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Beliefs about problem posing

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Beliefs about problem posing are rarely addressed in mathematics education. The study explored 43 senior preservice middle grades mathematics teachers' (PTs) beliefs about problem posing and their possible place in their mathematics-related belief system with two open-ended surveys and one problem posing task. In line with their mathematics-related beliefs, PTs believed that problem posing was important because it supported students' conceptual understanding. However, they believed that problems for the low-level students should require applying simple procedures and these students would not be able to pose problems. Problems including challenging tasks could be posed for high-level students and they had skills to pose such problems. PTs' problem posing beliefs may be linked more to their beliefs about students' learning than those about nature of mathematics and problems.

Keywords: Problem posing, preservice mathematics teachers, beliefs, belief system.

Introduction

Problem posing is (a) producing new problems by rearranging an existing situation to a problem or (b) revising existing difficult problems to formulate more solvable problems (Silver, 1994). It is an important, challenging, and cognitively demanding mathematical activity as it enhances students' conceptual understanding, reasoning and dispositions towards mathematics (Singer et al., 2013), and enables teachers explore students' mathematical understandings and experiences (Silver, 1994). Hence, there is a demand for teachers to encourage students for posing problems (Crespo & Sinclair, 2008; Rosli et al., 2015). Inservice and preservice teachers' mathematics-related beliefs influence their response to the demands for meaningful mathematical experiences and their conceptualization of teaching and learning mathematics (Philipp, 2007). Despite the importance of problem posing in mathematics education and of inservice and preservice teachers' beliefs, beliefs about problem posing have received less attention (Li et al., 2020) within the studies about mathematics-related beliefs.

Middle school mathematics in Turkey aims to enhance students' conceptual understanding of mathematics by guiding them to construct new mathematics concepts on existing ones, focusing on conceptual understanding, emphasizing connections to other content area and real-life, and utilizing multiple representations (Milli Eğitim Bakanlığı (MEB), 2018). This approach is supported by mathematics teacher education programs that train teachers specifically to teach middle school mathematics. Problem posing is an effective tool to enhance students' and preservice and inservice teachers' conceptual understanding of mathematics, yet it is addressed less in teacher education programs compared to problem solving (Crespo & Sinclair, 2008). Teachers' beliefs about problem posing may influence their tendencies to utilize problem posing in their teaching (Li et al., 2020). Therefore, this study investigated preservice middle school mathematics teachers' (PTs) beliefs about problem posing in the beginning of a short problem posing module to draw conclusions for how these beliefs might be related to each other within the web of PTs' beliefs and inform the teacher education programs with the following research question: How are PTs' beliefs about problem posing related to their mathematics-related beliefs and beliefs about problem solving?

Belief Systems

It is often accepted that beliefs are held in *belief systems* including the following characteristics as identified by Green (1971): (i) Beliefs are located in *clusters* in which we hold non-conflicting beliefs; (ii) some beliefs are constructed in relation to others in our belief system, which makes them *quasi-logical*, where we have *primary* beliefs and their *derivatives*; (iii) strong and important beliefs are located at the *center/core* of the clusters and those that are less strong at the *peripheral*; and (iv) we hold some beliefs *evidentially* (based on evidence/reason and can be changed by rational criticism) and some *non-evidentially* (not evidence-based and not changed easily). He suggested that teaching should aim for developing evidential beliefs and a quasi-logical belief system as logical as possible.

Beliefs in a cluster may be considered as a web of primary and derivative beliefs (Beswick, 2018). If we have experiences that conflict with our existing beliefs, we (our belief system) tend to adapt to these new beliefs or try to explain our existing beliefs in different ways as an attempt to limit the conflicts. This adaptation takes place when we reflect on our experiences and reorganize our understandings from our observations (Philipp, 2007). In the end, we have new beliefs and a new (sub-)web of beliefs linked to the existing web, which illustrate adaptation of our belief system when there are new experiences and knowledge (Beswick, 2018).

Beliefs about Problem Posing

Previous studies revealed that PTs considered having problem posing skills as important and beneficial for teachers and students especially to increase students' interest in mathematics (Hošpesová & Tichá, 2015). They also believed that problem posing increased students' mathematical thinking, made students the owners of their mathematical knowledge, and served as an effective process to assess their learning (Hošpesová & Tichá, 2015). PTs tended to believe that problem posing could help students develop a better understanding of mathematical concepts, think beyond solving and posing problems, become more creative, and develop positive attitudes when they worked on the problems posed by students (Grundmeier, 2015). PTs mostly believed that the connection between real-life and mathematics would be useful for students while posing problems, but they did not focus on the benefits on students' learning (Lee, 2012).

When PTs posed problems, they paid attention to the difficulty of the problems for the age and grade level of students, authenticity of the problems, mathematical concepts in the problems, and realistic and understandable nature of the problems (Rosli et al. 2015). Yet, analysis of their problems showed that the problems they posed were not in high quality (Hošpesová & Tichá, 2015), did not focus on conceptual understanding but promoted memorization (Crespo & Sinclair, 2008), and their solution required one or two arithmetic operations (Lee, 2012). PTs did not increase the reasoning and thinking skills required to solve the problems; rather, they added another operation for solution to increase the complexity of the problems (Leavy & Hourigan, 2020). It is possible that since PTs tended to pose problems similar to the ones they solved earlier in teacher-assigned tasks and textbooks because they were generally were not given opportunities to pose problems (Crespo & Sinclair, 2008).

Studies showed that training on problem posing provided teachers and PTs with new perspectives on problem posing (Barlow & Cates, 2006; Leavy & Hourigan, 2020). Teachers developed favorable beliefs about employing problem posing in the classroom and tended to do so in their lessons because

they observed that problem posing helped their students construct mathematics knowledge and increased their interest in mathematics and discussions in the lessons (Barlow & Cates, 2006). However, teachers and PTs believed that problem posing might be confusing for students in the beginning, consume time, result in ineffective mathematics lessons, and students might be more inclined to pose simple questions but not problems that require reasoning (Hošpesová & Tichá, 2015).

PTs can develop beliefs to support their practices for effective learning, such as beliefs about problem posing that will enable them to integrate problem posing effectively in their future teaching, when teacher education programs address their existing beliefs and aim for new ones (Philipp, 2007). However, this process requires knowing how PTs' existing mathematics-related beliefs are organized, where their (probably derivative) problem posing beliefs might be placed within their belief system in relation to the other beliefs, and what possible belief connections should be targeted while designing problem posing experiences for them. Therefore, the study focused on PTs' beliefs about problem posing in the beginning of a short module aiming to provide initial ideas on problem posing.

Method

A basic qualitative research design was employed to explore PTs' beliefs (Merriam, 2009) by open-ended surveys and problem posing tasks, which PTs provided written responses. The problems posed by PTs during the implementations were not the foci of the study and therefore, not reported.

Context and participants

The study was conducted in a four-year middle-grades (5 to 8) mathematics teacher education program in Turkey with 43 PTs (6 males and 42 females) who were taking the Practice Teaching course and consented for the study. Practice Teaching is offered in the last semester of the program and required 60 hours of observation of and teaching middle school mathematics lessons, and two-hour University component every week to discuss PTs' experiences and observations at schools.

Instruments and procedures

PTs participated in a four-hour module (2 two-hour meetings) on problem posing that took place at the University component of the Practice Teaching course. The study focused on the first two-hour meeting where PTs completed two open-ended surveys and one individual task. Surveys targeted PTs' thoughts, experiences, and observations about teaching and learning mathematics, problems, problem solving and posing. The reason for targeting a broad area of beliefs was to understand PTs' larger web of mathematics-related beliefs and possible derivative beliefs about problem posing within (Beswick, 2018). The task asked PTs to pose problems for students with different levels of knowledge and skills, and reflect on the process. Information about the data collection procedure is in Table 1.

Data analysis

Data analysis started by extracting belief statements after repeated careful reading of all responses (Beswick, 2018) where PTs expressed their understandings and reasons (Philipp, 2007) for the question content. I developed a code list based on the studies about PTs' mathematics-related beliefs in these programs (e.g., Haser & Doğan, 2012) and based on my repeated reading of the data with room for emerging codes especially for beliefs about problem posing. While I kept a list of possible

codes from the literature (above) for problem posing, my focus was more on codes extracted from the data because I tried to see the connections among the beliefs within this specific data set. After determining the initial code list, I coded the data with some emerging codes, revised my coded data for consistency, and listed examples of belief statements for the codes. After coding was completed, I determined the themes and revised the examples of belief statements and data to ensure an adequate representation of PTs' beliefs by the themes. I summarized the themes narratively to see a clearer picture of connections which revealed a web of PT's beliefs grounded in data and their conflicting beliefs. I revised the conflicting beliefs one more time in detail for a better interpretation.

Table 1: Examples of questions, (*) notes, and the number of PTs responded

Survey 1: What are the behaviors of a middle school student with an in-depth understanding of mathematics concepts? / Based on your observations at schools, what can you tell about what teachers pay attention to in their teaching? (43 PTs, 12 questions)
Survey 2: Based on your observations at schools, how do the middle school teachers address problem solving in their teaching? / Can middle school students pose mathematics problems? Why? (43 PTs, 12 questions)
Task: Pose a problem which requires the multiplication of two simple fractions (and if necessary, other operations and thinking) and which low-/mid-/high-level students can solve by spending some effort. Please indicate the grade level. (*)Adapted from Singer et al. (2011). Targeting a 6th grade objective (MEB, 2018), 39 PTs, 3 questions)

Findings

Mathematics-related beliefs

PTs believed that mathematical knowledge was about understanding the reasons and connections between the mathematical concepts by asking questions and thorough thinking. School mathematics had certain “definitions”, “important terms, [and] proofs” of key importance as they were connected to other concepts and daily life. Although PTs expressed the importance of these connections for mathematical knowledge, they did not mention abstract concepts and relationships. Rules, formulas, the reasoning behind them and their connections were also part of the mathematical knowledge.

Teachers should “conduct activities that would increase students’ interest”, ask “questions that would make students question,” and teach accordingly to ensure students’ learning. PTs believed that students should not memorize rules and formulas; therefore, teachers “should reach the rule by proving with the students.” Teachers should monitor students’ learning by questions “that require making connections with other concepts” and by asking them to try different ways, generalize, build patterns, give counter examples, and use multiple representations. PTs criticized teachers’ practice of solving several questions in the lessons. They would like to ask questions “that students will inquiry the most [and] will build cause and effect relationship” after “teach[ing] the rationale behind the concepts”. It is better if students solved questions after the lesson because “they may forget easily and not be able to see their missing [knowledge] if they do not solve questions.”

PTs believed that knowing fundamental concepts and building relationship between them were important for students. These required reasoning, questioning, and deeply thinking about the reasons

and meanings. They stressed the use of formulas, but were against memorizing them. PTs believed that students, who paid attention to their teachers with utmost care and spent effort on learning during the lesson, would learn mathematics effectively.

Beliefs about problems, problem solving, and problem posing

Mathematical problems were real-life situations for PTs which required “understanding and thinking” of mathematical knowledge and “building relationships and logical inferences”. Problems could be solved in different ways with multiple results, but not immediately by a procedure or a formula. Middle school level problems were the same except for the level of the mathematical knowledge. Problem solving at schools, as observed by PTs, was the activity where teachers first asked a question and answered it, and then asked a similar question to the students to be solved in a similar way. “Unfortunately” they observed that some teachers assigned closed-ended drill-and-practice questions, which did not match PTs’ definition of problems, and students were not asked about their reasoning.

PTs would like to employ problem solving in the future with problems that would target students’ mathematical thinking or understanding of mathematical concepts and more difficult than what students were used to solve. PTs’ problems would require attention to and discussion of the problem situation, and the daily life contexts to be solved. Students were the problem solvers, not the teachers.

I would like to consider a situation that actually requires a solution. I would like students to follow a solution [strategy] by considering what kinds of external factors would influence the results they would reach had they been in such a situation, instead of [asking a problem] that they would reach a single result directly by multiplication [and] division.

On the other hand, PTs did not have much experience in problem posing in their courses and only some of them were able to design and ask problems in real classroom settings, which received positive reaction from students and collaborating teachers. They gained initial ideas for their problems from course books or internet resources. Some focused on connections between the mathematical concepts and real life problems, and they employed real data.

The problems PTs posed in the study provided data on PTs’ thinking about problems for students with different levels of knowledge and skills, and the factors they would consider while posing problems. PTs posed problems requiring more/difficult operations for high-level students and those requiring “implementation of a known procedure” or “a memorized operation” for low-level students, contrary to their definition of problems and statements disapproving memorization. Although they indicated in the survey that problems did not have immediate solutions, some PTs posed problems that “would not lead [students] to difficulty and enable them to understand the concept and the question.” They did not take into account the properties of problems that they identified when they posed problems especially for low-level students. For example, one PT wrote a mathematical expression of multiplication of two simple fractions without a problem statement and stated that “[Students] will only implement what they have learned. Knowing the rule would be sufficient.”

PTs stated that they prioritized students’ knowledge, possible difficulties, the national curriculum, own experiences from the courses, and daily life connections and links to other mathematical concepts while posing problems, and targeted students’ thinking. The problems they posed for low-

level middle grades (5 to 8) students included fundamental concepts most of which were covered in the elementary grades (1 to 4), such as the meanings of fractions and fraction multiplication, part-whole relationship, fair sharing, and part-of-a-part. Their problems for high-level students were about connections among the mathematics concepts and the meaning of other fraction operations. Some problems required more/difficult operations as students' level increased: "I identified simple, medium [and] difficult level operations and I identified the context of the question for these."

Beliefs about students' problem posing

Many PTs believed that students had the capability to see the mathematics in daily life because they had to deal with several problems in their daily lives, which illustrated the contexts for students' problems. They indicated that students were creative, better and quick learners, and their prerequisite knowledge was sufficient to pose problems.

They have reasoning, they are involved in real life. They gain experiences [and] deal with troubles. Don't these show that they have a natural tendency to solve problems? [So] they can pose problems about some of the obstacles/questions that are in front of them while they improve their lives.

Yet, several PTs believed that middle school students could pose problems only "if [teachers] enable them to understand and conceptualize the topics [effectively]." Some believed that students' understanding of problems were limited and "they cannot examine what should be thought [and] whether it can be solved in multiple ways or not while posing the problem" because "posing a problem is more difficult than solving that problem" and students were "not used to posing problems". Therefore, "[problems] should be posed only by the people who know the concepts well."

Discussions and Conclusions

PTs' problem posing beliefs and mathematics-related beliefs seemed to be in line. They believed that problem posing provided students with deep thinking of mathematics concepts in context, and their connections to each other and daily life. These beliefs were probably derivative of PTs' primary beliefs because (a) PTs' experiences were not sufficient yet to build primary problem posing beliefs and (b) they had some observations and experiences of teaching mathematics in classrooms on which they were able to elaborate to develop new understandings of problem posing.

Even though PTs identified certain characteristics for problems, their related problems mostly lacked these characteristics. Their problems for high-level students were different from those for low-level students in terms of the number of operations involved in the solution, but not the thinking level, as can be seen in other studies (Leavy & Hourigan, 2020). PTs believed that problems should point to the links among the mathematical concepts, but they illustrated this link in their problems only for high-level students to some extent, not for low-level students. These differences showed that some of PTs' problem posing beliefs might be connected to their beliefs about students' learning more than to their beliefs about the nature of mathematical knowledge. They might develop contradictory beliefs within the same belief system in different clusters (Green, 1971) with an effort to reach stability by adapting to the new problem posing experience (Beswick, 2018).

PTs' beliefs about problems to be posed to high-level students (who are good at reasoning and dealing with challenge) seemed consistent with their beliefs about problems (challenging situations that

required thorough thinking) and teaching and learning mathematics (mathematical knowledge is not for memorization). These beliefs, however, conflicted with their beliefs about problems to be posed to low-level students (simple problems as they could handle only certain and memorized mathematical procedures, but not challenge). PTs possibly observed certain practices at schools where teachers posed several questions solved by routine procedures and believed that this practice was better for low-level students, even though they criticized it. They might also tend to replicate the problems they have seen in mathematics lessons and resources (Crespo & Sinclair, 2008) due to their insufficient problem posing experiences. PTs might believe that certain less risky practices would be better in classrooms when they lacked problem posing experiences; therefore, tended to pose easier problems for low-level students. Although PTs prioritized approaches that would lead students to conceptual understanding of mathematical concepts, they might keep a consistent belief system by thinking about low-level students as a special case that required a different understanding, and accordingly holding beliefs about low-level students in different clusters (Green, 1971).

Longer and intensive problem posing experiences would provide PTs with opportunities to develop more consistent web of beliefs. The beliefs documented here revealed that when PTs did not have sufficient problem posing experiences, they tended to develop derivative beliefs, which should be explored and addressed carefully when providing PTs with problem posing training. It is advisable that such training for PTs should include more and varied experiences of posing problems and require reflection on experiences and rationale. Further studies should explore PTs' initial beliefs about problem posing in-depth before they are trained for it and how their problem posing beliefs are linked to their beliefs about students, problem solving, and teaching and learning mathematics while preparing for and teaching through problem posing.

It should be kept in mind that the instrument in the study revealed certain beliefs of this specific group of PTs, but not the others, limiting the conclusions of the study. The analysis presented here was an initial exploration of PTs' beliefs about problem posing with a more general perspective. Their actual belief system could be revealed when a further analysis for each PT is conducted to map their belief systems. The present data set would allow only for a limited view of this map. A more comprehensive and detailed illustration of belief system can be possible with additional data set including task-based interviews with PTs and observation of their problem posing practices in the classroom.

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