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# A resource approach to mathematics teacher education

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*The relation between pre-service teacher education and the day-to-day practice of mathematics teachers in schools is a recurring topic in research on teacher education. Furthermore, the Norwegian teacher education programme has been criticised for polarity between theory and praxis. The following pilot study investigates what aspects of working with resources in university coursework pre-service mathematics teachers perceive as valuable preparation for teaching mathematics. I found that university coursework can alleviate the gap between theory and praxis through resources for teaching mathematics. Working with resources can provide learning opportunities that pre-service mathematics teachers experience as practical and useful.*

*Keywords: Documentational approach to didactics, resources, mathematics teacher education, preparedness, theory-praxis gap.*

## **Introduction**

Novice teachers leaving the profession is a challenge in many countries. In Norway, nearly 30% leave their teaching position within the first four years (Boyesen, 2021). The relation between pre-service teacher education and the day-to-day praxis of mathematics teachers in schools is a recurring topic in research on teacher education, and there is a call for better alignment of the knowledge development that takes place within universities and schools (Winsløw et al., 2009). Norwegian teacher education has been criticised for polarity between theory and practice, and developing a teacher education programme which better prepares pre-service teachers for their professional lives is pivotal (Mosvold et al., 2018).

In Norway, pedagogical content knowledge and, in this case, mathematical knowledge for teaching has been prominent in teacher education. However, the past decade witnessed a shift of focus from mathematical knowledge for teaching to core practices in the work of teaching mathematics (e.g., leading group discussions), both in the Norwegian teacher education programmes and in the international research literature on teacher education (Mosvold et al., 2018). Nevertheless, several researchers have expressed concern about this shift in ignoring content knowledge while highlighting the teachers' practice (Hovtun et al., 2021). I suggest that both approaches hold promise with respect to educating mathematics teachers who are well prepared for their profession.

Paving the way for a more extensive Ph. D study, the research question for this pilot study is: What aspects of working with resources in university coursework do pre-service mathematics teachers (PSMTs) accentuate as beneficial in preparing them for teaching mathematics in elementary school? In line with the shift towards core practices, the participants in this study showed an inclination towards teaching and experiences derived from praxis and school placement. However, as will be elaborated upon in due course, they also expressed appreciation of university courses that facilitate connections between knowledge acquired in university and in school placement.

The overarching goal of my inquiry is to contribute to mitigating the transition from pre-service teacher to novice teacher, aspiring to retain novice teachers in teaching positions. I contend that working with resources "hands-on" in mathematics teacher education can be beneficial in addressing the gap between theory and praxis, albeit not confined to core practices but rather as an activity that might amalgamate core practices and mathematical knowledge for teaching.

## **Theory**

I propose taking a resource perspective on the issue of connecting theory and praxis. Hence, I chose to use the documentational approach to didactics (DAD) as my framework. Moreover, for the pilot, I rely on the concept of schemes as retained in the instrumental approach by Vergnaud (1998), which draws on activity theory and sociocultural theory of teaching and learning (Gueudet, 2017; Gueudet & Trouche, 2009). Further, I consider the process of instrumentation (how an artifact influences the subjects' activity) and instrumentalization (how the subject shapes the artifact) as operationalising the PSMTs' interplay with resources.

Pepin (2018) argue that research on curriculum materials (in mathematics education) to date has largely focused on how these support student learning with less attention to teacher learning. She further claims that simply exploring how teachers interact with curriculum resources "as is" is not sufficient (e.g., investigating schemes of use), but that extending the research area by also addressing ways of how teachers can usefully work with these "tools" is equally important (ibid., p. 366). For this purpose, Pepin distinguishes between curriculum materials and educative curriculum materials. The latter are materials designed to support teacher learning as well as student learning. Retaining this perspective on teacher learning, I contend that this is also pivotal to pre-service mathematics teachers learning and teacher education. However, as described by Pepin (ibid.), this perspective focuses largely on materials designed for teachers (e.g., by researchers/ teacher educators) to further help teachers design learning sequences. In comparison, the tentative position derived in this pilot study is that (student) teachers learn from choosing, transforming resources, implementing them, revising them, etc. This is in line with the perspective proposed by Gueudet and Trouche (2012) in terms of taking a broad view on resources as anything likely to re-source the teachers' practice. Hence, I argue that involving PSMTs in all aspects of working with and developing resources, not limited to curriculum resources, is pertinent. Therefore, I consider the DAD framework apposite for this study as it regards teachers' work with resources as pivotal in their professional activity and professional growth (ibid.).

Thus, following the DAD, the aspects accentuated by the PSMTs in preparing them for teaching mathematics are operationalised through the concept of schemes, and studying the first three elements of a scheme. According to Gueudet (2017, p. 200), a scheme has four parts:

- An aim (can correspond with the goal in activity theory);
- Rules of action: regular ways of acting for the same aim;
- Operational invariants of two kinds: theorems-in-actions (propositions considered as true by the subject) and concepts-in-action (concepts considered as relevant for the subject);
- Possibilities of inferences: the subject can adapt his/her activity to the special features of a given situation corresponding to the same aim.

Delineated by Vergnaud (1998), a scheme developed by a subject is associated with a class (“set”) of situations which in turn correspond to the same aim of the activity. In this case, the aim I have chosen to consider (inferred from the data material) is *knowing how to teach mathematics* corresponding to the situation of being part of a teacher education programme attending classes on mathematics education. Hence, the activity considered in this case is at a rather general level and associated with a “big scheme” (Gueudet, 2017). One might be critical of this level of generality. However, as I have chosen to focus on consistent schemes in all the participants' responses, I suggest that a “big scheme” is applicable for rendering the different aspects in this context.

As the participants in this pilot study have not been followed and observed over time, rules of action represent what the PSMTs perceive to be regular ways of receiving lectures and teaching in their education programme. This is operationalised through their answers in this inquiry, consisting of a questionnaire and written reflections. Moreover, rules of action also represent omissions in relation to the same education programme as evidence of what the PSMTs would have preferred their education to contain. The operational invariants are inferred from these rules of action as the reasons/beliefs behind said omissions and experiences. As long-term follow-up was not done, the last element of schemes, possibilities of inferences, is beyond the scope of this interim study.

Finally, the process of instrumentation/instrumentalization details the PSMTs' interaction with resources. Resources in this case, denotes an activity that, in their own view, prepares the PSMTs for teaching mathematics supporting their learning process for the aim presented above.

## **Methodology**

Since 2017, the Norwegian teacher education has been a five-year education programme (previously four years), and the graduates receive a master's degree. In their fourth year, the student teachers choose their specialisation. The participants in this pilot are in their fourth year. They specialise in mathematics education, which also entails having at least 60 ECTS credits in mathematics from previous courses, which are didactical courses focusing on the learning and teaching of mathematics in primary schools. The data collection for the pilot was based on the participants' efforts in coursework in the fourth year of their educational trajectory.

As I try to solicit the opinions and experiences of the participants, the pilot study is conducted from an interpretivist position (Scheiner, 2019). Through a qualitative case study involving seven student mathematics teachers, I investigated what aspects of working with resources this group of PSMTs accentuate as beneficial in preparing them for teaching mathematics in school. The participants attended a course on problem-based mathematics education at a university in Norway. In this course, the PSMTs are introduced to a variety of resources on the subject of problem-solving and exploration as work methods in mathematics. At the end of the semester, the PSMTs were required to write down some overall reflections relating to their experience in this course, at which point they had only had one week of school placement. Seven responses were produced and gathered from the participants. Further, the participants took part in a qualitative questionnaire with open-ended questions at the end of the academic year. The questions focus on resources and the feeling of preparedness for teaching mathematics in elementary school. When completing the questionnaire, the PSMTs had been in school placement for five weeks, giving them opportunities to test and use their resources.

The overarching aim of the pilot was to gain an account of what the PSMTs regard as important in their mathematics teacher education. Simultaneously, I also ventured to discern what encourages or stymies their vocation for teaching mathematics. The concept of schemes as described within the DAD is pertinent for both objectives. Moreover, the data have been analysed using a qualitative content analysis approach where the theoretical categories (Kuckartz, 2019) are derived from the concept of schemes, namely aims, rules of action, and operational invariants, and further, the resources associated with the aim. The data analysed are the participants' answers to the given questionnaire and their written reflections from the coursework. The questionnaire was in Norwegian while they wrote the “overall reflections” in English.

As the data material in this pilot is limited, a discussion on trustworthiness is apposite. It is important to observe that this paper should be viewed in the context of a report on a pilot study. With respect to transferability, the sampling has indeed been purposeful as these participants are the predecessors in line of study to the participants selected for the main study. Hence, despite the lack of “thick” descriptions, I argue that the collected data give a reasonable foundation for interim inferences. Further, some degree of triangulation is applied by juxtaposing the overall reflections and the questionnaire.

## **Results**

Because the scope of this pilot is not to study the resources used in a specific teaching unit nor the PSMTs documentation work per se, I do not present the data using specific tools from the DAD, such as a document's table. Rather, in the following, I present excerpts from both the PSMTs' reflection notes and their answers in the questionnaire, emphasising shared features or attributes in the PSMTs' schemes that might inform the overall goal of better alignment of the knowledge development happening within schools and universities. For the situation of studying to be a mathematics teacher, the aim at the crux of all the PSMTs responses was knowing how to teach mathematics. In this paper, I have chosen to focus on this aim as it is salient both for theoretical and practical knowledge development.

### **Aim: Knowing how to teach mathematics**

In the questionnaire, the PSMTs were asked what they had found useful in their teacher education so far and what made them (better) prepared to teach mathematics:

- PSMT 1: Lectures in didactics of mathematics, so how to teach mathematics.
- PSMT 2: Another thing that I think has been very useful is learning about different strategies for teaching which we can use to plan good lessons for teaching mathematics.
- PSMT 4: More school placement and to learn more about ways to teach [mathematics]<sup>1</sup>.

These excerpts serve as examples of an attribute present in all the participants' responses, without exception, namely the expressed desire to know more about how to teach mathematics to children in elementary school. Typically, they emphasised learning about methods for teaching mathematics as particularly useful so far in their teacher education and simultaneously as something they wanted to

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<sup>1</sup> Words enclosed by brackets are added for the sake of context when not evident from the excerpts themselves.

learn more about to be more prepared for teaching. Several of the PSMTs put this in contrast to what they discern has been the norm in their teacher education:

PSMT 3: They often focus more on teaching us mathematics and not on how to teach mathematics.

PSMT 7: The introductory course was mainly doing mathematics. A lot of the curriculum was repetition from upper secondary school.

As demonstrated in the quotes above, some of the participants criticised previous mathematics courses in their teacher education programme, claiming that they focused too much on learning mathematics (which they, e.g., PSMT 7, described as a repetition from upper secondary school) and not on teaching mathematics. Triangulating these results with the PSMTs' "overall reflections", the PSMTs have a want for understanding how the knowledge acquired in their teacher education programme can have practical applications:

PSMT 3: This is the first course [problem-based mathematics education] that has allowed us to use what we already know and have learned before to make something that is going to be useful for us as teachers in the classroom.

The PSMTs express a lack of relevance and a call for practicable knowledge in their teacher education. Investigating this further, I now turn my attention to the rules of action and operational invariants interconnected with the aim described in this section and proposed resources thereof.

### **Rules of action and operational invariants**

A possible operational invariant inferred from what has been explored thus far can be that the PSMTs consider themselves proficient in mathematics and that teacher education should focus on learning how to teach mathematics in elementary school practically. The rule of action in the PSMTs experience has, to a large extent, been learning mathematics. However, drawing on the operational invariant expressed above, what they would have liked to be the rule in their education is rather learning practical ways to teach mathematics. Examples they list, among others, are teaching methods, how to introduce new mathematical topics to young students, learning about how to differentiate their teaching to include all the students in a class, and learning about students with learning difficulties in mathematics.

In the question of what could help them to be better prepared for teaching mathematics, five out of seven participants answered more school placement:

PSMT 5: We could use more school placement, and to work with connecting what we are taught [in university] to practical situations and how to use it.

When comparing with the "overall reflections", all the participants have contemplated the connection between what they learn in university and in praxis. Drawing on these responses, another operational invariant is deduced. The connection between what is taught at university and the field of praxis is, in many cases, not apparent to the PSMTs. Hence, learning in and from praxis is seen as more relevant and useful. Albeit this is by no means a new discovery (Hammerness, 2013; Solomon et al., 2017), it is still salient in context of this study as an overarching goal is to find a viable way to attend to the gap between theory and praxis. The PSMTs then perceive theoretical learning as the rule of action while they prefer learning in and from school placement. However, when they receive help in connecting the two, they are more positive as elaborated upon in the following section.

## **Resources: What helps the PSMTs in learning how to teach mathematics**

With respect to preparing them for teaching mathematics, the PSMTs endorse activities that help them connect theoretical and practical considerations – here represented by two of the participants reflections:

- PSMT 6: Some of the [problem-solving] task [sic] has been hard to find the “right” learning goal [from national curriculum], and then we had some discussions around different goals. This has been so learning full [instructive] for me<sup>2</sup>.
- PSMT 7: It has been exciting to read about what authors think students think and feel about problem-solving, and afterwards go out and actually see that a lot is true. I use a lot of what I read, and have used it both when planning a lesson plan, but also during planning in practice [school placement] and as a substitute teacher.

All the participants accentuated how working practically with tasks in coursework at the university helped them connect research on mathematics education to practical situations. Further, as PSMT 6 exemplifies, connecting this activity to the national curriculum was experienced as positive and instructive. Through a process of instrumentation, problem-solving help the PSMTs reflect on practical and theoretical considerations for teaching. Moreover, as PSMT 7 shows, it leads them to enact what they have learned in coursework in their own teaching and thus, appropriating the knowledge from their coursework (instrumentalization). This holds implications for mathematics teacher educators as it exemplifies what activity PSMTs might experience as meaningful and relevant in their further work as pre-service teachers.

Moreover, the PSMTs appraise problem-solving tasks as an asset both for their understanding of students (experience how the students might think/feel) and as a method for teaching which supports them in several ways:

- PSMT 1: Having the opportunity to work practically with tasks we can bring to the students ourselves has given me a greater experience of what it is like to work with such tasks with the students.
- PSMT 7: Now I am so lucky that I have many problem-solving tasks in my folder, as well as which learning goal they are under. This helps me a lot later. I have already planned to have a folder with problem-solving tasks that are sorted by year level. This way they are easy to retrieve when needed.

Thus, in working with resources for problem-solving, some of the PSMTs' requests for their education appear to be met (instrumentation) e.g., learning methods for teaching (problem-based teaching) and ways to differentiate their teaching through using mathematically rich tasks. Moreover, as PSMT 7 shows, it can also affect their overall practice as mathematics teachers.

## **Discussion and conclusion**

The RQ of this study was the following: *What aspects of working with resources in university coursework do pre-service mathematics teachers accentuate as beneficial in preparing them for teaching mathematics in elementary school?* I venture that explicitly working with resources for teaching mathematics might be one feasible way to align better what pre-service mathematics teachers learn in university coursework with what PSMTs themselves accentuate as beneficial in

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<sup>2</sup> I have not corrected their English writing to prevent conveying misconstrued reproductions.

preparing them to teach mathematics in elementary school (as expressed by PSMT 7 above). The PSMTs in this study express discontent with the current allocation of theoretical and practical learning in their mathematics teacher education, claiming that praxis and school placement provides more relevant and applicable knowledge. The PSMTs express that when entering mathematics teacher education, they expect coursework and teaching to give instruction on how to teach mathematics to students in elementary school. In contrast, their experience is that more attention is given to teaching them (the PSMTs) mathematics rather than how to teach the subject themselves (e.g., PSMT 3). However, when they work with resources for problem-solving in university coursework, they perceive this activity as practical, instructive, and relevant to their future professional work as mathematics teachers. Following this, I claim that working with resources in university coursework can be auxiliary in amending the strategy of mathematics teacher education.

Further, two important aspects of working with resources is evident from the participants' responses. First, it is the mathematical content of the problem-solving tasks themselves and the opportunities these provide in the teaching of mathematics as expressed by both PSMT 6 and 7. They appreciate both solving the tasks themselves in preparing them for what meets the students, but also how these tasks facilitate learning with respect to the national curriculum. Hence, they regard content knowledge as relevant in preparing them for teaching mathematics. Second, they accentuate the methods for teaching that problem-solving tasks provide, such as problem-based teaching, differentiation, and the more general activity of gathering problem-solving tasks they can use later in their own teaching. As PSMT 3 exemplifies, such activity is “useful for us as teachers in the classroom”. This suggests that working with resources may also promote core practices in university coursework.

A recent study in Norway shows that student teachers often see theory and praxis as separate entities and that theory does not have a place in discussing their experience when in school placement (Brekke & Leikvoll Eide, 2021). The results presented in this paper are in line with these findings and show that PSMTs need help establishing meaningful connections between knowledge acquired in university and in school placement. Evidently, these results are the expressed experiences of a small group of PSMTs. However, from what has been conveyed in this pilot I consider it reasonable to suggest that there is an imbalance between teaching related to praxis and strictly teaching on mathematical content in their educational trajectory. Moreover, Hammerness (2013, p. 411) found that when asked about learning opportunities grounded in practice, teacher educators, in line with their student teachers, saw the school sites as the places that provided those opportunities suggesting a discrepancy in the structure of the teacher education and how student teachers prefer to be educated. As seen from the results of this pilot study, the PSMTs accentuate working practically with resources for teaching mathematics in university coursework as meaningful and relevant to them, hence, supporting their vocation for teaching. Thus, resources for teaching mathematics can provide learning opportunities grounded in practice while at university, showing that coursework can also contribute to alleviating the gap between theory and praxis. How such work can facilitate professional growth (as described by Gueudet and Trouche (2012)) for PSMTs in mathematics teacher education requires further study, and is the scope of my ongoing PhD study.

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