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Socio-mathematical norms regulate whole-class discussion

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This study identifies existing socio-mathematical norms through student and teacher responses in mathematical activity in grade eight in Sweden. The data consist of one video observed research lesson, and the analyses contribute to methodological identification of socio-mathematical norms through student responses and connected teacher activity during dialogues. Through these dialogues, the study identifies how students and the teacher expressed aspects of mathematical knowledge in their responses based on socio-mathematical norms in this classroom.

Keywords: Mathematical reasoning, socio-mathematical norms, student responses.

Introduction

Socio-mathematical norms (SMN) are jointly agreed upon between teachers and students in the mathematical classroom, constituted through participation (Cobb, 2002). SMN are common cognitive and social structures that frame what is communally valued by the classroom community (Cobb, 2002). These are shaped and reflected by cultural traditions that emerge specifically in mathematics classrooms through the configuration of activities. Focus is directed towards what knowledge is rewarded during the learning process (Yackel & Cobb, 1996). What distinguishes a mathematics classroom from another is the structure of classroom norms, not their existence or absence (Yackel et al., 2000). According to Yackel and Cobb (1996), the teacher has a significant role in suggesting SMN, which the students can collectively accept. For example, the teacher can do so based on the type of solution and arguments from students (McClain & Cobb, 2001), which influences the type of mathematical knowledge to be valued in the activity.

Some previous studies have identified limitations where social classroom norms limit the establishment of SMN when the focus is more on how teachers or students should behave democratically in an activity (Fredriksdotter et al., 2021; Kazemi & Stipek, 2001), instead of promoting discussions or arguments for which a mathematical model is best suited for the task (Cobb & Yackel, 1996b). SMN frame both the knowledge and structure for interaction (Cobb & Yackel, 1996a). However, sometimes the teacher will intend to gather short student responses with a character of correct answers. Responses like that contrast with occasions where the student response is more comprehensive, containing conceptual understanding and elegant reasoning (Kilhamn & Skodras, 2018).

Methodologically it is challenging to detect classroom norms. These are often indirect and implicit but somehow obvious to participants through their participation in practice (Cobb, 2002). Different perceptions of existing norms may coexist within the classroom (Cobb & Yackel, 1996b). One way to methodologically detect classroom norms is when teachers and students explicitly discuss expectations (Cobb, 2002; Wester, 2015). Another opportunity to identify SMN occurs through participants' reactions when existing norms suddenly get challenged (Cobb, 2002). These reactions

will be indicators when existing norms are broken. The current study will contribute to identifying SMN through video observation of interactions between students and teachers in a mathematical teaching activity. Instead of methodologically using teachers' intentions to promote SMN (Wester, 2015), this current study elucidates student response through dialogical patterns in the classroom community of practice aiming to identify SMN. We address the following research question: How can student response and connected teacher activity be used to identify socio-mathematical norms (SMN)?

Method

We analysed a whole-class discussion (50 min) to answer this study's research question. To capture students' responses and connected teacher activity, the students were video-observed during a whole-class discussion in a grade 8 classroom in Sweden (one teacher and 25 students). The video observation was transcribed verbatim and analysed by the two authors. We have used an abductive approach during the analysis of empirical data and use of the framework (see Table 1). First, we divided the whole-class discussion into parts based on dialogue patterns. Each pattern consists of the students standing in front of the class discussing their own solutions to a given task (see Figure 1). Students' responses in the discussion, together with the related teacher activity, form the unit of analysis. By studying several dialogue patterns in the whole-class discussion during the same lesson, the opportunity for detecting how SMN directs and influences the mathematical teaching activity regarding the structure of the whole-class discussion increases. According to Kilhamn and Skodras' (2018), framework dialogue patterns are illustrated in the operationalising approach (see Table 1).

In operationalisation, different characters of reasoning are elucidated, and these have been inspired by Hjelte et al. (2020). Reasoning can be characterised as either general or domain-specific for mathematics (Hjelte et al., 2020). General reasoning is like a logical chain attempting to reach a solution, using necessary mathematics regardless of domain. Domain-specific reasoning belongs to a specific mathematical domain. Hjelte et al. (2020) suggest the following mathematical domains in research-based studies: *Spatial reasoning*, *Informal Inferential Reasoning*, *Additive, multiplicative and distributive reasoning*, *Algebraic reasoning*, *Proportional and covariational reasoning*, *Quantitative reasoning*, and *Transformational reasoning*. Different types of reasoning may be connected to each other and form a network of reasoning.

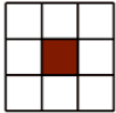
The operationalisation of the framework also focuses on different characters of mathematical knowledge as procedural or conceptual, based on Yackel and Cobb (1996), concerning SMN. Procedural knowledge focuses on solving the already constructed mathematical calculation structures, while conceptual knowledge means that the student knows mathematical concepts and their relations between each other (Yackel & Cobb, 1996). Knowledge is valued as SMN in category LC through the operationalisation procedural, while conceptual understanding is valued as SMN in category HC.

Table 1: Operationalisation of Kilhamn and Skodras’ (2018) framework according to student response and associated SMN

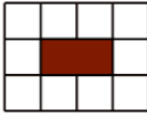
Level of response	Student’s response associated to SMN
F Factual	An accepted answer related to the task or to teacher’s questions. No attempt at descriptive or explanatory response. Ex: Name, identify, repeat, recall known facts or procedures, “correct” answers wanted. (often short answers, including yes/no)
LC Low-level Conceptual	The descriptive response values procedural knowledge (Yackel & Cobb, 1996) containing expressions of step-to-step solutions and may contain general reasoning (Hjelte et al., 2020)
HC High-level Conceptual	The explanatory response values conceptual knowledge (Yackel & Cobb, 1996) and is based on domain-specific reasoning (Hjelte et al., 2020).
E Evaluative	Apply and evaluate domain-specific reasoning (Hjelte et al., 2020) in relation to mathematical content containing in specific task. Different domain-specific reasoning forms a network for explanation.

Ethical procedures of informing participants of their rights to refuse to participate and obtaining verbal and written consent for their voluntary participation were followed using Swedish guidelines for research (Vetenskapsrådet, 2017). Furthermore, the collected data were anonymised and coded to protect participants’ confidentiality, and the researchers had no teaching or grading role for the students involved. In addition, the names of the participants and other sensitive data were removed from the stored data, and these data were secured in a safe locker to avoid unauthorised access.


Eva and Ali put white tiles around flowerbeds as the pictures shows.



Figur 1



Figur 2



Figur 3

3. How many white plates are used in figure n?

4. How long will the flower bed (dark tiles) be if they have 158 white tiles to lay?

Figure 1: Task

Result

Analysis of this whole-class discussion reveals the following lesson sequence of dialogue patterns based on the operationalised framework: F-F-LC-LC-HC-LC-F. Three dialogue patterns are

emphasised (bold) to answer the research question: How can student response and connected teacher activity be used to identify socio-mathematical norms (SMN)?

Dialogue 1 (student's response level LC)

- 66 Teacher: We will stick to problem 3 (How many white plates are needed in figure n?) Namely the formula (point to the formula that the students wrote on the board: $2 * n + 6$) and then I take problem 4 (How long will the discount be if they have 158 white tiles to put?) which was also to find out how many or which figure how many blacks what discount is it that has 158 tiles around it and then we have student A and student B, welcome.
- 67 Student A We have taken 2 times n plus 6 (read out what is written in the formula $2 * n + 6$ on the board for the teacher).
- 68 Teacher. How have you used this since, in task number 4 where what have you done?
- 69 Student B 2 was the difference, after all, any number plus 6 (rename the formula for the teacher).
- 70 Teacher. How have you used this since how have you done to calculate, ie which figure you are looking for?
- 71 Student A We took 2 times 54 plus 6.
- 72 Teacher Hm
- 73 Student A. It will be 158 minus 6 it was the edges and then 152 divided by 2 and it will be 76.

The teacher (line 66) begins by inviting students to the board and asks them to focus on parts 3 and 4 in the task (Figure 1). Through the introduction (line 66), the students explicitly request no specific student response. Still, it is up to the students themselves to interpret what is expected of them when they stand by their solution at the board. Student A responds by reading out the expression for the formula without mentioning the answer to part 3 in the task (line 67), which constitutes a limited student response. The situation that, as a student, sometimes occurs at the board is not new for these grade eight students. Students have support for their interpretation of what is expected of them at the board based on previous similar experiences, which follows prevailing classroom norms for the situation and similar contexts. Pretending that the students' responses do not contain the direct answer to the task may mean that the students have interpreted that the teacher is looking for something other than just the answer to part 3 in the task, of which the indication of the formula becomes their attempt at what can constitute an acceptable response (level F).

The teacher follows up students' short responses by asking a more investigative question (line 68), aimed at part 4 in the task, which the students also did not answer with their short response. This how-to question has the potential for student responses at higher levels than F. Student A's response continues as before by repeating the formula in different ways (line 69 and line 71), although the formula alone still does not answer the teacher's consecutive how-questions (lines 68 and 70). The student's response remains at level F.

Only when the teacher (line 72) does not provide the expected feedback on the given student response (based on the formula), do the students develop their responses to a description at the LC level. Student A describes how they arithmetically worked their way to answer part 4 in the task. LC level because the student in their response expresses different steps in their calculation against the answer. In their arithmetic description, the student also names a geometric representation (the edges of the figure) to support the calculation. Therefore, we cannot answer whether these students are able to

make an explanatory response at the HC level or if the students' interpretations of the norm system prevent them from providing an explanatory response.

In writing, the students show the way to the answer on the board and orally express what they value is most interesting and of the highest mathematical quality (the formula). These responses show how these two students interpret prevailing SMN. Since the teacher is repeatedly not satisfied with the students' responses, the students finally succeed in changing the content in their responses (line 73). Based on its arithmetic calculation, the steps to the answer are described (level LC), and no further teacher questions are then asked.

Dialogue 2 (student response level LC)

80 Teacher: Now there are two solutions left that do not do exactly the same [...] can you come up and tell?

Already in the introduction (line 80) of dialogue 2, the teacher expresses that one should compare two different solutions. This teaching document challenges the accounting students to develop their response since they have different solutions that must be compared in an investigative way.

83 Student C: May I draw?

84 Teacher: Yes

85 Student C: Ok, then we'll see (draw a rectangle and another rectangle inside) if you have, so to speak, the whole figure here, i.e. 158, (draw around the inner rectangle) then you take minus 2 and you get (draw again) you get the cubes. And then you get 156 which is this and that (shows in the drawing) then you divide by 2 so you get 78 and 78 is then this one down as well (shows in the drawing) and since we knew there was one on each side (shows in the drawing) instead of taking it minus 2 you know yes it is the solution (aimed at the teacher).

90 Teacher: (fills in more the four boxes in the middle) these are the discount in the middle and this is what you want to find out so C removes this and that piece (point to the boxes next to the right and left of the discount) to remove them simply and then he has left the top row and the bottom row (points in the rectangle) and then he divides them by two and then he only gets one row (points to the bottom row) but that row is two pieces longer still than the one in the middle so then he takes minus two again. Are you in? Turns to the class and then you did a formula of this (turns to student C).

Contained in the introduction (line 80) of dialogue 2, the teacher expresses that one should compare two different solutions. This teaching document challenges the accounting students to develop their response since they have different solutions that must be investigated. Student C asks if it is ok to draw (line 83). After the teacher's approval to draw, the student enters a dialogue with the teacher using some verbal caution and then tries to describe the chosen calculations based on a drawn picture. Through general reasoning, the student uses, without verbal expression, concepts in the mathematical domains geometry (perimeter) and algebra (Figure n). Based on the student's response, there are possible conditions to end up at the HC or E level. However, expressions in students' responses are not explanatory and contain domain-specific reasoning. Instead, this student's response is a step-by-step description, strengthened through the drawn picture, of the path to the task solution (level LC).

In line 93, the teacher once again verbally expresses the student's description of the solution based on the picture. For the context, the teacher possesses more functional language than the student, and the teacher's revoicing of C's description becomes more accessible to the listening students.

However, in terms of content, the teacher's narrative is on the same level as the student's (level LC) as it intends to explain the path to the solution. As the content of the teacher's responses remain at the LC level, the prevailing SMN in the internship community are confirmed for the context.

Dialogue 3 (student response level HC)

D: accounting student, Ex: students in the class.

- 97 Teacher: [...] can you tell us how that formula came to be?
100 Teacher: Look at what you have done (point to the formula $Y = (1n + 2) * 3$ and student D looks) $1n$ plus 2.
101 Student D: In order to ...
102 Teacher: What is 1 and why 2?
103 Student D: Hm.
104 Teacher: Can you tell others what he did? (turns to the class)?
105 Student E1: 2 is probably the difference between the different ones.
106 Student E2: n is well that in the middle.
107 Student E3: n is well the figure plus ...
108 Student E1: not always.
109 Student E2: Now we'll find out in the middle.
110 Student D: Wait n is it (points to the flowerbed in the middle, the four coloured squares in the rectangle) plus 2 is equal to it (points to the bottom row) and
111 Student E2: Why multiplied by 3?
112 Student E3: Y is the whole set.
113 Teacher: Yes exactly, he has taken the middle plus the edge pieces (points) and so he has called it y . So he puts someone else's letter. One row is (points) and taken it three times, there are three rows, but after doing that you have to remove what is in the middle to get what is around.

In line 97, student D is asked to unpack the formula they have formulated. Since the student cannot answer the question, it is reformulated by the teacher (lines 100 and 102). When the student is still unable to formulate a response (line 103), the teacher (line 104) turns to the whole class and asks them to investigate what is identified in the formula. This teacher's action breaches norms that open the norm system for investigative conversations. This invites interactions from students who have previously only listened and not been involved in the dialogue. Student D and some listening students then formulate a joint understanding of the formula (lines 105–112). As a collective unit, they unpack the formula and express the meaning of the parts in words. Each of the individual students' responses contains short responses and statements. Through joint interaction in the investigative conversation, the short contributions together explain domain-specific reasoning in algebra reasoning and spatial reasoning. In the common dialogue (lines 105–112), there is a knowledge of mathematical content based on spatial reasoning linked to numbers (numerical), which are allowed to meet algebraic reasoning about the variables and constants of the formula to create a common relational understanding of what the formula expresses. Despite students' mathematical knowledge, their verbal response does not contain expressions of reasoning and mathematical ideas. Instead, the students express different statements about how different parts of the formula can be interpreted in their responses. It does not become an open dialogue about which mathematical ideas and mathematical reasoning are behind the various statements. Instead, statements that are not met during dialogue with alternative statements are silently accepted. Therefore, the common student response remains on the HC level. Both the students' knowledge and the teacher's activity – to encourage students into

exploratory conversations – contain the potential for HC and level E. However, there is a lack of support in the SMN for students' responses to be able to reach level E in this situation.

Also, in this episode, the teacher sums up the discussion by re-telling and verbalising the jointly expressed opinions by the students. The teacher thus responds to students' responses at both the LC and HC levels.

Discussion

Through an operationalised framework (Table 1), it becomes possible to categorise students' responses in the dialogue patterns. The level of student response and connected teacher activity is used to identify the prevailing SMN. Based on an analysis of each dialogue pattern, a lesson sequence for the whole lesson is possible. In the current lesson sequence (F-F-LC-LC-HC-LC-F), F and LC levels became the most common student response. This means that prevailing SMN supports dialogue patterns at the F and LC levels through this classroom discussion but lacks support in SMN for both the HC and E levels. However, one dialogue pattern still ends at the HC level during this lesson. This exception happened when the current teacher opened up the prevailing norm system for that moment. After this particular dialogue pattern, the following dialogue patterns once again returned to the normative for F and LC levels. This then saw the identification of the F and LC levels around the mathematic knowledge valued in the activity (McClain & Cobb, 2001).

A complementary methodological way to identify SMN can be done through potential norm breaks. According to Cobb (2002), another way to identify SMN are manifested through participants' reactions when existing norms are challenged. In dialogue 1, the teacher's action (line 72) challenges students' perceptions of their existing valued response. Instead of the students becoming frustrated at expressing the algebraic formula once again (level F), the teacher's activity leads to the students developing a response containing a description of the calculation steps (level LC). In dialogue 2, the student explicitly asks the teacher for permission to break existing SMN about valid representations (line 83). This student's response aims to precede that there will be no reactions to the drawing. This student response leads to be descriptive or explanatory and contains reasoning. In this episode, the student response still ended up at the LC level by being descriptive and containing general reasoning (Hjelte et al., 2020). In dialogue 3, the teacher insinuates norm break (line 104) by challenging the prevailing SMN to change temporarily. This teacher activity made it normative for other students to become temporarily involved in a joint explanation activity. Through the mentioned potential norm breaks, we will receive similar SMN as we would when analysing through the framework (Table 1).

The operationalised framework contains four "levels" of student responses, and these levels in our approach are somehow considered misleading. This is because they are more about four different structures, with different teaching intentions linked to different SMN due to classroom community of practice (Cobb & Yackel, 1996a). From this point of view, we will consider the analysis through the framework as non-normative for teaching, rather a contribution to developing and challenging teacher practice. Based on the findings in this research study, our implications for practice indicate that SMN impact and even regulate the opportunities for developments in the mathematical community of practice. Therefore, implications for practice will be to contribute through this framework teachers' professional learning development about their own teacher practice according to deeper insights in

different SMN. We suggest further research in this area, with the aim to contribute to professional teaching-learning development. Such research could also contribute to further elaboration of the operationalisation of the framework and analysis through the framework according to SMN.

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