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“Counting with all children from the very beginning”: One attempt to promote early arithmetical skills based on part-whole thinking

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Currently, it seems to be difficult for teachers to create **substantial learning environments** of high quality to manage the arithmetical heterogeneity in First Grade. The 13 **substantial learning environments** based on part-whole thinking of the large-scale project “Counting with all children from the very beginning” might be one attempt at that. Following design-based research approaches, the **substantial learning environments** were first developed and then implemented in five mainstream classes with a total of 96 students in the Beta-cycle over a ten-week period. Data from the second cycle is the basis for the results presented in this paper. Preliminary results indicate that not all teachers succeeded in identifying **problematic areas** in their children’s arithmetical learning, especially in the instances of children who have underachieving mathematical skills and children with special educational needs (SEN).

Keywords: Early arithmetic education, part-whole thinking, computation without counting, inclusive mathematics education, concepts for mathematics education

Preliminary remarks

Every child has the right to access education of good *quality*, as international organisations such as UNICEF, UNESCO or the European Union affirm. Thus, the focus regarding inclusion of all children in developed countries is now on promoting the quality of the teaching and learning situation (Florian, 2008). However, there seems to be uncertainty in German-speaking countries about how to create inclusive learning environments of high quality for *all* children in mainstream classes, not only, but especially in early arithmetical education (Korff, 2016). Hence, the present research project seeks to design **substantial learning environments** based on *part-whole thinking* for First Grade.

In order to allow others to make comparisons between inclusive school education in different nations, the author of this paper draft the current situation that teachers might face regarding inclusive practices in early arithmetical education in *German-speaking* countries, with a focus on Austria, where the research of this paper was conducted.

With the passage of time, the Austrian education system, like that of many other countries, has made great progress as far as the issue of inclusion is concerned. In particular the ratification of the UN Convention on the Rights of Persons with Disabilities in 2008 sparked debate about *inclusive* education (as in many countries) and the necessary measures for the common compulsory school system. Nonetheless, *even today*, Austria, like many other countries in the world, still focuses on a multi-track approach: Parents are free to decide whether to send their child to a (local) mainstream school or a special school. In total, in **2020-21**, approximately five percent of the children in compulsory education belong to the group denoted as having special educational needs. (Statistik **Austria, 2021**). It should be made clear that these data refer *merely* to the positioning-oriented definition of inclusion (see below).

By the time children enrol in compulsory school in German-speaking countries (between the ages of six and seven), they have received considerably different targeted *early mathematical education*, depending inter alia on different educational plans in kindergarten and the mathematical knowledge of the kindergarten staff. The educational plans in kindergarten differ within and between German-speaking countries – in Austria, the educational document consists of a few pages, and early mathematical education is dealt with in a few words. The year before school enrolment all children will already have attended kindergarten. By the end of First Grade, it seems to be widely accepted within the relevant German literature, that all students should learn how to *compute without counting* fluently with numbers up to 20 (see also Häsel-Weide & Nührenbörger, 2013). Therefore, it seems to be helpful if teachers first get children to grasp that numbers (at least up to 10, if not up to 20) are composed of other numbers and then, based *merely* on that knowledge, initiate their understanding of addition and subtraction (see also Schipper et al., 2015; Gaidoschik, 2019a; 2019b).

The author of this paper is aware that in the field of *international* mathematics education, designing **substantial learning environments** of high quality in early arithmetical education is a central element for colleagues from other countries, where there are also likely to be challenges for teachers to teach mathematics inclusively. Thus, the following considerations concerning the implementation of part-whole thinking in early arithmetical teaching may apply to other nations as well.

Considerations on inclusive education in early arithmetical teaching

In the international inclusive education community, there is *no* clear working definition of inclusion. Consequently, in many nations inclusive practices are handled very differently, and these practices can differ also between schools in one country, or even within the same school (see also Booth & Ainscow, 1998). To specify inclusion, it has been defined as both a one-dimensional and/or a multidimensional oriented approach. The former means physical positioning and teaching of children in an ordinary class (Vislie, 2003), the latter represents a wide plurality with other qualities (for example a positive learning environment to promote all children's learning development) in addition to positioning. Many authors agree that inclusion in developed countries is now *not* merely about physical positioning, but has to comprise several aspects. Empirical research has also shown that positioning alone is *no guarantee* of meeting the needs of all children (see also Mitchell, 2014; Vislie, 2003).

The international *mathematics* education community will certainly respect the multidimensional oriented approach of inclusion. To understand how pupils learn, the theory of constructivist learning is vital. The students actively construct knowledge, and of course, there is huge variation amongst children with regard to how quickly and how sophisticatedly they are able to solve mathematical problems: "Children with different developmental backgrounds may well be able to get the same answers on an arithmetical task, but how they do so might differ significantly" (for more details of the discussion between constructivist views and other views on mathematics education theory and research, see Steffe & Kieren, 1994, p. 719).

Thus, in recent decades, some considerations on general inclusive (mathematics) didactics and *few* substantiated didactical concepts for inclusive mathematics education in primary school have been developed *in the mathematics education community*, but I am not aware of any (apart from the

following), that *explicitly* deal with the utilization of the concept of numerical part-whole thinking in First Grade.

Considerations on the TIGER concept (Gaidoschik, 2019a)

All children in First Grade classes should be aware of “numbers as compositions of other numbers” (Resnick, 1983, p. 114), also termed the “part-whole schema” (Resnick, 1983, p. 115). Resnick describes this goal as “probably the major conceptual achievement of the early school years” (1983, p. 114), therefore a preferable alternative to computing by counting. According to this concept, children should learn to divide numbers in terms of part and whole relationships for all numbers at least up to 10. Children thus can be led to understand that any whole number can in fact be divided into smaller numbers, and so the number 7 for example, can be divided into 2 and 5. When put together, the parts 5 and 2 fulfil the requirement of being equivalent to the whole number 7. With that understanding, they might be able to solve addition ($2+5$; $5+2$) as well as subtraction problems ($7-5$; $7-2$) – without counting.

The *utilization* of the concept of numerical part-whole thinking in First Grade seems to be the standard in international mathematics education as in for example the “Number framework” (New Zealand Ministry of Education, NZME, 2008) and the “Mathematics programme of study” (UK Department of Education, 2013). Quite when this concept is taught in the academic year can differ across nations. Part-whole thinking is incorporated as *early as possible* in the school programme in these guidelines for teachers’ actions. The New Zealand Number framework explicitly states:

It is important for you to recognise that part-whole thinking is seen as fundamentally more complex and useful than counting strategies. One reason is that counting methods are strictly limited, whereas part-whole methods are more powerful. Counting strategies are an inadequate foundation for these ideas, and this means that for counters, many advanced number ideas are inaccessible. Therefore, your major objective is to assist students to understand and use part-whole thinking as soon as possible. (NZME, 2008, p. 7)

Current models and learning trajectories of these and other guidelines name different numerical stages for counting strategies, and give recommendations for teachers to follow and to work with their classes from one stage to the next. However, Gaidoschik (2019a) describes in detail a structure-genetic didactical attempt as an alternative approach to the already existing ones. The concept he has named TIGER (Gaidoschik, 2019a) attempts to show how to teach children addition and subtraction on the basis of *part-whole thinking*. The author of this paper will provide a more detailed explanation of the TIGER concept and list some examples of the learning situations of the current project.

The TIGER concept of part-whole thinking

The TIGER (“Teile im Ganzen Erkennen und damit Rechnen”) concept, created by Gaidoschik (2019a), focuses on solid number concepts in early arithmetical teaching. The 13 learning situations of the current research project are based on these ideas. Drawing on this concept, the learning situations consist of the following three fundamental aspects, whereby the researcher (Gaidoschik, 2019a, p. 424) recommends teachers “work with children in all three Fields A to C more or less *concurrently*” and “it would not help to stick to one single theme for too long in a row [...] the sites relate to each other”:

Firstly, (A) a child needs to acquire a solid understanding of counting competences and should thus master the significance of cardinality as a means to understand "how many of whatever". This includes teaching the "counting principles", defined by Gelman and Gallistel (1987). The **substantial learning environments** that have been designed try to allow all children to consolidate their *individual* counting competences in the counting activities, for example by counting the number of children present in the class. Counting is used almost every day: Is anyone absent today? Is the number different from yesterday and if so, how has it changed? Of course, First Grade classes will be quite heterogeneous in their counting competences: *Many* children will already have no problems with counting (forwards and/or backwards), while some children will need more counting activities than others. As a benchmark, all children should (at an earlier or later stage) be able to count easily up to at least 10, both forwards and backwards (Gaidoschik, 2007).

Secondly, (B) it is important that a child is able to judge very small sets without counting – through direct pattern recognition - and uses *subitizing* for smaller quantities (up to 4) and uses *perceptual subitizing* for quantities greater than 3 or 4. Perceptual subitizing is only possible for children and indeed us adults if bigger sets are presented in a structured way, so that counting can be avoided (see also Clements & Sarama, 2009). The learning situations that the author of this paper has created include activities to teach part-whole thinking, including in particular activities *using fingers*, recommended by Gaidoschik (for more details of fingers as a very useful "material", see Gaidoschik, 2007). Thus, for example, children should learn at the beginning to show the right number of fingers without actually counting them, and without extending them one by one. As a starting point we should focus on (at least) all numbers up to 5. In the following activities, structured dot patterns, dice, then ten-frames are useful materials in the learning situations. All in all, the use of materials in the current research project is important for children to acquire *mental pictures* and finally automatized knowledge but the use of materials is limited to a *few* structured arithmetical materials, as mentioned above. Note that all the activities with material should be presented in a *structured* way so that children are able to identify quantities (first for quantities up to 5 and later for all numbers at least up to 10) at a glance. The focus of such early activities is to teach children the interpretation of these structures, their relations to 5 and 10; and especially their part-whole *compositions* (5 as consisting of 2 and 3, or of 4 and 1, etc.).

Thirdly, (C) a child should *compare* quantities and numbers. On the basis of one-to-one matching up, children should learn and/or consolidate that no counting is needed for pairing the items for number comparison such as identifying without counting (Gaidoschik, 2007). The learning situations focus on activities that foster children's ability to compare quantities. Thus, inter alia in the learning activity "Throwing Tiles" they use one-to-one matching without needing to count the tiles, they create rows and then compare in pairs their quantities. In "Finger use" they should "show the right number of fingers without counting them" and then compare the fingers with other quantities and/or numbers.

Organising the created **substantial learning environments in the classroom**

To achieve a *balance* between children learning individually and learning together in (early arithmetical) education, which is the general consensus within the mathematics education community in German-speaking countries (see for example, Häsel-Weide & Nührenbörger, 2015), the current

learning situations are divided into: (I) Whole-class instruction; (II) Individual Work; (III) Teamwork. Each teacher decides, depending on the class situation, which organisational structures are best and for which children. Presumably, for some children it might be useful at some point to be given additional or different activities to practice and/or consolidate their arithmetical knowledge. Nevertheless, as Feuser argues (1997), all pupils should, as much as possible, be working on the same activity even if it is being carried out on a different level. The learning situations in the current paper respect these ideas and try to give specific ideas for teacher's action in classes.

The research project, research method and research question

To be able to examine the effectiveness of the learning situations created, collective case studies following the framework of design-based research approaches (Euler, 2014) have been conducted. In the Alpha-cycle, conducted in the school year 2019-2020 with 45 First Grade students in two Tyrolean mainstream classes, video-recorded conclusions were drawn for the implementation and further development of the **substantial learning environments**. These results were processed in an additional cycle. The Beta-cycle, which is relevant for the present data analysis, was carried out in five Tyrolean primary school classes in the school year 2020-2021 with 95 students over a 10-week period in First Grade in inclusive mainstream classes. Before starting the classroom implementation, all teachers were instructed in using and adapting the learning situations. To generate insights into children's learning and their mathematical thinking, each lesson was video recorded, and framed as the basis for data analysis using "Qualitative Content Analysis" (Mayring, 2015).

The pairs of children analysed are, as far as possible, unchanged, so that conclusions can be drawn regarding the progress of their learning. Furthermore, "Assessment of Teaching-Learning-Situations in Mathematics of the Early Grades" (Steinweg, 2010) is used for data analysis, as an additional level of analysis. Steinweg's idea of dimensions focuses on the *teachers'* possibilities for action in First and Second Grade. The combination of both gives indications for (further) development of the **substantial learning environments**.

The questions guiding the **paper** are: (1) Do the analysed transcripts of the substantial learning environments based on part-whole thinking indicate any difficulties of understanding for many children and/or a particular group of children with certain arithmetical knowledge? (2) Are there difficulties in teaching the substantial learning environments based on part-whole thinking; and if so, are these observed in *several* scenes?

Preliminary results

It should be made clear that it is not the intention of the author to draw conclusions from specific scenes or class observations. The idea is not that a learning activity should be done just once in a class or that *all* children should be able to solve the arithmetic task (at the same time). Thus, the dimensions of Steinweg (2010) – consistent with the author's intention – are considered 'competence-oriented', that is to say, the (analysing) focus is more on the teacher's achievements than on their failings. When Steinweg's dimensions are used in a way that *several* scenarios of teachers' actions in classes are analysed, conclusions can then be drawn to give teachers targeted indications for the work in daily classes.

Drawing on these ideas, preliminary outcomes of the analysis of *some* transcripts using “Qualitative Content Analysis” (Mayring, 2015) indicates content related **problematic areas** in early arithmetic education: These are problematic areas of arithmetical knowledge that might be common to many children and/or a particular group of children. To illustrate by means there are some examples. In “Throwing Tiles” children take turns throwing 10 reversible tiles, which have a red side and a blue side. Then together they have to match one red with one blue tile, using one-to-one matching without counting. They create two rows to identify who has more tiles, and how many more there are of one colour compared to the other. Regarding Steinweg’s idea of mathematical dimensions (2010), *not* all teachers were able to foster children’s arithmetical competences and not all teachers were *consistently* able to identify these **problematic areas of arithmetical knowledge**. In some sequences, for example in this sequence “Throwing Tiles”, some teachers did not realize that children require a solid understanding for one-to-one-matching up as a basis for further arithmetical strategies. Especially for underachieving children in mathematics and children with SEN the understanding and utilization of the one-to-one-matching process was *consistently* difficult in several instances on several days.

Again, it is the same group of children that seem to have difficulties in the following scenarios: One of the advanced activities of “Finger Use” is to work out the 5-plus-x structures of the numbers 6 to 10. The children play in pairs, and one says a number, for example 7, and the other child, with hands underneath the table, without looking at their fingers, has to show the right number of fingers without actually counting them, and without extending their fingers one by one. Their partner then checks the solution. In further exercises, they have to describe what they would do, without actually doing it. For example, “Seven. I have to show one full hand and two fingers on the other hand; in total that makes seven”. Drawing on Steinweg’s idea of mathematical dimensions (2010), even in this activity not all teachers were able to foster the use of fingers. For these children it might be worthwhile to give more thought-provoking stimuli to acquire part-whole thinking. Thus, the base numbers 5 and 10 (5 fingers on one hand, 10 fingers on both hands together) should be focused on first.

Thus, a *preliminary indication* of the analysed transcripts is that particular attention should be paid to the mathematical thinking of underachieving children and to children with SEN, and it is evident that teachers seem to find it especially difficult to meet the learning needs of those children.

Closing remarks

The current research project on early arithmetical teaching in First Grade, presented in this article, is *one* attempt to face challenges in teaching (arithmetic) inclusively. As has been outlined, the **substantial learning environments** that have been created are based on the TIGER concept (Gaidoschik, 2019a). The author of this paper agrees with the ideas of many researchers in the relevant mathematics education literature, that children who need special support in learning (mathematics) do *not* learn differently, nor need completely different *concepts* than are already recommended for all children (see Gaidoschik, 2019a, 2019b; Moser Opitz, 2008; van de Walle, 2004). Thus, their learning behaviour is not completely different from that of their classmates. Of course, some mathematical content might be acquired by some children at a later stage than other children, but all children should learn them at *some* point during their mathematical education. From

the structure-genetic didactical analysis approach this idea seems logical because the mathematical content is the same for all children.

Nevertheless, empirical evidence (see for example, Pfister et al., 2015), as well as *some* interim results of this research project indicate how difficult it is for teachers to implement already *existing* concepts of mathematics education in their classes, even for those who participate voluntarily in in-service development programmes.

Thus, one conclusion of this research is that it is essential to enhance teachers' knowledge of relevant pedagogical methods *and* content, so that teachers can then fulfil all pupils' differing learning requirements. The other conclusion is that data analysis has also shown that for teachers to get a collective picture of their students' arithmetical knowledge, it might be worthwhile to identify **problematic areas of arithmetical knowledge**. The awareness of these **problematic areas of arithmetical knowledge** is crucial for *quality* teaching in inclusive mathematics education. Of course, this places high demands on the teacher's abilities.

Yet, in further data analysis something that needs to be evaluated is whether such **problematic areas of arithmetical knowledge** involve many children and/or a particular group of children in *different* classes that participated; and whether it was *consistently* difficult for the teachers to identify these **problematic areas of arithmetical knowledge** and to foster children's competences. This would allow us to develop further indications for the work with teachers and children.

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