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Strategies exhibited by students with autism spectrum disorder and their typically-developing peers when solving a generalization task: an exploratory study

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This paper focuses on algebraic learning and explores the strategies followed by 17 students with Autism Spectrum Disorder and 17 typically-developing students to solve a generalization task in a functional thinking context. We design and administer a questionnaire with different questions about specific (consecutive and non-consecutive) cases and the general case. Success in the proposed task was higher in the group of typically-developing students. Different strategies were encountered, predominantly (1) modeling with drawing and (2) operations, with neither group having much success obtaining the general term. We discuss the implications for teaching students with autism.

Keywords: Autism spectrum disorder, functional thinking, generalization, primary education, strategies.

Introduction

In the context of mathematics education in Spain, the official primary school curriculum establishes the requirement for students to successfully solve situations involving both numbers and their relationships (BOE, 2014). Along these lines, the early algebra presented is intended to introduce algebraic thinking in levels before secondary school (Blanton et al., 2015).

Some studies on functional thinking have focused on studying the strategies shown by students when solving generalization tasks that involve functional relationships. For example, Blanton and Kaput (2004) noted differences between the various strategies used depending on the age of the participants. In Early Childhood Education, students used counting and addition strategies, while some fifth graders managed to establish multiplicative patterns through words and symbols. Cañadas and Fuentes (2015) also studied the strategies shown when solving a functional relationship and concluded that the six- and seven-year-old students participating in their study responded by using strategies such as counting with drawings, direct answer, associating elements in groups and others.

Although the official curriculum in Spain is standardized and intends for all students to achieve the same goals by the end of the primary school, the reality is that classrooms are increasingly heterogeneous in terms of their students' characteristics. This diversity of students includes those with Autism Spectrum Disorder (ASD), which is a developmental neurobiological disorder that manifests itself during the first years of life and lasts throughout the entire life cycle. Its main symptoms are: (a) persistent deficits in social interaction and communication and (b) restrictive and repetitive patterns of behavior, interests or activities (APA, 2013). In addition, people with ASD may exhibit resistance to change, a tendency to maintain routines, deficits in executive functions, and difficulties

in understanding spoken and figurative language, and in inferring the mental states of other people (Ozonoff & Schetter, 2007).

Students with ASD are increasingly enrolled in mainstream schools alongside their typically-developing (TD) peers (Barnett & Cleary, 2019). Some of the deficits of the disorder, such as those in executive functions or verbal comprehension, may interfere with learning, particularly in learning mathematical concepts (e.g., Chen et al., 2019; Polo-Blanco et al., 2019; in press). Specifically, deficits in abstract reasoning may limit the ability to generalize (Minshew & Goldstein, 2002; Ozonoff & Schetter). Although there are studies that analyze the understanding of pre-algebraic functional tasks by TD students (Cañadas & Fuentes, 2015; Morales et al., 2018), there are few analogues for ASD students (Barnett & Cleary, 2019; Goñi-Cervera et al., 2021; Polo-Blanco et al., under review).

Considering the above, we expect that students with ASD will rely often on basic strategies like the use of modeling with manipulatives when solving a task in a functional context, and that they will show difficulties when generalizing the functional relationship.

Objectives

This study is part of a larger one whose main objective is to describe the mathematical abilities of students with ASD and their relationship with cognitive variables, such as executive functions. This work focuses on algebraic learning and explores the strategies employed by TD and ASD students to solve a generalization task in a functional thinking context in order to observe similarities and differences between the two groups of students in terms of the strategy use, and to describe possible difficulties in the ASD group.

Methodology

This research of an exploratory nature (Yin, 2017) relied on cases and controls, and compared the solution strategies of students with ASD matched with their TD controls when solving a task that involves the functional relationship $f(x) = 2x + 2$.

Participants

The participants were 34 students in grades 1 through 4 (6 to 9 years old) from 12 schools in Cantabria (Spain). Of these, 17 had ASD diagnosis and 17 were TD at the time of the study. The students were paired: each ASD with each TD control from the same school and grade as his/her ASD student pair. An inclusion criterion for participants from both groups was to have IQ equal to or greater than 70 as measured by WISC-V. Ten of the participants were enrolled in 1st grade (A1, A2, A3, A4, A5, and their respective TD pairs T1, T2, T3, T4 and T5), two (A6 and T6) in 2nd grade, six (A7, A8, A9, T7, T8, T9) in 3rd grade and 16 (A10, A11, A12, A13, A14, A15, A16, A17, T10, T11, T12, T13, T14, T15, T16 and T17) in 4th grade. The arithmetic mean and standard deviation of the IQs for the ASD group were 87.35 and 10.22, and 103 and 13.12 for the TD group.

The participants with ASD were recruited in different ways, by advertising the project through social media, the press, associations, guidance staff in schools and hospitals. Once an ASD participant was

recruited, a TD student from the same school and class to act as a control was sought via family members or the school's guidance staff.

Information gathering tool

A task used in Carraher et al. (2008) and Merino et al. (2013) whose structure involves the function $f(x) = 2x + 2$ was adapted and implemented. The adaptation consisted of using simple language, helping the students read the statement and guiding their work. The task began by presenting, as an example, a square table with four people around it and two square tables together with six people (Figure 1). They were then asked how many people could be seated if 3, 4, and 5 tables (consecutive terms) were joined, if 8, 18, and 100 tables (non-consecutive terms) were joined, and if any number of tables (general term) were joined. The task was given to the students in printed form so they could solve it individually in writing, or both in writing and orally, and they had manipulatives (blocks) that they could use if they wanted.

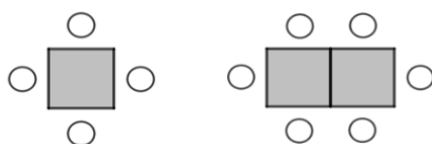


Figure 1: Introduction of the task with people arranged at one and two tables

The students answered the task individually in a classroom free of distractions, with only the interviewer present. Prior to this task, the students had solved three other tasks with the same interviewer in a previous session. First, the interviewer created a climate of trust with the students, letting them play with the blocks or drawing. If the student was not comfortable, the session was postponed. The solution process was videotaped and transcribed for later analysis. Both the written and oral responses were analyzed.

Analysis categories

Based on the strategies defined by Morales et al. (2018), the following categories of answers were established: (a) no answer: if the student does not provide a response, either verbal or written, or does not know the answer; (b) direct answer: if the student provides an answer that is difficult to justify or has no apparent relation to the task; (c) given number: if the student provides as an answer the number given in the statement; (d) modeling with manipulatives and counting: if the student models the situation using the available manipulatives; (e) modeling with drawing and counting: if the student models the situation using drawings; (f) counting: if the student performs the same actions as in modeling, but without using manipulatives or drawings; and (g) operations: if the student performs additive or multiplicative calculations orally or in writing.

Results

Next, and to simplify, we show the frequency of the strategies used by the participants in those questions that involve consecutive, non-consecutive and general terms (see Table 1). The results show how often a certain strategy is used by ASD and TD students. The numbers in parentheses indicate the number of right answers. The column "students" shows the number of students who used this strategy at least once.

Table 1: Frequency of Strategies and Success in Solving the Task $f(x) = 2x + 2$

Strategy	Autism spectrum disorder students				Typically-developing students			
	Case							
	Consecutive	Non-consecutive	General	Students	Consecutive	Non-consecutive	General	Students
No answer	0(0)	8(0)	4(0)	5	0(0)	8(0)	2(0)	4
Direct answer	9(2)	7(1)	6(0)	10	0(0)	1(0)	0(0)	1
Given number	3(0)	3(0)	1(0)	1	0(0)	2(0)	0(0)	2
Modeling with manipulatives and counting	0(0)	0(0)	0(0)	0	6(3)	4(1)	2(0)	4
Modeling with drawing and counting	15(12)	7(2)	2(0)	6	21(17)	14(7)	3(0)	9
Counting without manipulatives	3(3)	2(0)	0(0)	1	0(0)	0(0)	1(0)	1
Operations	21(0)	24(0)	4(0)	11	24(6)	22(5)	9(1)	14
Total number of strategies	51 (17)	51 (3)	17 (0)		51 (26)	51 (13)	17 (1)	

In what follows, the strategies are exemplified, focusing on the two most frequent in both groups. We also show some examples concerning other less frequent strategies.

As Table 1 shows, the *operations* strategy was the most frequent, in both groups of participants with ASD (49 occasions among all the sections) and the TD students (55 occasions among all the sections). However, no student with ASD used this strategy successfully, compared to three 4th-grade TD students (T10, T12, and T15) who correctly answered twelve times.

The students with ASD who used the *operations* strategy chose, for the most part, additive operations. For example, A1 exhibited the strategy by using an incorrect additive operation, answering for 100 tables: “200. Because $100 + 100$ is 200”. Student A8 also used an incorrect additive strategy, answering all the sections of the task by adding six to the number of tables given. Among the ASD students, only A13 resorted to multiplication (for example, for 18 tables, he multiplied 10 by 18 and for the general term, answered: “you have to multiply it by 10”). Two other students with ASD used multiplicative reasoning, although without explicitly stating the multiplicative operation. For example, A15 answered for the general term, “I counted by twos”, and A11 used expressions such as “four times four” or “you add ten times four”.

In turn, 14 TD students used the *operations* strategy. They proposed correct and incorrect additive operations. For example, T12 responded correctly to the consecutive terms, looking at his previous

answer and adding two. On the contrary, T1 used an incorrect strategy, because when asked "How many people can be seated if 100 tables are joined?" he described being unable to draw the 100 tables and referred to how "3 + 2" people sit at each table. A transcript of the conversation between the interviewer and the student (Figure 2) is provided below:

Interviewer: What if we have 100 tables?

Student T1: Well... two and three. Two and three and two and three and two and three

Interviewer: How would we put that?

Student T1: We have to put: three, two, three, two, three, two, three, two... in all of them. [...]

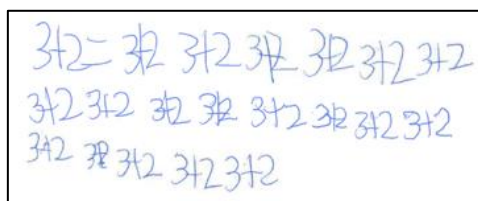


Figure 2: Operations Strategy for 100 Tables (T1)

Among the 14 TD students who used *operations*, 10 of them used some multiplication throughout all the sections, and of these, 7 used multiplications exclusively. Thus, T10 used a multiplicative operations strategy by reasoning "you have to multiply the number of tables by 2 and then add 2 for those at the ends", when asked for the general term.

The *modeling with drawing and counting* strategy was the second most frequent strategy, both for ASD (24 times) and TD (38 times) students. In addition, this strategy was the one that led to the most correct answers, especially in the consecutive cases.

Figure 3 (a and b) shows the solution for four and eight tables given by student A7. The first solution is correct, while the second is incorrect, as it places more guests than there should be at the ends. Figure 3 (c and d) shows the solution for 18 tables and the general term for student T7. The solution for 18 tables is incorrect, as he forgot to draw the people at the ends. The solution of the general term shows that T7 determined the number of people by drawing and counting them.

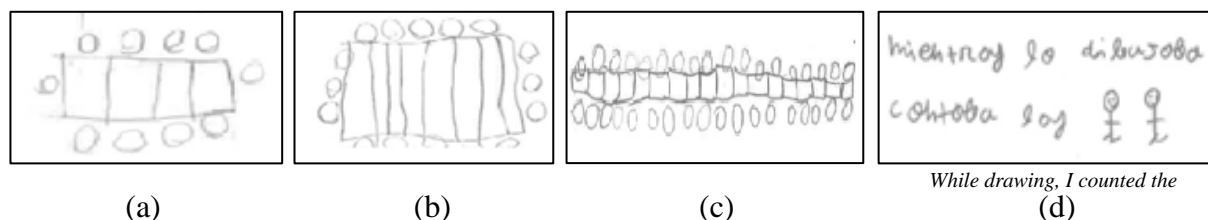


Figure 3: Modeling Strategy with Drawing

Other less frequent strategies

The remaining strategies were less frequent in the two groups of students. For example, on up to 12 occasions, four TD students used the *modeling with manipulatives and counting* strategy, which was not used by any student with ASD. Student T16 used the blocks made available during the task, modeling the situation for the different numbers of tables. The modeling was correct for 3, 4, 5, 8 and 18 tables (Figure 4, a and b), and incomplete due to insufficient blocks for 100 tables (Figure 4, c).

In addition, T16 also used blocks, particularizing to represent "any number of tables" and respond to the general term (Figure 4, d). He said: "counting these pieces... The middle thing was the tables and those who are here next door were the people".

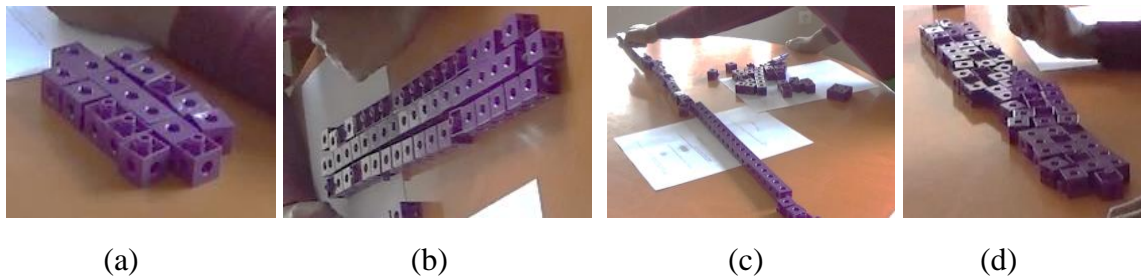


Figure 4: Modeling with manipulatives strategy for 5, 18, 100 tables and general term (T16)

Another infrequent strategy in ASD students was *counting without manipulatives*. Two students (A1 and T4) used a counting strategy for some questions. Thus, to determine how many people could be seated if 4 tables were joined, A1 swayed to the right and left, and each time his body tilted to one side, he counted one. Each time his body tilted to the middle he said "a gap here" meaning it was a table and no one sits in a table. He said: "one, a gap here, two, a gap here, three, a gap here...". Student T4 also employed a counting strategy by responding to the general term by writing "using the tables and counting them".

Regarding the *direct answer* strategy, it was only used by one TD student and for just one question, while 13 students with ASD gave these answers on 22 occasions. After being unable to draw 100 tables, T4 replied: "A thousand... I don't know. Or 50 or so... A thousand".

In the group of students with ASD, an example of the *given number* strategy was provided by student A12, who did not justify his answer to the problem in any section. However, his answers show a relationship $p = t$, where p is the number of people and t the number of tables. His answers for the terms $t = 3, 4, 5, 8, 18$, and 100 were $3, 4, 5, 8, 18$, and 100 , respectively. In addition, for the general term he wrote the number zero. This could be related to the absence of a specific number of tables or people in the statement. Students T6 and T16 also responded with the same number of tables to the question that involved 100 tables.

Conclusions

With this exploratory study, we are contributing to the start of an investigation comparing algebraic thinking in both TD and ASD students in primary education. Success in the proposed task was higher in the TD group, although it was accessible to some of the students with ASD.

In both groups of students, the predominant strategies were *modeling with drawing and counting* and *operations*. The task increased in difficulty as the number of tables rose, with the question involving the general term proving very difficult. Contrary to what we expected, *modeling with manipulatives* was not used by any student with ASD. One reason that could explain this is that the material provided (blocks) was not adequate to represent the situation. Given the type of literal thinking common in ASD students (Happé, 1995), they may have had difficulty imagining that the blocks represented the tables and people around them. However, they did not show difficulties in implementing the modeling strategy through drawings. The frequent use of a *direct answer* strategy among students with ASD

and given that in most cases it did not lead them to the correct answer, could be associated with a poor understanding of the task. The preference in ASD students for following additive strategies agrees with previous studies carried out with TD students and enrolled in lower grades (Blanton and Kaput, 2004). In addition, the choice of strategies based on drawings agrees with previous research involving 5- and 6-year-old TD students (Cañadas & Fuentes, 2015) and it is in line with other works about problem solving by ASD students (Polo-Blanco et al., 2019).

As a future line of research, the sample could be expanded to delve into possible differences within the group with ASD, and to see if subgroups of normal-performing and low-performing students are identified, in line with previous work (Chen et al. 2019). The aim of this study is to enhance the research on mathematical learning in students with ASD. In particular, this study serves as an aid to those teachers who work with ASD students enrolled in mainstream classrooms who follow the official curriculum, alongside their TD peers.

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