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# Inclusive math practices in primary school

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*In the last 10 years inclusive settings in mathematics have increased in German primary schools. In essence, inclusive mathematics education means giving all students equal opportunities for mathematical education, regardless of their subjective learning requirements and learning potential. In our research project we reflect on the one hand central design principles for developing substantial teaching units for inclusive classes. On the other hand, we analyze the emergence and development of practices focusing on the teaching processes of thematizing and negotiating mathematical contents. For this purpose, 22 class discussions from lessons were examined with regard to the reconstructable practices.*

*Keywords: Inclusion, practices, natural differentiation, multiplication, discourse.*

## Introduction

“Inclusive schools aim to involve all learners in quality learning experiences which empower them to become active participants in a more equitable system” (Scherer, et al., 2016, 640). Inclusive education in math classes aims at breaking down barriers and creating universal approaches to math learning. Regarding mathematics education, the main aim is to enable all children developing basic mathematical competences in interaction with others.

Even though the primary school in Germany traditionally sees itself as a school for all, the teachers (children, parents, ...) are shaped by a system that is traditionally and currently geared towards segregation and is changing very slowly to an inclusive school. In subject didactics, an understanding of inclusion seems to be established in everyday teaching that is based on a one-sided deficit conceptualisation of the concept of inclusion and reduces inclusion to “compensatory support measures” for precisely those children who seem to have deficits. But inclusive education does not mean that only few children need special support. Rather, inclusion means paying attention to each individual person with his or her individual prerequisites and potentials. Different children cannot and do not have to achieve the same goals. Inclusive education demands from all children exactly what they can achieve. Inclusive teaching engages children to bring in their own needs and interests. They are also allowed to set their own priorities within a certain framework and develop a personal educational profile through their chosen topics and subjects. The diversity of children is also reflected in the fact that they learn differently — at different speeds, in different ways and with different prerequisites. Therefore, inclusive mathematics teaching aims to offer learning opportunities that are individually adaptable and enable mathematics learning in community with others, regardless of competences, learning requirements, interests, and development potentials (Scherer et al., 2019).

In our paper we focus on the ongoing practices in inclusive math education with respect to the research question addressed: “Which practices characterise the plenary in introduction and reflection and how far do they correspond to the normative aims?”

## **Theoretical framework**

In many cases of everyday math teaching in inclusive classes, however, teachers cope with the heterogeneity of the children mainly by using individualised working materials organized in station work or plan work. This organisation at first glance promises differentiated learning progress for each child, as well as optimal development of potential: each child works on a prescribed (sometimes also designed by the child) learning plan at different places and with different levels of support. The children's learning activities only take place side by side. This poses the risk that the learning quality of the lessons is reduced and that the children cannot optimally develop their potential. But mathematical learning processes are particularly dependent on content-related social negotiation processes. Therefore, children need to share mathematical discoveries and to present one's own reflections as well as to communicate and explain them to others (Steinbring, 1997). This is not possible, if children have to learn more or less on their own.

Consequently, there is a need for the design of substantial teaching materials and for the research of practices of math teaching and collaborative math learning processes in inclusive settings.

## **Principles and learning situations of inclusive math education**

The intertwine dimensions of personhood, sociality, and complexity link subject-specific and social participation. In recent years, different guiding principles have emerged for the design of inclusive mathematics teaching (Häsel-Weide & Nührenbörger, 2021). We refer to four design principles that aim to consider the different potentials of the learners and to connect to them in a targeted way:

- (1) accessibility to the common subject matter for all children,
- (2) subject learning at different levels,
- (3) active exploration of content connections and finally
- (4) initiation of common learning phases for all children with social negotiation processes of communicating, representing, and arguing.

Reflecting these four principles substantial learning environments and the task formats on which they are based (Wittmann, 1995) are a basis for the design of mathematical learning situations in primary schools. The environments consider the idea of differential sensitivity, i. e. reflective perception of the heterogeneous competencies of children in a concrete learning situation. The influence of academic and content language for the understanding of mathematical concepts is highlighted as well as the use of appropriate material.

But fundamental for successfully initiated joint learning processes are not only substantial task formats, which focus on the basis staff. The support of teachers seems to play a special role for the learning processes. Especially children with difficulties in learning mathematics seem to be dependent on impulses that support the solution process (Korten, 2020). These challenge them to think about mathematical patterns and structures as well as their own and others' thought and solution processes. But not all impulses do this in the same way. Pfister et al. (2015) investigate the scaffolding

processes of teacher in inclusive classes. They see differences in the way teachers stimulate the interaction and active the children regarding to the mathematical subject. The video study of Krähenmann et al. (2019) shows, that teachers have difficulties in creating common and at the same time differentiating learning situations. Teachers either provide common but not differentiating learning opportunities for a very heterogeneous learning group or differentiating learning opportunities where joint learning did not take place.

### **Social and subject-matter practices**

Practices are established as a “theoretical construct that allows us to talk explicitly about collective mathematical development” (Cobb, 1998, p. 34). Characterising inclusive practices, a distinction can be made between normative pedagogical practices and the practices that can actually be reconstructed in the observation of teaching. Normative inclusive practices are e.g. formulated in the Index for Inclusion and describe how practices should be. The dimension of “developing inclusive practices” contains the following three aspects: (1) “Lessons are planned with the diversity of students in mind”, (2) “Lessons strengthen the participation of all students” or even (3) “Students learn together” (Booth & Ainscow, 2002). “Mathematical practices include problem solving, sense making, reasoning, modeling, abstracting, generalizing, and looking for patterns, structure, or regularity” (Moschkovich, 2015, p. 1068).

These practices can be distinguished from concrete teaching practices, which Hirschauer (2016) describes as “ways of doing”. He defines 'practice' as a physical consummation of social phenomena such as “types of activities, ways of acting, patterns of behavior, forms of interaction” (Hirschauer, 2016, S. 46; transl. by the authors). Practices are analytical units and describe structures, customs, or things themselves which influence teaching and shape learning.

### **Design of the Study**

In our project IGEL M (Inclusive practices in shared learning opportunities in mathematics) we develop existing learning environments further by conducting didactic teaching-learning experiments for inclusive learning settings and by analyzing mathematical learning processes from a qualitative perspective (Häsel-Weide & Nührenbörger, 2021). We focus on the mathematical practices in inclusive lessons. The analysis follows an interactionist perspective, focusing on the classroom microculture and mathematical practices.

We accompany classes during their time in primary school and visit them 4-5 times a school year. In cooperation between the teacher of the class and the authors of the paper we plan learning environments, select material and develop further. The lessons, carried out by the teacher, are videographed and used to a) answer the research questions as well as b) to reflect jointly the quality of the learning. Actually 22 lessons are videographed, each subject was realized in two consecutive lessons. In detail, we tackle three research questions in the project, but in the following, we focus on the first question:

RQ1) Which practices characterise the plenary in introduction and reflection?

RQ2) Which practices characterise the collaborative work of children?

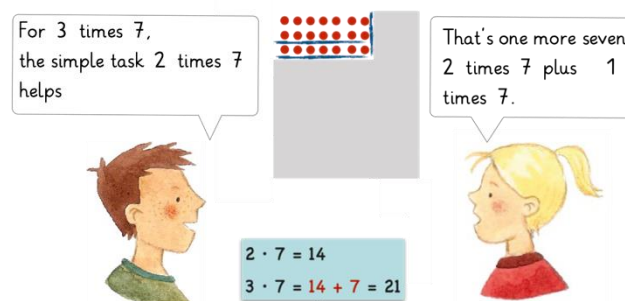
RQ2) Which mathematical understanding can be reconstructed? How far differences the understanding?

The reconstruction of the practices bases on the videographed teaching and learning situations. Corresponding transcripts were interpreted by a group of researchers finding different typifications. The interpretation of the negotiation processes follows four analytical steps: (1) Video scenes are transcribed. (2) In discursive analyses of the researchers involved, these are paraphrased and interpreted by means of turn-by-turn analysis. (3) In discourse, plausible typifications of practices are elaborated by categorising patterns of interaction or activities. (4) The practices are compared comparatively with other analyses of other scenes and examined in a generalised way (Voigt, 1994).

## Analysis of an episode

### Learning environment

This paper is about a common task to explore the distributive relationships between so-called easy and more difficult tasks. In essence, the students first must recognise the doubling tasks (multiplier 2) and solve the neighboring task (the multiplier 2 is increased by 1) with the help of this (see Figure Fig. 1).



**Fig 1: Neighboring tasks. Deriving a solution for a simple task for calculating a difficult task**  
(Nührenbörger et. al., 2017; Illustration by K. Mosen, PIKAS)

The basic material for all students is therefore not limited to recognising simple multiplication tables, but to exploring the structural relationships. Children with mathematical learning difficulties have great issues in seeing the structure, understanding the operation, and deriving results. According to the idea of sensitivity of different competences, this must be considered in inclusive classes. For this purpose, both representation-sensitive and language-sensitive supports are offered to enable access to the common learning object. For example, students can show the simple painting angle task on the 100 field and then move the angle down one line. Moving the painting angle down represents increasing the multiplier by 1, so that the product increases by the multiplicand once.

This multiplication relationship can be supported linguistically by picking up on speaking in groups (see example of students) and by verbalising the move of the angle. If not all students in an inclusive class are already working in the number range up to 100 or if, for example, the whole multiplication table still seems too complex, the distributive relationships can also be explored in qualitatively differentiated tasks with a structurally reduced field (the 25 field, consisting of 5 times 5 points, see Figure 2).

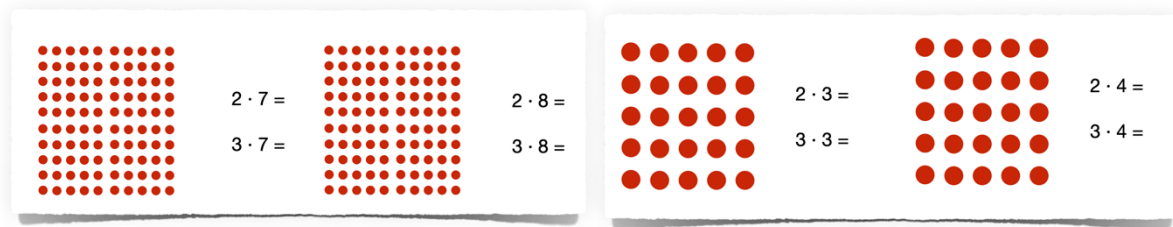


Fig 2: Neighboring tasks at the 100 field and 25 field

In an inclusive class, these tasks can be worked on individually. On the other hand, the students can also work cooperatively in pairs on such a task: For example, one child looks for and shows the easy task, the other notes it down. Then the more difficult task is derived from it by shifting the painting angle and noting and calculating the task in relation to the previous one.

### Plenary practices

At the beginning of the lesson, the teacher highlights the aim of the lesson and links it to previous related topics (see transcript). She reminds the students that simple tasks have already been used in addition. The teacher also points out that the result should be found with the help of simple tasks, not by counting.

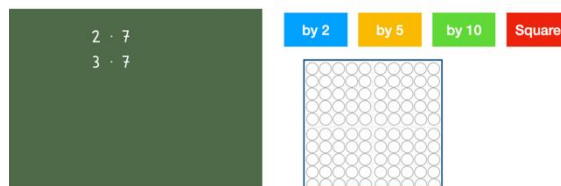
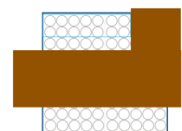
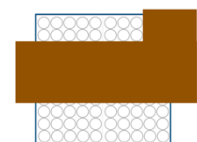
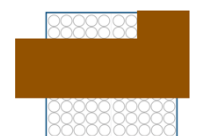


Fig 3: Blackboard picture for the neighboring task

- Teacher: Today we calculate difficult multiplication tasks. You may remember from the plus tasks, we did that too. If you can do a simple multiplication task and you know it well, it will help you to solve a difficult task. Then you don't have to start counting or calculating all over again. Yes? And today, I want to show you how this simple task can be a help. Ok? (*goes to the blackboard and points to the tasks 2 - 7 and 3 - 7*). If you have two tasks, think for yourself which of the two tasks is a simple task. Josie.
- Josie: Two times seven
- Teacher: Why? (*she marks the task 2 · 7 with an x*)
- Josie: Because it is a task by two
- Teacher: Ok. Good.
- Josie: Shall I tell the result?
- Teacher: I will mark first and then you can tell me the result, ok? (*she takes the multiplication-angle and a pen*). How many are in a row? Frederic
- Frederic: Seven
- Teacher: (*puts the multiplication-angle on the dot field*) Josie, now.
- Josie: Fourteen.
- Teacher: So, now we have to think. The next task is called what? The difficult multiplication task. Lea.
- Lea: Three times seven
- Teacher: What do I have to do to get three times seven?
- Lea: One to the right
- Teacher: (*moves the angle; a couple of children raise both arms*)
- Lea: (*names a child*)
- Child: Now the tasks are called two times eight. You have to move one to the left and one down.
- Teacher: So you mean I should go back first and then down. I'll use a different colour, I think you can see it a bit better then. What has been added? (*points on the third row of the field*). Doreen.
- Doreen: Seven once again



The teacher always switches between different the symbolic and verbal form of task presentation and the iconic dot field. Josie must wait before she is allowed to give the answer, because the task should first be presented in dot field. Even after the difficult tasks has been solved, she asks the children to look again to the added quantity on the dot field. In doing so, the teacher highlights the connection between the representation. This explicit connection important especially for children with difficulties. The teacher asks the children how the angle must be shifted, too. Leading the conversation, the teacher picks the children. Most of them put up their hands. With this practice she decides which children in participate in the classroom conversation. But the practice “veto” differs from this routine. If the children do not agree with an answer, they rise both hands. Then the student (Lea), who gave the disputed answer chooses someone, to put forward their argument. Similarly, the practice “help” works, which can be reconstructed if children could not solve the tasks or answer questions. Asking for help they pick up a classmate who answers for them (Jonas):

After both tasks are solved and both results are called, the teacher initiates a discussion how to calculate  $14 + 7$  without counting. Jonas probably remembers the solution already given but had difficulty formulating his way of calculating.

- Teacher: Ok. How do you calculate? You want to tell me the solution right away, don't you? How much is that? Do you know, Jonas?
- Jonas: Twenty-one
- Teacher: Can you tell me how you can calculate this cleverly without counting. (..)? How far do you calculate first? (..). How did you calculate it? (...) Do you know? Would you like to get some help?
- Jonas: Ahmad.
- Ahmad: First I calculated fourteen and then I added seven.

$$\begin{aligned} 2 \cdot 7 &= 14 \\ 3 \cdot 7 &= 2 \cdot 7 + 1 \cdot 7 \\ &= 14 + 7 \\ &= \end{aligned}$$

The teacher seems to be aware that there may be a difficulty in adding the multiplicand to the product of the simple task. So, she asks for a clever, non-counting strategy to solve the addition. She picks up Jonas, who mentioned the result, but did not explain the process. Probably he remembers the solution, which has already been mentioned in the interaction. Jonas asks Ahmad for help, who himself mentions the task without explaining a clever strategy for calculating. Bruno explains later: “You take fourteen plus six, then it's twenty. Then you have to add one more because there was one left and then it's twenty-one”.

In summary, the plenary is characterised by the teacher's effort to connect easy and difficult tasks in an understanding-oriented way. The teacher also addresses skillful calculation strategies in addition. The new subject is connected with past subject. These practices address children with difficulties and allow them to participate without making the address explicit. But, the teacher uses (only) an example for the class discourse with fits to the regular stuff and probably overtaxes children, who work in the reduced field. However, the simple task chosen is one with 2, which should already be familiar to all children as a doubling task.

## Results

The practices that are evident in this selected scene are typical of inclusive math practices that could be reconstructed in plenaries at the beginning of inclusive class (Häsel-Weide & Nührenbörger, 2021). Those discourses are characterized by the following practices:

## **Presentation of the subject**

- *Explication of the subject*: The subject and the aims of the lesson are explicated and, in many times, classified as simple, basic or elaborated. This may give orientation for all children in an inclusive class, but could also categorise different children regarding to the used material.
- *Presentation of the subject in different forms*: The didactic-normative inclusive practice of presenting a mathematical object on different levels of representation is established as a basic practice for all pupils and all subjects.

## **Initiation and moderation of subject-related discourses**

- *Initiation and decision of participation by the teacher*: The participation of different children is initiated and controlled by the teacher, picking up students in connection with explicitly formulated questions and impulses.
- *Self-responsible participation practices*: The practices of “veto” and “help” were agreed upon in the class between teacher and children as conversation practices and now moderated by the children themselves. So, they take responsibility for the joint learning process. These practices include the opportunity to make alternative interpretations or to pick up and explain mistakes. Children with good mathematical understanding thus remain involved in the conversation and children with difficulties have the opportunity to decide for themselves when to ask for help. All learners in the inclusive class are involved, empowered and supported.
- *Creating subject-related discourses*: The conversation is condensed and directed towards mathematical aspects that the teacher considers important for all learners. Nevertheless, the children are asked to argue and to verbalise freely.

The reconstructed inclusive mathematical practices move in the field of tension between multi-layered-structural explorations and discursive discussions on the one hand, and condensed, focussed treatments on other. In the analysed scene, no obvious assignments of level are made, nevertheless, a closer analysis shows corresponding to other studies (e.g. Straeler-Pohl, et. al. 2014) that higher-performing children are asked for explanations and help by the teacher, but also by their classmates. Lower-performing children are just only asked to help with routine tasks or tasks to secure the basic material. In this sense, the aim that “all students are empowered to engage meaningfully in mathematical practices, for such engagement is the source of agency and identity” (Schoenfeld, 2020, p. 1173), has not yet been fully achieved.

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