroom with their knowledge to make sense of what they have observed in order to make follow-up decisions. Based on the concept of teacher noticing (Sherin et al., 2011), we described such ability as teachers' competence of *analysing classroom situations* (e.g., Friesen & Kuntze, 2020). Numerous studies have shown that teachers' analysing of classroom situations informs their decision-making and is highly relevant for instructional quality and student learning (Sherin et al., 2011). A teacher who is competent in analysing classroom situations regarding the use of problem-solving strategies is consequently able to identify the problem-solving strategies used by his or her students and to

evaluate the strategies' potential for solving a particular task. Such competence is an essential prerequisite for supporting students in choosing and applying problem-solving strategies, for helping students where they struggle and for engaging them in reflection on the use of different strategies. In the following section, we describe the potential of using vignettes for developing and evaluating pre-service teachers' professional growth in this context alongside with the cartoon-based approach we take in the framework of the ERASMUS+ project coReflect@maths (Ivars et al., 2020).

Vignette-based learning and the potential of cartoons for teacher education

Using representations of classroom practice, so-called *vignettes*, in the form of short video clips, teacher-student dialogues or cartoons, has proven to be an effective approach in teacher education and corresponding research (e.g., Buchbinder & Kuntze, 2018). Vignettes provide the opportunity to engage in classroom practice without the pressure to act and can help future teachers to analyse classroom situations and to discuss and reflect on alternative approaches. In this way, vignettes can become both, representations of practice *and* theory: As learning opportunities, vignettes are often used to illustrate particular aspects of quality teaching or where norms are challenged and can therefore support future teachers in applying their theoretical knowledge to teaching practice in a safe context. Vignettes have, therefore, also proved to be an effective method to assess development in teacher education courses (Jeffries & Maeder, 2005). Correspondingly, questions regarding possible designs of vignette-based learning and testing environments have become essential and the potential of cartoon vignettes has gained increased attention. Cartoon vignettes combine numerous advantages ascribed to vignettes in the formats video and text: They allow the systematic, theory-based design and variation of classroom situations where individual characteristics that are important to comprehend a classroom situation can be added easily (Friesen, 2017). Compared to the formats of video and text, cartoon-based vignettes can be equally suitable to assess and foster teachers' competence in analysing classroom situations (Friesen, 2017). The design and evaluation of cartoon-based learning material is a main objective of the ERASMUS+ project coReflect@maths and the present study has been conducted in the framework of this project. The following section describes the organisation and contents of the university course in which the study was conducted as well as the design and use of the cartoon-based learning material.

The university course: organisation and cartoon-based learning material

The present study was conducted in a one-semester university course (April-July 2021) for pre-service teachers studying to teach mathematics at primary schools. The course took place at a University of Education in the south of Germany where primary school comprises the first four years of schooling for students aged between six and ten years. Due to the pandemic situation, the course had to be delivered completely online with meetings organised as video calls and study material provided via the university's online learning platform. The course covered different topics related to the teaching and learning of mathematics at the primary-school level. Problem solving was one unit within the course covering four weeks and three group meetings (3 x 90 minutes). The problem-solving unit contained key topics with special regard to the primary-school context (e.g., what are mathematical non-routine problems, problem solving according to Polya, problem-solving strategies, how can learners be supported). Figure 1 (on the left) shows the organisation and contents of the unit. In order to support the pre-service teachers' learning about students' problem-solving strategies, we

designed two types of cartoon-based vignettes: (1) short cartoons with the aim to develop the participants’ professional knowledge about different problem-solving strategies and (2) more complex cartoons with the aim to develop their competence to analyse primary-school students’ use of such strategies in classroom situations. Figure 1 (on the right) shows one of the short cartoons illustrating the problem-solving strategy “work backwards”. Based on Haering (2016) who describes various problem-solving strategies used by primary-school students, 16 of such short cartoons were designed, each depicting a problem-solving task and how one or several students solve that task using a particular strategy. The other cartoons of that type show strategies such as: guess–check–revise, draw a picture, act out the problem, use objects, solve a simpler problem, make a table, look for a pattern, make an organized list, write an equation, etc. (cf. Charles et al., 1992).

Week I	Week II
online meeting I: introduction of unit; release of online learning material and study plan	online meeting II: questions, group work
Week III	Week IV
self-study time	online meeting III: presentation of tasks, questions, course evaluation
between meetings: self-study time with online material supported by study groups and online forum	

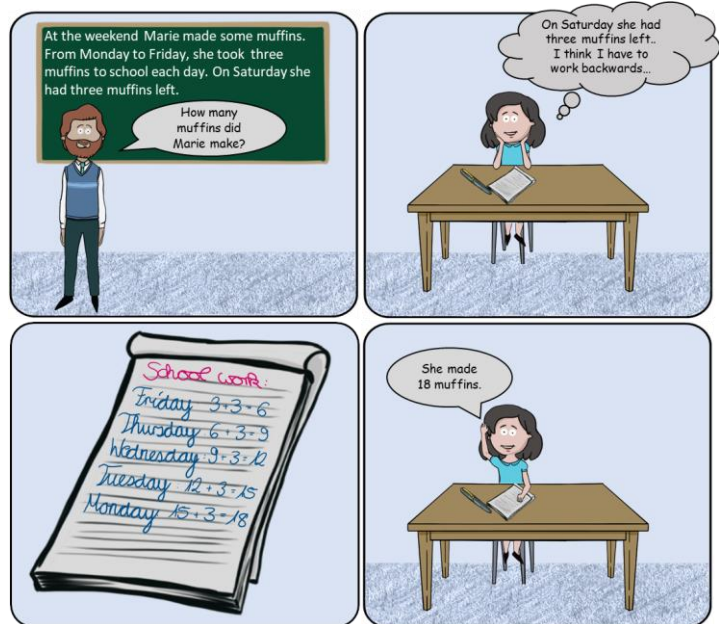


Figure 1: Organisation and contents of the unit on problem solving (left);

Short cartoon vignette illustrating the problem-solving strategy “work backwards” (right);

(cartoon designed by Alyssa Knox based on Haering (2016); characters drawn by Michael Weninger)

During the unit, the pre-service teachers worked on different tasks provided with the vignettes. They explained, e.g., the strategies used in the cartoons and verified their use with the cartoon characters’ solutions or comments. They were also asked what alternative strategies students could have made use of for solving the given task and to compare and reflect on the potential of different strategies.

Figure 2 shows one of the more complex cartoon vignettes. These vignettes were designed based on lesson transcripts by Rasch (2016), representing typical classroom scenarios around primary-school childrens’ solving of non-routine word problems. Four vignettes of that type were used in the unit about problem solving, each showing one or several students who use various strategies while solving

a non-routine task, who struggle in choosing and/or applying a strategy and who are supported by their teacher. The participants of the course were asked to identify the strategies used by the cartoon students and to evaluate their potential for solving the task. They were also asked to make assumptions about reasons for the students' difficulties in solving the task and to verify their answers with the students' use of strategies as shown in written solutions, use of material or comments. Finally, the participants were given the task to evaluate the teacher's support and to think about alternative approaches. For sharing these approaches, they received a cartoon with empty speech bubbles that could be adapted. In this article, we focus on the students' use of problem-solving strategies.

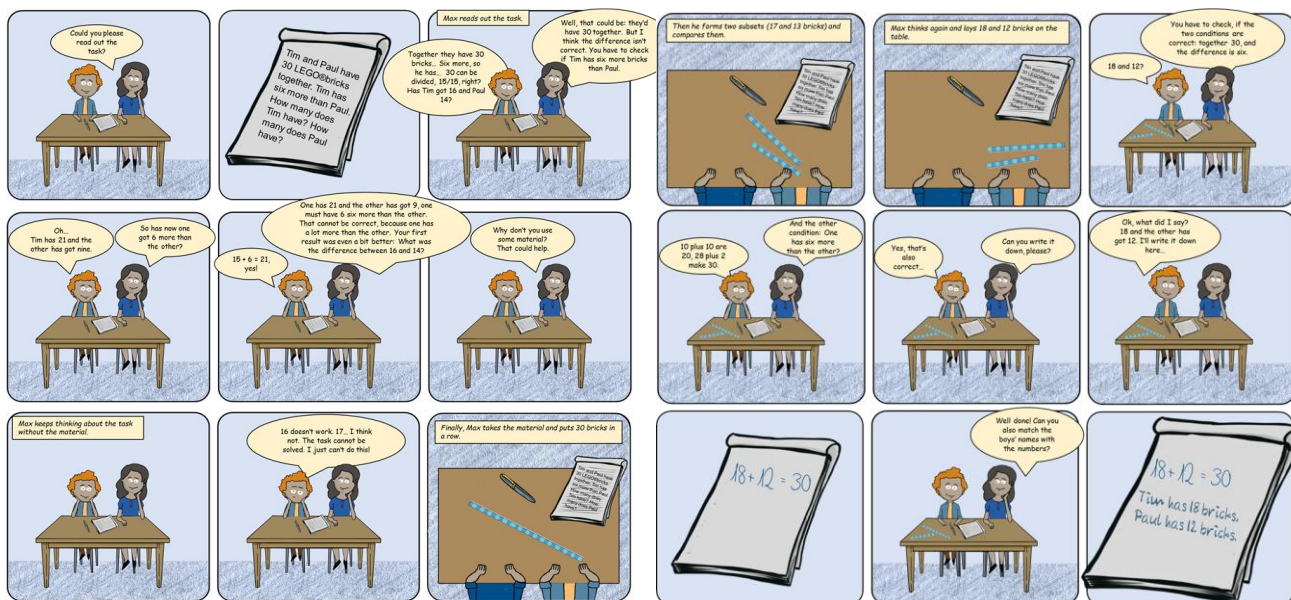


Figure 2: Complex cartoon vignette illustrating different strategies and need for learning support (designed by Alyssa Knox based on Rasch, 2016; cartoon characters drawn by Michael Weninger)

Research interest and research questions

This study aimed at exploring the potential of cartoon-based vignettes for pre-service teachers' professional learning regarding students' use of problem-solving strategies. Accordingly, we addressed the participants' own perceptions regarding their learning with the cartoons and their competence to analyse students' use of problem-solving strategies in classroom situations at the end of the unit. The research questions were the following: (1) How do the participating pre-service teachers perceive the potential of cartoon vignettes in terms of their professional learning? (2) To what extent are the participating pre-service teachers able to identify the strategies used by students when analysing a complex cartoon vignette? (3) Can they suggest alternative strategies?

Sample and methods

The course evaluation took place at the end of the unit on problem-solving after the fourth online meeting (cf. Figure 1). The pre-service teachers received an online questionnaire and were asked to analyse a complex cartoon vignette and to evaluate the cartoon-based learning material in terms of their professional learning. They could use their course material and as much time as they needed. $N=42$ pre-service teachers gave consent to have their data analysed in an anonymised way and

participated in the study. The items for the pre-service teachers' evaluation of the cartoon vignettes in terms of their learning (RQ 1) are displayed in Table 3. Since participants' engagement with vignettes is an essential pre-requisite for analysing them (cf. Friesen, 2017), items 1–3 addressed the pre-service teachers' perceived motivation, authenticity and immersion when working with the cartoons. Items 4–5 evaluated to what extent the participants felt supported by the use of cartoons in their learning about problem solving and items 6–7 evaluated the perceived potential of cartoons for teacher education in a more general way. Figure 2 shows the cartoon used for the course evaluation (RQ 2+3) together with the following task: *In the presented situation from a second grade classroom, Max is working out the solution to a problem-solving task while his teacher is supporting him. Please read through the situation and answer the following questions: What problem-solving strategies does Max use to solve the task? What other problem-solving strategies could he have used to find a solution?*

Data analysis and results

Related to the first research question, Figure 3 displays the results of the participants' evaluation of the cartoon vignettes in terms of their learning. Regarding pre-service teachers' engagement with the cartoon vignettes, they rated their perceived authenticity, motivation and immersion on average as positive. The pre-service teachers perceived the cartoons as supportive for their learning regarding students' different strategies and difficulties in solving problems. They evaluated the use of cartoons in teacher education as helpful and as a good connection to practice.

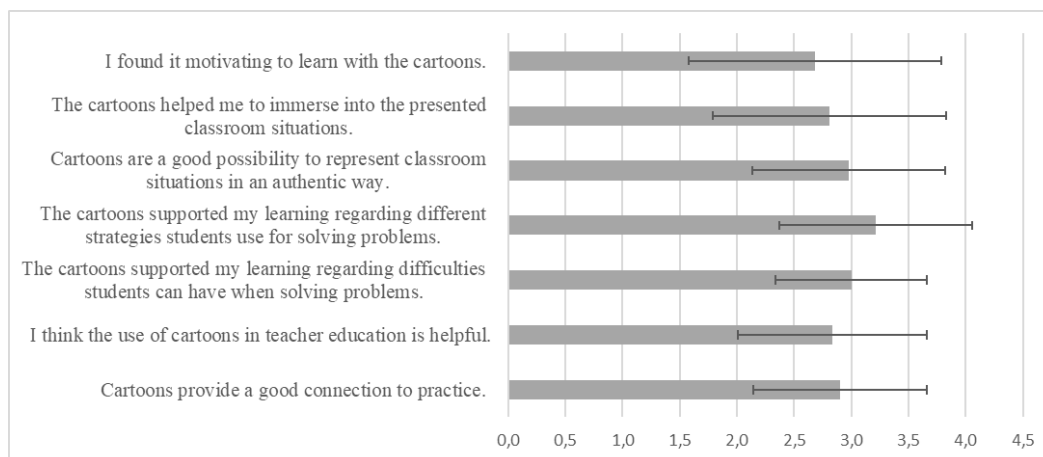


Figure 3: Pre-service teachers' own perceptions of their learning with cartoons (means and standard deviations; 0 = I strongly disagree; 4 = I strongly agree.)

To answer research question 2, an a priori analysis of the cartoon vignette used for the evaluation (see Figure 2) was conducted. In this way, the strategies used by the student in the cartoon vignette (e.g., work backwards, use objects, guess-check-revise) and further potential strategies for solving the given task were identified (e.g., try systematically, make a table). The pre-service teachers' written analyses for the complex cartoon vignette were coded accordingly. The findings show that all of the participants were able to correctly identify at least one strategy used by the student in the complex cartoon; 32 pre-service teachers (76.2%) were able to correctly identify two or three strategies and eight pre-service teachers (19.0%) were able to correctly identify four or five strategies. There were,

however, also participants who identified strategies that were not appropriate (e.g., “write an equation” although the student did not use the equation to solve the problem, but only when writing down the answer). In this context, 21 pre-service teachers (50.0%) described one or two such unsuitable strategies and five participants (11.9%) matched three or four strategies in an incorrect way. Although not explicitly asked for, providing explanations for the choice of the strategy was found to be significantly correlated ($r(40) = .41, p = .008$) with the number of correctly identified strategies. All of the participants were able to suggest between one and four suitable alternative strategies for solving the task in the vignette. However, 27 participants (64.3%) suggested also one or two unsuitable alternatives; 11 participants (26.2%) suggested three or four.

Table 1: Sample answers showing the relation between correctly identified strategies and explanation for the choice of strategy by referring to relevant events in the vignette

<p>(...)“Try systematically” as he goes through examples in order (first 16, then 17, then 18) and comes to a result this way (PST #2)</p>	<p>(...)“Look for a pattern” as he always has to check for two numbers (difference 6 and result 30) (PST #2)</p>	<p>(...) When he works with the objects later, he also tries systematically, because he notices for 17 and 13 that the difference is still too small and moves one brick from the bar with 13 bricks to the bar with 17 bricks. (PST #22)</p>
--	--	---

Summary and discussion

Vignettes provide safe environments for future teachers’ analysing of classroom situations and can support them in connecting classroom observations to theory – an essential prerequisite for making sense of what has been observed when teaching and for making suitable follow-up decisions (Buchbinder & Kuntze, 2018). This study shows that cartoons hold great potential in this context: they can be purposefully designed to represent key aspects of practice and theory on the learning of mathematics while considering different levels of complexity (cf. Friesen, 2017). Although the study has limitations (e.g., no control group or pre-post design), it encourages further research into the use of cartoons in teacher education: The participants perceived the cartoon vignettes as a valuable learning opportunity, not only in the context of problem solving but also for teacher education in general. The findings show that pre-service teachers’ analysing results were more often correct when they explained them by referring to relevant events in the given classroom situation. When working with vignettes, validating observations with specific classroom events should be part of that work and pre-service teachers need to be supported in a systematic way. Whereas numerous studies have shown the importance of teachers’ analysing for instructional quality and student learning (Sherin et al., 2011), this study contributes by showing how pre-service teachers can learn to analyse in a university course on problem solving and how different types of cartoon vignettes can support their learning.

Acknowledgement

The project coReflect@maths (2019-1-DE01-KA203-004947) is co-funded by the Erasmus+ Programme of the European Union. The European Commission’s support for the production of this publication does not constitute an endorsement of the contents, which reflect the views only of the authors, and the Commission cannot be held responsible for any use which may be made of the

information contained therein. We would like to thank our colleagues from the coReflect@maths project for their feedback and collaboration: Ceneida Fernández, Pere Ivars and Salvador Llinares (University of Alicante, Spain), Sebastian Kuntze and Jens Krummenauer (Ludwigsburg University of Education, Germany), Karen Skilling (University of Oxford, UK), Libuše Samková (University of South Bohemia in České Budějovice, Czech Republic), Lulu Healy (King's College London, UK) and Ralf Erens (University of Education Freiburg, Germany).

References

- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389–407. doi.org/10.1177/0022487108324554
- Buchbinder, O., & Kuntze, S. (Eds.) (2018). *Mathematics teachers engaging with representations of practice*. Springer.
- Carlson, M., & Bloom, I. (2005). The cyclic nature of problem solving: An emergent multidimensional problem-solving framework. *Educational Studies in Mathematics*, 58, 45–75. doi.org/10.1007/s10649-005-0808-x
- Charles, R., Lester, F., & O'Daffer, P. (1992). *How to evaluate progress in problem solving*. NCTM.
- Elia, I., van den Heuvel-Panhuizen, M., & Kolovou, A. (2009). Exploring strategy use and strategy flexibility in non-routine problem solving by primary school high achievers in mathematics. *ZDM Mathematics Education*, 41, 605–618. doi.org/10.1007/s11858-009-0184-6
- Friesen, M. (2017). *Teachers' competence of analysing the use of multiple representations in mathematics classroom situations and its assessment in a vignette-based test*. PHB.
- Friesen, M. & Kuntze, S. (2020). The role of professional knowledge for teachers' analysing of classroom situations regarding the use of multiple representations. *Research in Mathematics Education* 22(2), 117–134. doi.org/10.1080/14794802.2019.1710555
- Häring, G. (2016). Problemlösen lernen [Learning to solve problems]. *GS Mathematik* 50, 32–35.
- Ivars, P., Fernández, C., Llinares, S. et al. (2020). A digital tool to support teachers' collaborative reflection on mathematics classroom situations: The coreflect@maths project. *ICERI 2020*.
- Jeffries, C., & Maeder, D.W. (2005). Using vignettes to build and assess teacher understanding of instructional strategies. *The Professional Educator* 27(1&2), 17–28.
- Liljedahl, P., Santos-Trigo, M., Malaspina, U., & Bruder, R. (2016). *Problem solving in mathematics education*. Springer. doi.org/10.1007/978-3-319-40730-2_1
- Rasch, R. (2016). *Textaufgaben für Grundschul Kinder zum Denken und Knobeln* [Word problems for primary school children. Solve mathematical problems - develop strategies.] Klett.
- Schoenfeld, A. H. (1983). The wild, wild, wild, wild, wild world of problem solving: A review of sorts. *For the Learning of Mathematics*, 3, 40–47.

- Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 334–370). MacMillan. doi.org/10.1177/002205741619600202
- Sherin, M. G., Jacobs, V. R., & Philipp, R. A. (2011). *Mathematics teacher noticing. Seeing through teachers' eyes*. Routledge.
- Verschaffel, L., De Corte, E., Lasure, S., Van Vaerenbergh, G., Bogaerts, H., & Ratinckx, E. (1999). Learning to solve mathematical application problems. *Math. Thinking & Learning*, *1*(3), 195–229. doi.org/10.1207/s15327833mtl0103_2