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# Gaze pattern analysis to reveal student difficulties in interpreting kinematic graphs

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*Graphs are an important interdisciplinary and everyday tool for visualizing and interpreting information and communication processes and, thus, an essential part of 21st Century Skills. In particular, linear functions are a fundamental component of school and university education, but students often have difficulties interpreting this type of function, in mathematics as well as in physics – especially in kinematics. This paper presents first results of an eye-tracking study, which compares learners' visual attention during the interpretation of linear graphs in mathematical and physical contexts.*

*Keywords: Eye tracking, graphs, context, problem solving.*

## Introduction

Graphs are a typical representation to show the dependencies between different quantities and therefore play an essential role in conveying information in the natural sciences. They are typically introduced in mathematics in secondary school and are part of both the school and university curriculum in different STEM contexts (Glazer, 2011; Leinhardt et al., 1990). Beyond the purely mathematical understanding of the functional relationship, graphs are an important tool, e.g., for visualizing trends in measurement data and are thus an interdisciplinary tool for interpreting quantitative information in the context of 21st Century Skills (National Research Council, 2012).

In recent decades, however, research has shown that many students have difficulties interpreting graphs (Glazer, 2011). This is especially true for kinematic graphs (Beichner, 1993; Ivanjek et al. 2016; McDermott et al., 1987). Since the cognitive processes involved in graph interpretation are closely linked to visual perceptual processes, e.g., extracting relevant information from graphs, eye tracking opens up the possibility of gaining insights into learning or problem solving processes involving graphs (Klein et al., 2021; Susac et al., 2017).

In this work, we use eye tracking to investigate visual strategies of students while solving line graph problems in a mathematical and kinematical context. To this end, we recorded their gaze data to investigate their visual attention while solving item pairs of a test instrument validated by Ceuppens et al. (2019) that require the same mathematical solution procedure. On a selected item pair, we show that gaze behavior differs significantly between the mathematical and kinematical contexts.

## State of Research

Students' learning difficulties concerning graphs in the contexts of mathematics and physics can be grouped into three main aspects: First, although mathematics and physics are deeply connected (Redish & Kuo, 2015), students have difficulties linking both disciplines adequately (Ivanjek et al., 2016). Reasons for this are a domain-specific learning (Pollock et al., 2007) and a lack of the ability to transfer knowledge from mathematics to physics (Christensen & Thompson, 2012). Second, students have problems using multiple external representations competently (e.g., Ainsworth, 2006), especially when switching between different representations back and forth (Even, 1998). Third, in particular the high degree of abstraction of graphs poses difficulties since learners need to relate mathematical entities to real world processes (McDermott et al., 1987).

Eye tracking is a method to investigate visual attention by recording eye movements. The eye movement can be described as a sequence of fixations (eye stop points) and saccades (jumps between fixations). Predefined areas in the field of view, so-called areas of interest (AOI), are used to define eye-tracking metrics such as the total visit duration (TVD, cumulated times between first fixation in and first fixation outside an AOI) and transitions (saccades between AOIs). To get an adequate understanding of line graphs, learners have to extract information from the graph and combine this with prior knowledge. Such processes of information extraction and constructing meaning with graphs are described by the Cognitive Theory of Multimedia Learning (CTML, e.g., Mayer, 2009). CTML describes three main processes concerning problem solving and learning: selection (extraction of sensory information from graphs), organization (building a coherent internal representation through information structuring) and integration (combining internal representations such as axis values or axis intervals in graphs with the long-term memory).

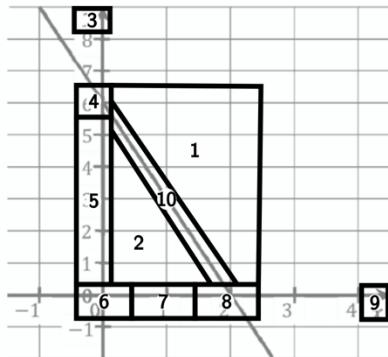
The following connection between CTML and eye tracking allows a theory-based interpretation of the eye-tracking data: Gaze durations (TVD, fixation durations) are associated with processes of the selection and organization of information extracted from the processed material, gaze shifts (transitions) are related to integration processes (e.g., Alemdag & Cagiltay, 2018). In summary, eye tracking is a non-intrusive method to obtain information about visual attention and cognitive processing in problem solving processes. So far, to our knowledge, there is no eye-tracking study of the comparison of visual attention processes on linear graphs in the context of mathematics with other contexts, especially with physics in high-school. In general, there is a gap in analyzing mathematical problem-solving processes and representations using eye tracking in secondary school (Strohmaier et al., 2020). This study aims to fill this gap.

## Research Questions

For our eye-tracking study, we used the validated test instrument of Ceuppens et al. (2019) because it contains items in both kinematical and mathematical contexts. While Ceuppens et al. (2019) examined 9<sup>th</sup> grader, we used the test instrument with students in the entry phase of upper secondary school. The aim was to check whether the difficulties in interpreting linear graphs are still present in the upper grades. The background is that the competent handling of graphs in general and of linear graphs in particular is assumed in the upper school and can be seen as a basis for the development of

M8

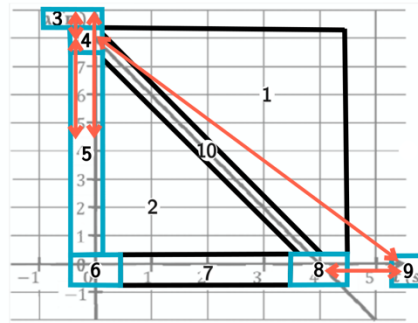
The figure shows the graph a function  $f$ .



Determine the slope. Explain your answer.

K8

A car is moving along a straight road. The graph shows the position  $x$  of the car as function of the time  $t$ . The position is in meter, the time in seconds.



Determine the velocity of the car. Explain your answer.

**Figure 1: Isomorphic item pair no. 8 from the test instrument used; on the left: mathematical context (M8), on the right: kinematical context (K8). The diagram-area of each item is divided into AOIs 1-10. AOIs with significantly longer TVDs than in the isomorphic item are marked in blue. Red arrows indicate transitions between AOIs that appear significantly more often than in the isomorphic counterpart**

many new learning contents in the STEM context. The research questions of the study were as follows.

RQ1: Do the difficulties of 9<sup>th</sup> grader, particularly in solving kinematic items, also occur with students in upper secondary classes?

RQ2: Do selection and organization processes analyzed by gaze behavior in solving isomorphic items differ between mathematics and kinematics contexts?

## Methodology

The test instrument consists of 24 items taken from a validated test by Ceuppens et al. (2019) and translated literally into xxx (details redacted for review). The instrument consists of pairs of items in mathematics and kinematics contexts that are isomorphic to each other, i.e., have the same surface features and require the same mathematical solution procedure. In order to avoid sequence effects, all items were presented to the students in arbitrary order and an alternating start either with physics items or mathematics items. In the context of this paper, we focus the eye-tracking based analysis on item pair number 8 (cf. Figure 1). Here, students evaluated the negative slope of a linear graph once for a mathematical function and once for a time-position function.

A total of 35 upper secondary students (14 male, 21 female, all with normal or corrected-to-normal vision) from a secondary school in xxx (details redacted for review) participated in the study. The students voluntarily took part in data collection either in free periods or in regular classes (with teacher permission). At the time the study was conducted, kinematics had already been covered in the courses of all participants. The students were rewarded with a 5 €-voucher for participation. Item pair no. 8 was completed by 24 students only. In the context of this paper, we are interested in why students with mathematical knowledge exhibit problems in the physics context. Therefore, we only

consider students who answered the mathematics item correctly and failed solving the physics item ( $N = 14$ , 8 female, 6 male).

The study took place in the school's library where two identical eye-tracking systems (Tobii X3-120) were set up. First, the participants answered a short questionnaire about their demographics. After that, a 9-point calibration was performed to obtain a full-customized and accurate gaze point calculation. Subsequently, the 24 items were shown on the computer screen (1920 x 1080 px; refresh rate 75 Hz). If students were ready to give an answer, they pressed a key to move to the next slide. After they had answered, they were asked how confident they were about the correctness of their answer (4-point Likert-type rating scale, ranging from very high confidence to no confidence). The students could take as much time as needed to answer a question. They did not receive feedback after completing a task and could not return to previous tasks.

*Performance data:* The answers were coded dichotomously (0 for wrong solution, 1 for correct solution) following Ceuppens et al. (2019). Answers for which the participants stated that they had guessed were marked as incorrect. To compare the difficulty of the items, the item difficulty  $P$  (proportion of participants who answered the item correctly) was calculated.

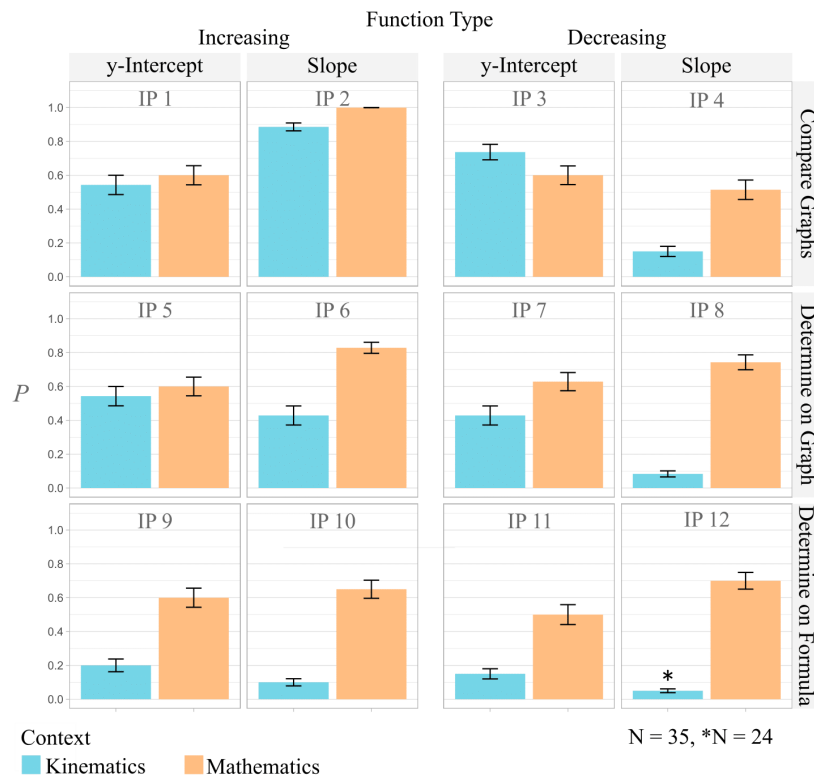
*Gaze data:* The data collection and the definition of AOIs was done with the software Tobii Studio. For the assignment of the eye-movement types, the default I-VT (Identification by Velocity Threshold) algorithm of the software was used (threshold:  $30^\circ/s$  for the velocity; Salvucci & Goldberg, 2000). One participant's results were excluded due to poor quality eye-tracking data. For the gaze data analysis, the items were restricted to the diagram area. The AOIs were chosen such that graph relevant structures are covered by one AOI each (cf. Figure 1). For example, areas of the axis intersection points or areas below and above the linear function are summarized in AOIs. The eye-tracking metrics TVD (selection/organization) and transitions (integration) were considered. A non-parametric Wilcoxon signed rank test was used to test whether the central tendencies of the TVD / number of transitions of the participants in the dependent samples (M8 correct, K8 incorrect) were different. The analyzed datasets met the assumptions required to perform the Wilcoxon test. A threshold of  $p = 0.05$  was used to determine the effect significance level within all tests performed. To control the false discovery rate due to multiple testing, the  $p$ -values were corrected using the Benjamini-Hochberg procedure. The effect size  $r$  (for non-parametric data, cf. Fritz et al., 2012) with 95% confidence interval (calculated using Bootstrapping with 1000 replications) was determined for all Wilcoxon tests with significant results and can be interpreted after Cohen's guidelines (small effect:  $0.1 \leq r < 0.3$ ; medium effect:  $0.3 \leq r < 0.5$ ; strong effect:  $0.5 \leq r \leq 1.0$ ; Cohen, 1988).

## **Results and Discussion**

In this section, descriptive results of all study participants are analyzed first. Afterwards, results of the analysis of the eye-tracking data for item pair no. 8 are presented. This item pair was selected for the eye-tracking based analysis because it shows the greatest difference in terms of item difficulty in the mathematics and physics contexts (cf. Figure 2).

## Descriptive results

The results of the item difficulty analysis for all test items are shown in Figure 2. The task description of the item pairs is placed at the top and right-hand side of the diagram. For example, in item pair no. 1 (IP 1), participants were asked to compare the  $y$ -intercepts of two linearly increasing functions in mathematics or the initial positions of two objects with linearly increasing  $x(t)$ -graphs, respectively. Item pair no. 6 required the determination of the slope of a linearly decreasing function or the velocity of an object with a linearly decreasing  $x(t)$ -graph, respectively. Except for item pair no. 3, the item difficulties of the mathematics items are higher than those of the respective isomorphic kinematics items. Extremely difficult items are kinematical items dealing with negative velocities and with the representation form formula ( $P_{K4} = 0.15$ ,  $P_{K8} = 0.08$ ,  $P_{K9} = 0.20$ ,  $P_{K10} = 0.10$ ,  $P_{K11} = 0.15$ ,  $P_{K12} = 0.05$ ) (Bortz & Schuster, 2010). Whereas the items K2, M2, and M6 are extremely easy items ( $P_{K2} = 0.89$ ,  $P_{M2} = 1.00$ ,  $P_{M6} = 0.83$ ) (Bortz & Schuster, 2010).



**Figure 2: Comparison of item difficulty level  $P$  of all item pairs (IP) in the test instrument**

### Item Pair No. 8

The results of the Wilcoxon test ( $p$ -values with effect sizes  $r$  for significant results) are presented in Table 1 together with the mean TVD for each AOI of item pair no. 8. All significant results relate to longer TVDs in item K8 than in item M8 and are visualized in Figure 1. The two axis labels (AOIs 3 and 9), the  $x$ -axis (AOI 5) and the intersections with the axes (AOIs 4 and 8) were viewed significantly longer in item K8.

The number of transitions between AOIs with significant TVDs in both items of item pair no. 8 were analyzed. Table 2 shows the results of the Wilcoxon test ( $p$ -values with effect sizes  $r$  for significant results) together with the average number of transitions between two AOIs.

**Table 1: Average TVD (Mean) per AOI with standard error (SE), adjusted  $p$ -values of the Wilcoxon Test and effect sizes  $r$  with 95% confidence intervals are given for all Wilcoxon tests with  $p < .05$**

AOI	Mean <sub>M</sub>	SE <sub>M</sub>	Mean <sub>K</sub>	SE <sub>K</sub>	$P$ (adj.)	$r$
3	.05	.02	2.22	.38	.004	.882 [.877; .882]
4	1.86	.37	4.30	.91	.020	.663 [.441; 1.000]
5	1.24	.19	2.29	.49	.011	.714 [.543; 1.000]
6	.11	.04	.73	.44	.011	.748 [.643; .863]
8	.92	.21	2.65	.69	.020	.663 [.471; 1.000]
9	.03	.02	1.13	.19	.004	.850 [.815; .914]

**Table 2: Average number of transitions (Mean) with standard error (SE), adjusted  $p$ -values of the Wilcoxon Test and effect sizes  $r$  with 95% confidence intervals for all tests with significant results**

AOI	Mean <sub>M</sub>	SE <sub>M</sub>	Mean <sub>K</sub>	SE <sub>K</sub>	$P$ (adj.)	$r$
3   4	.07	.07	5.53	1.01	.016	.852 [.793; .888]
3   5	.00	.00	1.00	.22	.018	.744 [.627; .878]
4   5	.87	.41	3.40	.87	.018	.736 [.529; .865]
4   9	.00	.00	.67	.27	.044	.653 [.421; .816]
8   9	.13	.09	1.73	.51	.018	.772 [.627; .888]

All significant results refer to more frequent transitions in item K8 than in item M8 and are visualized by red arrows in Figure 1. Transitions along the axes that link information between axis labels and other axis content occur significantly more often in item K8 than in item M8. Other gaze shifts along the upper  $x$ -axis area and gaze shifts between the  $x$ -axis intercept and the  $t$ -axis label also occur significantly more often in item K8 than in item M8.

## Conclusions and Outlook

In this study, a test instrument validated by Ceuppens et al. (2019) on linear graphs in a mathematical and kinematical context was used with upper secondary students. Our results confirm for the described sample from grade 11 the following main findings of the study by Ceuppens et al. (2019) with students from grade 9: Difficulties in the context of kinematics, with formulas and in the interpretation of functions with negative slopes. This shows that the difficulties also exist with older students and are not remedied by teaching in the classes in between. To gain more insight into the solution process, the eye-tracking data from a selected item were examined in more detail.

Using a selected pair of isomorphic items to quantify a negative slope, it was shown that gaze behavior differs between kinematical and mathematical contexts. In detail, the results of the eye-tracking analysis show a higher dwell time of the gaze from the axes in item K8 than in item M8,

which speaks for a stronger focus of the information extraction on the axis sections. In addition, changes of view between time- and position-axis happen more frequently in item K8 than in item M8, which means a stronger linking of the information on the axis sections. The analysis of the gaze data suggests that in the kinematical context mostly the attempt is made to form the quotient of place and time, which leads to the fact that the negative sign of the velocity is not taken into account and the task is thus solved incorrectly. The reason for this could be the physics lessons, in which an algorithmic procedure for calculating the velocity from the location and time data is taught, so that the application of mathematical procedures is no longer taken into account. These assumptions are supported by isolated interviews conducted as part of this study. For example, one student commented as follows in his description of the solution strategy for item K8: “The distance divided by time is the speed, so I just calculated at the distance 8 divided by time 4, 8 divided by 4 and that was 2 meters per second.” To remedy these transfer difficulties, teachers should be sensitized to this, and this should be done early in their curriculum. A targeted linking of mathematics lessons and physics lessons could on the one hand help to promote the transfer of mathematical procedures for solving physical problems, but on the other hand also connect mathematics to an application.

It seems that students struggle interpreting graphs independent of their age. As graphing is one important, ubiquitous and everyday 21st Century Skill, we will study this question in a broader sample. In addition, based on this important preliminary work, we will collect interview data for triangulation in a follow-up study.

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