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Problem-solving by students with Asperger's Syndrome

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This study considers the use of heuristic strategies by pupils with Asperger's Syndrome in a problem-solving workshop. The strategies aimed to mitigate learning difficulties associated with the syndrome as a result of deficits in central coherence and executive functioning by introducing manipulatives and visual organisers to support the processes of understanding and finding solutions to the problems. The different heuristics employed during the comprehension, execution and verification phases, in addition to the visual and tactile nature of the materials, promote diversity in terms of the chosen strategies, and stimulate the pupils' abilities. The limitations imposed by literal thinking and cognitive rigidity, alongside weak attention and organisation, can be reduced by a more schematic, as opposed to mimetic, visual content, and a more explicit sequencing of the problem statement.

Keywords: Problem Solving, Asperger syndrome, inclusion, heuristic, inclusive Mathematics.

Introduction

This study aims to contribute to inclusive education regarding the needs of children with Asperger's Syndrome (AS). Specifically, it considers how a problem-solving approach (PS) can be adapted for use with students with AS, and trials classroom-based strategies that can be applied in any classroom. The mathematical focus targets PS, as this approach allows for the incorporation of tables, diagrams and other visual material, encouraging the organisation of data and developing cognitive flexibility (Liljedahl et al., 2011). This enables the teacher to design an environment conducive to diversity of thinking through the application of different heuristics, generalizable to other contexts.

The study explores how specific pedagogical strategies might be deployed in order to minimise the characteristics of AS which impede mathematical learning, and maximise those that promote it (Roos, 2013). The strategies were trialled in PS workshops with children from the Huelva Asperger's Syndrome Association (AOSA) and the results were analysed. The research questions were the following: What strategies facilitate PS with children with AS? What heuristics do they bring into play when they attempt to solve problems?

Theoretical framework

AS is a pervasive developmental disorder within the autism spectrum. The majority of students with AS have average to above-average intellectual abilities, but present characteristic personality traits and abilities. Research by Happé and Frith (2010) suggests that people with AS present deficits in central coherence, which translates to an incapacity to infer the larger meaning inherent in a communicative situation. The result is that they tend to process meanings without reference to the broader context, which can be manifested in difficulties with understanding non-verbal messages. People with AS tend to prefer fixed routines and patterns of behaviour, usually showing emotional vulnerability and often have a limited tolerance of frustration (Carr & Seah, 2019). Their motor skills are characterised by a low degree of coordination and limited fine motor skills. Many people with AS

show weak executive functioning, which limits their flexibility of thinking, the capacity for internal representation, and organisation (Bae et al., 2015). By contrast, they show talent in certain areas, such that they may excel in terms of visual and mechanical memory, auditory perception, the ability to focus on detail, and pattern recognition (de Gimbattista et al., 2019). Into the bargain, those with savant syndrome display astonishing skills in different branches of the arts, and may present hyperlexia, hypermnnesia or hypercalculia (Happè & Frith, 2010).

With regards to these characteristics, we consider PS ideally suited to pupils with AS as it provides a narrative context in which they can participate, helping them to assimilate concepts and processes. Finding solutions to problems allows the pupils to try different strategies and ways of thinking. Within the confines of this study, it provides with an excellent opportunity for exploring what it is they need to understand and process from the problem statement in order to proceed to a solution.

Improvements in the PS of pupils with AS through specific strategies have been reported. Among these are interventions aimed at improving sequencing (Klaren et al., 2017), which suggests that structuring the solution into stages would be beneficial. In this regard, the strategies and heuristics described in the work of Polya (1945) can be highlighted for the support they offer in interpreting the information and organising the answers. Various studies illustrate the benefits of simplifying and contextualising problems in terms of pupils' interests, deploying a set of strategies for representing the information and providing stages for solutions (Delisio et al., 2018). Also worth noting are transversal methods, such as the TEACCH programme, which use augmentative and alternative systems of communication to support written language with pictograms (Bruno et al., 2018). Finally, the literature review by Kribbs and Rogowsky (2016) brings together references to PS which foreground visualisation as the key feature, an essential issue in eliminating and/or limiting problems deriving from comprehension of written texts.

Methodology

In collaboration with the AOSA and the INCLUREC Project¹, we organised a two-day mathematics PS workshop featuring problems adapted for an age range of 6 to 18. The AOSA psychologists grouped the participating students according to age and characteristics. There were varying levels of ability; one group included pupils with ADHD (Group 1); two included pupils with limited cognitive capacity (Groups 2 and 4). After problems selection, a set of manipulatives were made, designed to illustrate through pictograms the sequence described in the problems. These were accompanied by a visual organiser for recording the information and step-by-step solutions. The materials and problems were first trialled with a control group of schoolgirls not diagnosed with AS, of similar ages to those attending the workshop, aiming to gain experience of using the materials applicable to the workshop itself. The original selection included more problems than were ultimately used, as experimentation

¹ INCLUREC (from the Spanish for 'inclusion resources') is a Teaching Innovation Project (University of Huelva), which maintains a permanent workshop for developing and adapting teaching resources for students with special educational needs.

prompted a degree of reduction and adaptation to the needs of the pupils towards whom the workshop was directed (see Table 1 below, outlining the potential heuristics for each problem, according to each solving stage, drawing on Carrillo, 1998):

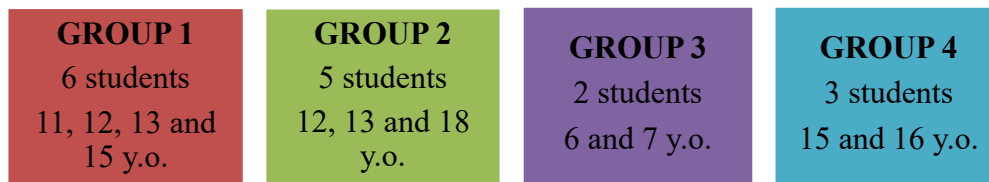




Figure 1: Workshop groups

Content: Those problems that were pre-algebraic in nature were simplified, and those involving fractions were discarded altogether, on the premise that the workshop was oriented towards the use of strategies rather than arithmetic. The topics chosen for the problems was especially significant, and featured situations of interest to the students relating to their everyday lives; they also paid special attention to the characteristic of people with AS regarding specific topics.

Resources: A laminated visual organiser was supplied, which allowed the students to place the illustrations in position with Velcro, with room in a table alongside for them to note down any information that would help them to understand and carry out the problem. The set of illustrations and organiser was complemented by differently coloured tongue depressors to represent the information in tactile form. In the case of the second problem, involving farm animals, the pupils were free to represent the elements involved however they wished, but unlimited numbers of coloured dot stickers were available for representing the animals. In both cases the material allowed different ways of reaching a solution, given out directly after the instructions had been carefully read and sequenced.

Table 1: Problems, workshop materials and heuristics arising at each stage

These figures are made from sticks which are all the same. How many sticks are needed to make 4 figures like these? And 7 figures?		1B (Group 3)  1A (Group 1, 2 and 4) 	Coloured tongue depressors Instructions with pictograms and illustrations Visual organiser Illustrations of 1-7 figures, held in place by Velcro
Heuristics and stages in the solution	<i>Identification and understanding</i>	Promoting perception: observing the illustrations and the information they represent. Representation: reproducing the information in the problem using depressors, pictures held in place by Velcro and a table for charting progress. Exemplification: the illustrations enable the pupils to see examples of how to construct the figures with the right number of sticks. Organising information: The organiser lets students note down the information alongside the illustrations.	
	<i>Planning, exploration and execution</i>	Finding regularities: Noting whether the number of sticks varies from one figure to another.	
	<i>Verification</i>	Generalising: finding a pattern which relates the number of sticks to the figures.	

2 (Groups 1, 2, 3 and a student from group 4) Juan lives on a farm with some chickens and rabbits. If he counts all the animals, he counts 10 heads and 28 legs. How many chickens are there on the farm? How many rabbits?		Coloured dot stickers Question statement with pictograms
Heuristics and stages in the solution	<i>Identification and understanding</i>	Representation: Extracting the relevant information from the question statement, supported by pictograms and illustrations.
	<i>Planning, exploration and execution</i>	Focus on a single variable or condition: Starting from the number of heads, discard the other information, which then allows the pupil to simplify the beginning of the solution. Trying numbers, randomly or systematically: Assigning a value to the number of chickens or rabbits. Foreground the progress made along the way: Setting sub-goals helps pupils discover the relationship between the number of legs and heads as the pupils draw them.
	<i>Verification</i>	Checking the consistency of the solution and the process: While the pupils are drawing the animals, they check their solution by repeatedly counting of the number of legs.

Data collection from the PS processes during the workshop was carried out by means of observation and video-recording, along with the students' own comments while they were working their way through the material. The subsequent transcription enabled units of information to be obtained relating to the application of the heuristics, explanations of the strategies and difficulties arising at the comprehension and solution stages linked to the characteristics of the students. These units were then categorised according to the heuristic used and the pedagogical aspect involved, allowing for a comparison between the workshop and control groups.

Results and discussion

Use of heuristics

All those participating in the workshop managed to establish different generalisation strategies in problems 1A and 1B, developed through finding regularities and patterns among figures. Some added 9 sticks for each additional figure, developing a factual generalization (Radford, 2001); and others focused on the shared stick forming part of the heads, multiplying the number of figures by 10 and subtracting from that the number of figures less 1 ($10n - (n - 1)$) through contextual generalization, since they could explain the construction of any given figure but did not manage to apply symbolic language (ibid., p. 83). In some instances, however, pupils had difficulty in generalising the pattern to larger numbers, such as 9 or 10. It was nevertheless very clear that the use of the manipulatives (i.e., the tongue depressors) and the table helped the pupils to sequence each step of the task. Ramón (Group 1) initially rejected the shared stick hypothesis. Instead, he noted down the data in his table as he counted the sticks for each set of figures, in order to establish a count back strategy, ignoring the visual evidence of the figures he had made. Unfortunately, his calculations were incorrect, largely because of his poor concentration skills due to ADHD.

Some students, such as Miguel Ángel and Juan (group 2), worked on a simpler mathematical plane by constructing successive figures and then counted the number of sticks. In this instance, the use of the table for keeping a running total of sticks for each new figure, enabled them to see the numerical pattern involved and anticipate the expected results. The implementation of this problem, albeit under

guidance, gave the students the freedom to choose whether to use the organiser or the depressors according to their needs (Figures 2 and 3).

heads	legs	animals
10		
19		
28		
37		
46		
55		
64		
73		
82		
91		

Figure 2: Generalisation in problem 1B

heads	legs	animals
10		
19		
28		
37		
46		
55		
64		
73		
82		
91		

Figure 3: Representation in problem 1A



In Problem 2, the pupils were given free rein to use the materials in any way they chose to represent the animals. The results (Figures 4-7) ranged along a scale from the more literal to the more figurative. What can be noted from these illustrations is that the problem is more clearly understood when the representation is figurative (Kribbs & Rogowsky, 2016), as it draws attention to the relationship between the problem data and encourages the pupils to focus on the essential concepts involved rather than a mental image of the animals. The children whose representations approximated to the literal of Figure 5 later found it much harder to substitute rabbits for chickens, and vice versa. This kind of difficulty was not seen in the control group.

Establishing sub-goals promoted the development of pre-algebraic thinking, in that the relationship between two unknown variables needed to be determined. Starting from only one of them (the number of heads), represented by drawings or with dot stickers, the relationship with the number of legs could then be established, through use of the heuristic “trial and error”, by which the number of rabbits and chickens was adjusted according to whether the total number was greater or smaller than that given in the problem statement. An unexpected strategy was the systematised trials (Figure 6), since pupils with AS tend to learn sequentially, have weak organisational skills, and lack flexible thinking (Bae et al., 2015), all of which were deployed in this solution.



Figure 4: Representing rabbits by drawing their heads and legs (Problem 2)

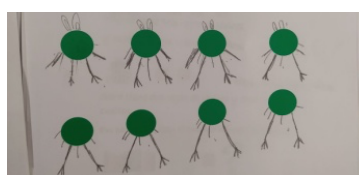


Figure 5: Substituting chickens for rabbits (Problem 2) resulted in 6-legged animals

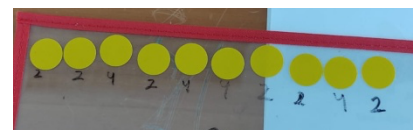


Figure 6: the number of legs is written underneath each sticker (Problem 2), alternating until the student hits the target

Educational aspects

Although the groups varied and there were differences in executive functioning, the key aspects in the teaching-learning process were the use of visual material and the sequencing of the task. Despite the organiser being available in Problems 1A and 1B, which allowed the students to separate out the information into rows and columns, and aids such as the depressors and illustrations on Velcro being

supplied, the initial phase of the problem was characterised in 80% of the cases by the wait for instructions or suggestions on how to proceed, such as whether errors could be erased (Gabriel, Group 4), and questions about using the material, and the steps to follow in the solution. The latter example refers to Cristóbal (Group 2), who, despite finding the pattern to follow in the problem, stopped after completing each row to wait for further instructions. This kind of behaviour may have been the result of over-monitoring on the part of tutors, teachers, and family members, by which students with specific needs inadvertently have their range of responsibilities much reduced, disconnecting them from their own learning (Roos, 2013). Mechanisms of self-instruction also occurred, by which pupils attempted to set goals and meet the aims of the problem, although in the majority of cases the pupils followed the steps of their chosen strategy to the point of forgetting why they were doing so, highlighting limitations in their short-term memory. This was the case with Jaime (group 1), who organised the information in Problem 2 into a table but was unable to establish the relationship between the heads of the animals and their legs (Figure 7). Using stickers to represent both animals indiscriminately only increased the degree of difficulty, and he became more frustrated when he found he could not apply his strategy, nor remove the stickers (“I’m very bad at maths because I complicate things a lot”). Although the pupils needed continuous guidance, and were limited by cognitive rigidity

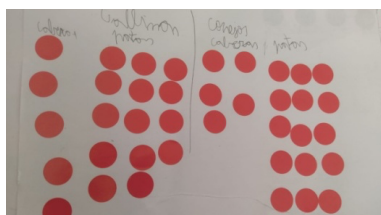


Figure 7: Tabular presentation of chickens and rabbits' heads and feet (Problem 2) using dot stickers

the visual organiser and tongue depressors helped to promote a degree of diversity of thinking (Problem 1). Participants constructed the figures and noted down the number of sticks in the table, repeating the process as many times as necessary until they recognised the pattern, some requiring just one construction, others needing more examples to complete or check their calculations.

- David (4): It's like it was the 9 times table. You work it out according to the people in the drawing and you add 10 from the first one, which is the odd one out and doesn't follow the pattern.
- Ignacio (2): I'm changing the numbers in the table. To this one (a number in the units column) I take one away and I give it to the other one (a number in the tens column).
- Tutor: What if you didn't know the previous one? 9 people, for example.
- Ignacio (2): Then I take away 2 and I give it to the other one.
- Marta (1): For each figure you add 9 sticks to the others because they share this stick (pointing to the illustration held in place by Velcro).
- Tutor: And how do you then know how many sticks there are in 7 people?
- Marta (1): I count the number of people there are ($\times 10$) and I take away the shared sticks.

The acceptability of each strategy proved to be key to combatting frustration, and was a significant motivating factor for a group of students who were used to a kind of mathematics in which there was only one way to find a solution. It is worth noting that during the carrying out of the workshop, the students in Group 4 decided on comparing strategies for themselves, out of curiosity. An atmosphere of expectation was also created by presenting the problems as challenges, graded according to difficulty, such as the progression from 1A to 1B. This intrinsic motivation, together with empathy, proved to be the keys to emotional control and self-esteem (Goleman, 1995).

With respect to the visual nature of the solutions, it was clear that the use of colour made the difference in both understanding and solving Problems 1 and 2 (Figures 8 and 9). Nevertheless, literal thinking once again influenced the pupils' capacity for visualisation: Gabriel (Group 4) rejected the pattern of the shared stick (Problem 1B) because of the thickness of the lines in the illustrations; while Salvador (Group 2) refused to try and solve Problem 2 because the shape of the stickers did not match his mental image of what rabbits and chickens should look like.

Salvador (2): I know that you are referring to that rabbits have four legs and chickens two, but this is not a rabbit (draws a rabbit in great detail on the page).

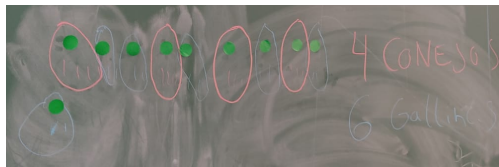


Figure 8: Using dot stickers and different colour chalks for each animal to improve counting and understanding (Problem 2)



Figure 9: Using differently coloured sticks to distinguish what needed to be added to each figure (Problem 1B)

Conclusions

The visual element and colours helped to reduce the difficulties associated with understanding the problems, by complementing the verbal meaning and reducing the decoding process (Delisio et al., 2018). Nevertheless, literal thinking became an obstacle when the choice of illustration or visual representation did not match the mental image of the pupils. Hence, it is important to transform illustrative representations into schematic ones (Kribbs & Rogowsky, 2016) which form part of the solution itself, and are not limited to decorative additions or demonstrations of artistic ability. Such representations could also help to overcome limitations associated with visuospatial deficits and weak mental flexibility (Liljedahl et al., 2011). On another level, orderliness and organisation were both important and necessary (Klaren et al., 2017), as a dynamic part of the processes of understanding and carrying out the problems and were encouraged by the staged solutions and by heuristic strategies such as representation and the different types of generalisations (1A and 1B). It is necessary, however, to develop the visual organiser and materials which make up for the deficit in the executive function associated with AS (Delisio et al., 2018), ordering each stage of the strategy without losing sight of the objective and the information in the question statement. This was made evident in Problem 2, in which the trial and error and the sub-goal, although applied correctly, ended up becoming disconnected from the process of the problem. We would also highlight the spontaneous use by some of the pupils of heuristics such as best guessing – in some cases systematised – which might on first consideration appear incompatible with the rigidity of thinking and lack of intuition associated with AS, but which was successfully deployed in the workshop.

Based on this experience, we are considering PS workshops as an impetus for the development of student autonomy, whereby the pupils make use of different materials, types of representation and strategies for themselves, and for developing self-regulation and feedback techniques associated to the metacognitive processes.

References

- Bae, Y.S., Chiang, H.M., & Hickson, L. (2015). Mathematical Word Problem Solving Ability of Children with Autism Spectrum Disorder and their Typically Developing Peers. *Journal of Autism and Developmental Disorders*, 45(7), 2200–2208.
- Bruno, A., Polo Blanco, I, González López, M.J., & Olivera, B. (2018). Strategies and representations in the resolution of arithmetic division problems in students with autism spectrum disorder: a case study. *Revista Nacional e Internacional de Educación Inclusiva*, 11(2), 159–172.
- Carr, M. & Seah, W. T. (2019). Mathematics education for students with Autism Spectrum Disorder: Where are we now? *Annales Universitatis Paedagogicae Cracoviensis: Studia and Didacticam Mathematicae Pertinentia*, 10, 17–39.
- Carrillo, J. (1998). *Modos de resolver problemas y concepciones sobre la matemática y su enseñanza: metodología de la investigación y relaciones*. University of Huelva.
- de Giambattista, C., Ventura, P., Trerotoli, P., Margari, M., Palumbi, R., & Margari, L. (2019). Subtyping the Autism Spectrum Disorder: Comparison of Children with High Functioning Autism and Asperger Syndrome. *Journal of autism and developmental disorders*, 49(1), 138–150.
- Delisio, L., Bukaty, C., & Taylor, M. (2018). Effects of a Graphic Organizer Intervention Package on the Mathematics Word Problem Solving Abilities of Students with Autism Spectrum Disorders. *The Journal of Special Education Apprenticeship*, 7(2), article 4.
- Goleman, D. (1995). *Emotional Intelligence*. Bantam Books.
- Happé, F., & Frith, U. (2010). *Autism and Talent*. Oxford University Press.
- Klaren, M., Pepin, B., & Thurlings, M. (2017). Autism and mathematics education. In A. Bikner-Ahsbabs, M. Haspekian, A. Bakker, & M. Maracci (Eds.), *Proceedings of the CERME 10* (pp. 629–636). CERME.
- Kribbs, E. & Rogowsky, B. (2016). A Review of the Effects of Visual-Spatial Representations and Heuristics on Word Problem Solving in Middle School Mathematics. *International Journal of Research in Education and Science*, 2, 65–74.
- Liljedahl, P., Santos-Trigo, M., Malaspina, U., & Bruder, R. (2016). *Problem Solving in Mathematics Education*. Springer.
- Polya, G. (1945). *How to solve it*. Princeton University Press.
- Roos, H. (2013). Inclusive Mathematics from a special education perspective – how can it be interpreted? In B. Ubuz, Ç. Haser & M.A. Mariotti (Eds.), *Proceedings of the CERME 8* (pp. 2860–2869). Turkey.
- Radford, L. (2001). Factual, Contextual and Symbolic Generalizations in Algebra. In Marja van den Huevel-Panhuizen (Ed.), *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 81–89). Freudenthal Institute, Utrecht University.