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Using Concept Cartoons in primary school teacher training: the case of a mathematics content course

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The contribution focuses on an educational tool called Concept Cartoons and the possibilities to use the tool in teacher education. It perceives Concept Cartoons as educational vignettes and show how they can be incorporated into mathematics content courses to help assessing future primary school teachers' knowledge on topics related to the primary school curriculum. The paper introduces one of the Concept Cartoons created on the topic of divisibility and a qualitative empirical study conducted with 67 future primary school teachers within a mathematics content course. The aim of the presented study is to observe and investigate the nature of knowledge displayed in written data collected via the Concept Cartoon. The results of the study confirm the potential that educational vignettes such as Concept Cartoons have in future primary school teacher education.

Keywords: Concept cartoons, educational vignettes, elementary school teachers, mathematics education, preservice teacher education.

Introduction

An integral part of primary school teacher education consists of mathematics content courses where mathematical content related to the primary school curriculum is reviewed and applied in contexts outside the primary school level. For instance, the primary school curriculum in the Czech Republic covers the four operations with natural numbers (addition, subtraction, multiplication, division) and their properties, and the secondary school curriculum in this area starts with prime and composite numbers, common multiples and divisors, and criteria of divisibility. So that the content course for future primary school teachers contains all these secondary school topics, to bring the future teachers a broader perspective and to engage them in intensive argumentation related to the mathematical content they are supposed to teach in their future school practice. Such an arrangement helps verifying that the future primary school teachers understand and comprehend primary school concepts properly.

This contribution focuses particularly on the topic of divisibility and on subject-matter knowledge (Shulman, 1986) of future primary school teachers. From the more detailed perspective of mathematical content, it focuses on conditional reasoning within the topic of divisibility – an area that appears to be difficult for future teachers as they often tend to handling the topic of divisibility procedurally rather than conceptually (Zazkis et al., 2013) and having deeply rooted misconceptions about argumentation that affect their conditional reasoning (Simon & Blume, 1996).

The following text describes a qualitative study of an explorative character that uses an educational vignette (Skilling & Stylianides, 2020) in the form of a Concept Cartoon (Samková, 2020) as a tool for collecting data. The study addresses the research question “*What kind of subject-matter knowledge can be revealed in future primary school teachers when using Concept Cartoons as a written assessment tool within a mathematics content course?*” The paper follows up on the contributions from previous ERME conferences where various educational vignettes were used in

teacher professional preparation: e.g. for investigating professional vocabulary of future teachers (Friesen et al., 2019), assessing how future teachers respond to hypothetical student ideas within primary school topics (Buforn et al., 2017; Samková & Hošpesová, 2015), how they respond to hypothetical learning support situations (Kuntze & Friesen, 2017) or what awareness they have about potential student ideas (Samková, 2019b). The presented study was conducted within the framework of the ERASMUS+ project *coReflect@maths* that aims at designing and evaluating vignette-based learning environments for various mathematics teacher education courses, with a particular focus on cartoon vignettes (Krummenauer et al., 2020).

Vignettes and Concept Cartoons

In this paper, educational vignettes are understood as stories representing school practice (Buchbinder & Kuntze, 2018), i.e. as representations of classroom situations or situations that relate to content taught and learnt in the classroom. In that sense, vignettes are rather short, descriptive episodes that may take the form of texts, single or multiple images, videos, or their combinations (Skilling & Stylianides, 2020). The protagonists of vignettes might be various combinations of teachers and students, e.g. a teacher with one or more students, one or more students without a teacher, one or more teachers without students. With future teachers as respondents of research or intervention, the purposes for implementing vignettes are wide (Herbst & Chazan, 2011); they usually lay in aiming for development or analysis of professional knowledge and skills such as noticing (Schack et al., 2017), professional vocabulary (Schleppegrell, 2007), etc.

Among vignettes, we may also include Concept Cartoons – individual pictures showing a content-related situation and a group of several children discussing the situation via a bubble-dialog. The opinions in the bubbles may be correct, incorrect, unclear or incomplete (Keogh & Naylor, 1993). Originally, Concept Cartoons were developed as a means of supporting the quality of discussion in primary school classrooms (Naylor et al., 2007), with the key aspects for the discussion being the absence of the teacher in the picture (i.e. the presence of just the peers) and the diversity of opinions given in the bubbles. However, Concept Cartoons may be created for different target groups and different purposes, including the target group of future primary school teachers and the purpose of analysing their pedagogical content knowledge (Samková & Hošpesová, 2015) or subject-matter knowledge (in this paper). Within this context, the protagonists in the picture may not be just children but also adults (future teachers, i.e. peers of the respondents). For the purpose of collecting data on teacher knowledge, Concept Cartoons are usually accompanied by some set of indicative questions, and this combination appears to be able to provide data that are ample and relevant (Samková, 2019a).

When creating a new Concept Cartoon, one has to choose the focus of the mathematical task in the background (calculation, proposition, application), its openness (e.g. single vs multiple correct solution procedures), determine the nature of correctness of individual bubbles (ambiguous, unambiguous, conditioned), and choose the form of texts in bubbles (results, procedures, statements); for more details on the typology of Concept Cartoons see Samková (2020). This study is based on a Concept Cartoon that has a group of future teachers as protagonists, a propositional task with multiple correct solution procedures in the background, four bubbles with unambiguous correctness, and texts in bubbles in the form of statements (see Figure 1).

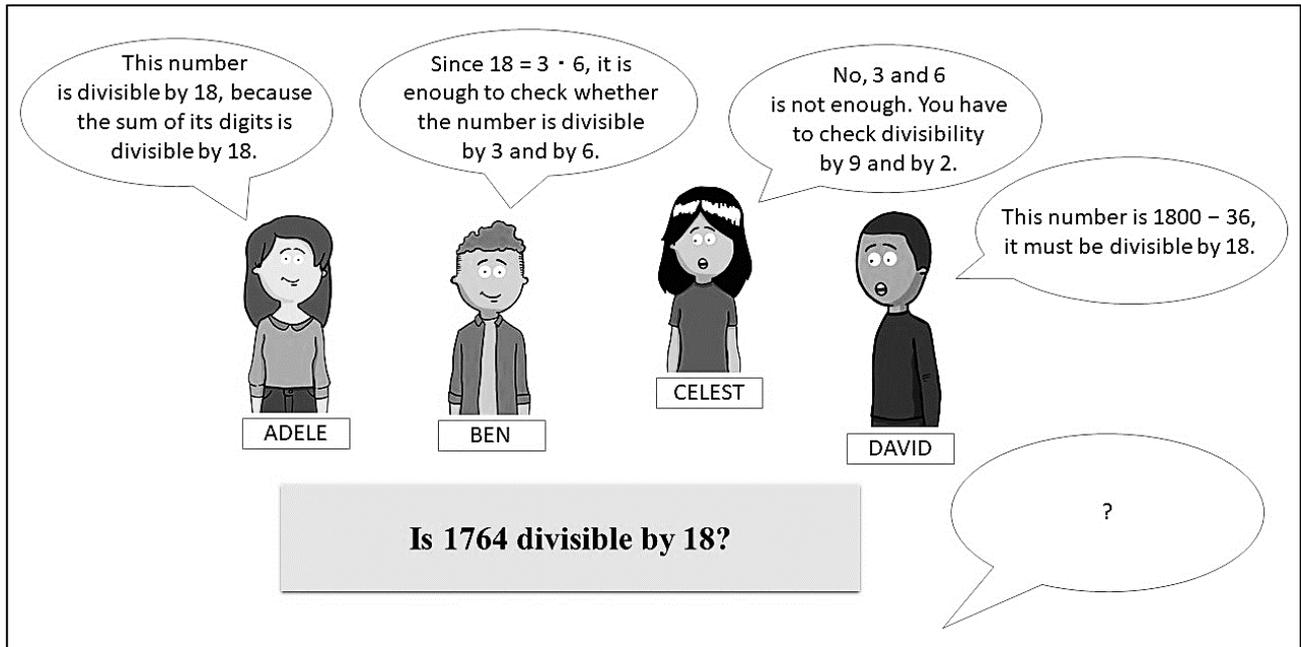


Figure 1: The Concept Cartoon on divisibility

The Concept Cartoon in Figure 1 presents two statements that are correct (Celest, David), and two that are incorrect (Adele, Ben). One of the statements (David) refers to a manipulation with numbers (based on finding a nearby multiple of 18 that is easily identified), while each of the other three statements (Adele, Ben, Celest) informally refers to an application of a general rule. The rules can be formally rewritten as follows:

- Adele: If the sum of digits of a given number is divisible by 18, then the number is divisible by 18.
 Ben: If a given number is divisible by 3 and by 6, then it is divisible by 18.
 Celest: If a given number is divisible by 9 and by 2, then it is divisible by 18.

For the rule behind the Celest bubble, the condition in the statement is necessary as well as sufficient, i.e., the rule is valid and can be also rewritten in the form of equivalence. For the rule behind the Ben bubble, the condition in the statement is necessary but not sufficient, since 3 and 6 are not coprime numbers; numbers 6, 12, 24, 30 are some of the counter-examples for the rule. For the rule behind the Adele bubble, the condition in the statement is not necessary (even the number 18 itself does not have the sum of digits divisible by 18) nor sufficient (swapping the order of digits does not change the sum of digits but may easily create a number that is not even and thus not divisible by the even number 18; e.g. 1467, 7641).

Such an arrangement creates an environment that challenges skills in conditional reasoning, by requiring proper differentiation between necessary, unnecessary, sufficient and insufficient conditions in an informally worded statement (Buchbinder & McCrone, 2019). The statement in the Adele bubble is also closely related to *overgeneralizing* – a frequent misconception consisting in improper use of analogical reasoning (Hemmi et al., 2017); here the overgeneralizing stems from criteria for divisibility by 3 and by 9 that are both based on the sum of digits.

- I would be more specific and decompose as $18 = 3 \cdot 3 \cdot 2$, it is enough to check whether the number is even and its sum of digits is divisible by 3.
- Celest: She just made another decomposition. She is not right, both decompositions are good.
- V31 Celest: I think she is not right. It is enough. She must decompose the 6.
- V65 Ben: I think that the divisibility by six is a little extra. It is enough to verify divisibility by two and by three.

In responses to the third question, 22 of these 33 respondents offered as their own solution the decomposition into $9 \cdot 2$ or $2 \cdot 9$, and 5 respondents offered the decomposition $3 \cdot 3 \cdot 2$ or $2 \cdot 3 \cdot 3$. On the other hand, there were respondents who remembered the coprime condition and pointed it out:

- V64 Ben: We cannot decompose this way. The numbers you decompose into must be coprime (cannot be divisible by the same number) \rightarrow 3 and 6 are divisible by 3, we do not want it.

Language

Some of the respondents displayed shortcomings in the language of mathematics that transpired in the form of an inaccurate terminology (V30), a shift of a meaning of a mathematical concept (V36), a shift in an interpretation of the text in a bubble (V47), or a combination of them (V34 – terminology & interpretation):

- V30 David: 1800 is the closest whole number.
- V36 Ben: The idea is good, but it is not sufficient to have the number divisible by 3 and by 6, it must also, after dividing by one of the numbers, be divisible by the other.
- V47 Celest: According to Celest, we have to check divisibility by 3, 6, 9, and 2. It is sufficient to check just divisibility by 9 and 2.
- V34 Celest: She thinks that we have to check all variants of multiples, but it is not true.

Argumentation modes

The third code category refers to modes of argumentation and logical aspects in general. Among the proper argumentation modes, it included counter-examples that some of the respondents provided as a reaction to Adele. These counter-examples referred either to a condition that is not sufficient (V14) or a condition that is not necessary (V29):

- V14 Adele: She tried to sum the digits of 1764 \rightarrow it came out 18, and $18 : 18 = 1$, so that she thinks this is a rule for divisibility by 18. However, when we take e.g. the number 4455, the sum of its digits is also 18 but the number is not divisible by 18.
- V29 Adele: Her opinion surprised me. The sum of the digits is 18, so it is divisible by 18, but I did not find this kind of criterion anywhere. ... I chose the number 126 (a multiple of 18) to check it $\rightarrow 126 : 18 = 7 \rightarrow 1 + 2 + 6 = 9 \rightarrow 9 : 18 = 0,5$. Other example: $1710 : 18 = 95 \rightarrow (1 + 7 + 1 + 0 = 9)$
 $118764 : 18 = 6598 (1 + 1 + 8 + 7 + 6 + 4 = 27 \rightarrow 27 : 18 = 1,5)$.
 In my opinion, it implies that we cannot use the sum of digits this way as decisive. It was just a coincidence that it worked out for her.

Surprisingly, none of the respondents offered a counter-example as a reaction to Ben. However, several of them provided to Ben an objection towards coincidence similar as the one by V29 to Adele:

- V46 Ben: That Ben's claim comes out in this particular case is, in my opinion, just a coincidence.

The improper argumentation modes included using an assumption that was not mentioned in the bubble (V18), overgeneralizing (V46/David) or following a rule in the bubble instead of verifying it (V46/Adele):

- V18 Adele: She is right, because if a number is even and a sum is divisible by 18 \rightarrow it's true.
- V46 David: Number 1800 is divisible by 18, number 36 as well, in this case she is right.
Check: $18 \cdot 4 = 72 = 100 - 28$
100 is not divisible by 18
28 is not divisible by 18
- Adele: In my case, it did not work out, which means that it was just a coincidence.
 $1 + 7 + 6 + 4 = 18 \rightarrow 18 : 18 = 1 \rightarrow$ she is right, it will work.

Alternative ideas

The last code category summarizes how respondents reflected the fact that there were alternative opinions shown in bubbles. Aside from the discourse between Ben and Celest that got assigned its own code category (*Coprime condition*, see above), there were also two different correct statements presented by Celest and David. Here, some of the respondents favoured the Celest's way based on a well-known rule (V13), others appreciated that David had managed without the rule (V29, V5); one of the respondents favoured both the statements (V44):

- V13 David: This procedure is logically correct, but might be time consuming. It is better to use divisibility criteria instead.
- V29 David: His opinion is interesting and might also be considered correct ... He came to the conclusion logically even without knowledge of the divisibility criteria.
- V5 David: Nice, quick reasoning!
- V44 (3) I myself would support both opinions (C and D). C is a classical method. For D, we have to think a bit but, for one, it is faster.

Discussion and conclusion

As illustrated in the previous section, using educational vignettes, namely Concept Cartoons, as an assessment tool within a mathematics content course for future teachers might bring a broad insight into various facets of knowledge that is more or less related to the mathematical content that the future teachers would teach in their future teaching practice. The environment consisting in a Concept Cartoon presenting various correct and incorrect opinions on a chosen topic (divisibility by 18) and a set of three differently aimed indicative questions has proved to be able to indirectly provoke reasoning of future teachers and obtain rather talkative responses from them (even if only in writing). These responses reflected in detail how future teachers reasoned about the topic, how they understood key concepts, what mathematical language they used, what kind of arguments they were able to provide, and how they reacted to various alternative ideas.

The results of the study highlighted the advantage that Concept Cartoons have over standard written tests: 33 of the 67 respondents labelled as correct a solution that was not correct (Ben, missing coprime condition), however, 22 of them offered as their own solution a solution that was correct. It is reasonable to assume that if only a standard test were used as a method of assessment (e.g. with a task "Is 1764 divisible by 18?"), these 22 respondents would succeed in the test and there would be no doubt about their subject-matter knowledge. Moreover, using the format of Concept Cartoons for

assessing subject-matter knowledge also allowed to learn about future teachers' mathematical language and argumentation. The findings of the study confirmed weaknesses in conditional reasoning (cf. Simon & Blume, 1996; Buchbinder & McCrone, 2019) as well as a tendency to overgeneralizing (cf. Hemmi et al., 2017), a tendency to handling the topic of divisibility rather procedurally than conceptually (cf. Zazkis et al., 2013), insecurities in mathematical language (cf. Schleppegrell, 2007). Such findings show that vignettes might be implemented meaningfully into teacher training not only to advanced courses focusing on pedagogical content knowledge and teaching practice (Buchbinder & Kuntze, 2018) but also to initial content courses.

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