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# Teacher Perspectives and Mathematical Knowledge for Teaching

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*In this study, we report on data from videotapes of two prospective secondary mathematics teachers' practicum teachings. The aim of the study was to investigate the relationship between prospective teachers' mathematical knowledge for teaching and their perspectives on mathematics, mathematics learning and mathematics teaching. We analyzed data using both the Teacher Perspectives and Knowledge Quartet frameworks. Results showed that prospective teachers having a progressive incorporation perspective or perception-based perspective depicted all the codes in Knowledge Quartet; indicating that they held mathematical knowledge for teaching. Yet, results pointed that they had different reasons behind their actions indicating their mathematical knowledge for teaching. We discuss implications of these results on teacher education.*

*Keywords: Teacher perspectives, mathematical knowledge for teaching, prospective secondary mathematics teachers.*

## Background and Rationale

Teachers' having mathematical knowledge for teaching is important (e.g., Rowland et al., 2005). In this regard, considerable amount of research conducted with both in-service and prospective teachers pointed that although focusing on what teachers have or lack has importantly informed the field (e.g., Wilson & Cooney, 2002), consistencies among knowledge, beliefs, and practices (prospective) teachers might hold need to be given further attention (Chapman, 2016).

Teacher perspectives as a robust construct is one of the ways to investigate such consistency. Researchers stated that perspectives "...can be thought about as ... paradigms with respect to the development of mathematical knowledge...the term paradigm emphasizes the existence of internally coherent systems..." (Simon et al., 2000, p. 599). That is, a teacher's perspective (i.e., meaning making systems) underlies the teaching practices that indicate not only what teachers think about, know, believe and do but also everything that contributes to their teaching (planning, assessing, interacting with students) (Simon et al., 2000). Therefore, teacher perspectives go beyond understanding particular knowledge and beliefs in the context of practice of teachers in transition (Simon et al., 2000) which have affordances and limitations on their teaching (Jin & Tzur, 2011). Particularly, doing research with in-service teachers, researchers reported on teacher perspectives in a continuum between traditional perspective, perception-based perspective (PBP) and conception-based perspective (CBP) on mathematics, mathematics learning and mathematics teaching (Simon et al., 2000; Heinz et al., 2000; Tzur et al., 2001). Later, Jin and Tzur (2011) have placed the progressive

incorporation perspective (PIP) between the (PBP) and the (CBP). Tzur et al. (2001) stated that teachers holding (CBP) act accordingly with the views of radical and social-constructivist epistemology such that knowledge is actively built by the learner; so knowledge is re-invented. And, individual and social mathematical learning are reflexively related. Therefore, CBP has two important aspects: First, teachers are aware of his/her current mathematical understandings being qualitatively different from their students' understandings (Jin & Tzur, 2011); and second, teachers focus on what students currently know rather than what they don't know (e.g., Heinz et.al., 2000). On the contrary, (PBP) like the traditional approach, views mathematics as part of the external world independent of the knower, compatible with Platonist view of knowledge (Jin & Tzur, 2011). Thus teachers expect their students to see mathematical situations in the same way as they do. That is, mathematics learning means coming to see a first-hand experience of mathematical reality shared by all through discovery. Mathematics teaching occurs through teachers creating situations that reveal the mathematical ideas. Teachers having this view, in contrast to having a CBP, focus on what students *do not* understand (Heinz, et.al., 2000). Jin and Tzur (2011) postulating an intermediate perspective (PIP) stated that PIP "underlies an integrated stance on knowing and learning---reflecting both 'existence outside the learner' (hence, teacher involvement) and 'dependent on what a learner knows' (hence, student active problem solving)". Mathematics learning is therefore an active mental process. Teachers' main goal is to create learning opportunities for all students to activate their existing knowledge related to the intended mathematics, the old incorporating the new rather than being transformed as in CBP. Thus, we concur that perspectives not only include the foundational knowledge one needs to hold but also how such knowledge is embedded into teaching.

**Table 1: Placing PIP within teacher perspectives (Jin & Tzur, 2011, p. 20)**

Perspectives	View of knowing	View of learning	View of teaching
Traditional Perspective (TP)	Independent of the knower, out there	Learning is passive reception	Transmission, lecturing instructor
Perception-Based Perspective (PB)	Independent of the knower, out there	Learning is discovery via active perception	Teachers as explainer (points out)
PIP (PIP)	Dialectically independent and dependent on the knower	Learning is active (mentally); focus on the known required as start, new is incorporated in to known	Teacher as guide and engineer of learning conducive conditions
Conception-based Perspective (CBP)	Dynamic; depends on the knower's assimilatory schemes	Active construction of the new as transformation in the known (via reflection)	Engaging students in problem solving; Orienting reflection; facilitator

This takes us to mathematics knowledge for teaching in action (MKT) depicted in the Knowledge Quartet framework (Rowland et al., 2005). Knowledge Quartet (KQ) framework is based on four

main categories (See Table 2). The foundation dimension relates to teachers' beliefs on the nature of mathematics and mathematics learning and teaching. It is also about teachers' knowing 'why' behind the mathematics they teach. Transformation regards teachers' presentation of ideas to learners in the form of illustrations, examples, explanations and demonstrations. Connection includes sequencing the material for instruction and awareness of the relative cognitive demands of different topics and tasks. Finally, contingency is the ability to make cogent, reasoned and well-informed responses to unanticipated and unplanned events during the lessons (Thwaites et al., 2010).

**Table 2: The Knowledge Quartet: dimensions and contributory codes (Thwaites et al., 2010, p.86)**

Dimension	Contributory Codes
Foundation (1)	(1.a) theoretical underpinning of pedagogy (1.b) awareness of purpose (1.c) identifying pupil errors (1.d) overt display of subject knowledge (1.e) use of mathematical terminology (1.f) adherence to textbook (1.g) concentration on procedures
Transformation (2)	(2.a) teacher demonstration (2.b) use of instructional materials (2.c) choice of representations (2.d) choice of examples
Connection (3)	(3.a) making connections between procedures (3.b) making connections between concepts, (3.c) anticipation of complexity (3.d) decisions about sequencing (3.e) recognition of conceptual appropriateness
Contingency (4)	(4.a) responding to students' ideas (4.b) deviation from lesson agenda (4.c) teacher insight (4.d) responding to the (un)availability of tools and resources

So, we hypothesized that the three dimensions, the nature of mathematics, mathematics learning and mathematics teaching expressed in the perspectives corresponds with the (KQ) framework (Rowland et al., 2005). This is because as well the theoretical knowledge and beliefs related to mathematics and mathematics education are handled in KQ, the theoretical knowledge possessed by teachers is transformed into teaching through connections and the existence of contingency moments revealing students' thoughts, mistakes and difficulties. This implies a link between teacher perspectives and MKT; yet, there is few studies focusing on their relationship (Karagoz Akar, 2016; Bukova Guzel et al., 2019). Also, we hypothesize that (prospective) teachers holding different perspectives might depict different codes in KQ (Karagoz Akar, 2016). By the same token, even if (prospective) teachers depict the same codes referring to their MKT they might do so with having different reasons (Weston, 2013). Therefore, we conjectured that once prospective teachers had different perspectives, their MKT would reveal itself at different levels during their teaching. Scrutinizing the coherency between teacher perspectives and MKT is important: it might help uncover the reasoning behind (prospective) teachers' MKT. Diagnosing the reasons might provide teacher educators with particular steps to follow towards establishing more sophisticated perspectives and a full grasp of MKT on part of not only (prospective) teachers but also in-service teachers during professional development studies. With the results reported in this study we also aim to contribute to the field in the following ways:

First, how the practices of prospective mathematics teachers with PBP and PIP before, during, and after teaching will be depicted. Secondly, how such practices comprehensively revealing the relation between these perspectives' characteristics and the codes of KQ through empirical data, including before-during-after teachings and interviews, will be shown. It is in this respect that, in this study, we investigated the following research questions: What are the indicators that prospective teachers have a particular teacher perspective? How is their mathematical knowledge for teaching revealed in the act of teaching? How are prospective teachers' perspectives reflected in their mathematical knowledge for teaching?

## **Methods**

### **Participants and Data Collection**

In the larger study, participants were six prospective secondary mathematics teachers who were senior students at one of the universities, in which the medium of language is English, in Turkey. We chose to work with these participants since they volunteered to participate in the continuing set of interviews and the practice teaching sessions till the end of the study. Data from six prospective teachers depicted that four of them hold PBP perspective and two of them hold PIP perspective. For the report in this paper, we specifically depict data from two prospective teachers, Alin and Meryem for illustrative purposes as the data from them were providing context to examine the relationship between the two perspectives (e.g., (PBP) and (PIP)) and (MKT). For the larger study, six prospective teachers' two practicum teachings and interviews were videotaped and transcribed. Also, we conducted pre-interviews on their lesson plans, observed the teachings and conducted post-interviews upon completion of the teachings within the same week. For instance, in the pre-interviews, we asked the rationale behind prospective teachers' choice of learning goal(s), the tasks and how they consider the tasks they have chosen would allow students to learn meaningfully. In this paper, the reports will be on Alin's and Meryem's practice teachings. Alin taught an 80-minute lesson to the 10<sup>th</sup> grade students at a private high school. Alin had created a task for her students to make sense of the graph of the function  $f(x) = ax^2 + bx + c$  in terms of the meaning of real coefficients  $a$ ,  $b$ , and  $c$ . Meryem taught a 40-minute lesson to the 11<sup>th</sup> grade students at a public high school. Meryem had modified a task for her students to make sense of piecewise functions.

### **Data Analysis**

We analysed the data using coded analysis (Clement, 2000). First, based on previous research results (e.g., Jin & Tzur, 2011; Simon et al., 2000) we created a chart regarding teacher perspective characteristics. Following, each researcher read the lesson plans, transcripts from the pre-interviews, the practice teachings and the post-interviews line by line, looking for participants' explanations regarding their perspective. Using the characteristics of teacher perspectives, we looked for their existing meanings. Once we spotted a line of explanation regarding their meanings in any of the data sources in terms of mathematics, mathematics learning, and mathematics teaching, we also checked other sources to possibly provide further evidence of such meaning. Then, we came together to have a consensus on the data set and our analyses going back to the whole data set to challenge our conjectures. Following, we created the table showing the frequencies of the teacher perspectives. The reason was that in different data sources, the same characteristic has been represented more than once.

For instance, the code PIP.1a1 was depicted five times within the lesson plan. Secondly, engaging in the same process using the codes from KQ we examined both this same data set and read further each of the data sources line by line to determine their MKT. Then, again we came together to have a consensus on the whole data set to challenge our conjectures. For example, the code KQ.1a was depicted five times in all the data sources for Alin. Finally, we wrote the narratives.

## Results

Data showed that, Alin had a PIP and Meryem had a PBP. Also, compared to Meryem's practice teaching, the number of KQ codes from each data source from Alin's practice teaching indicated that Alin had shown more actions regarding the MKT in every aspect of the teaching. Now, based on some data from Alin and Meryem (the pre-interview data), we explicate how different perspectives and MKT are depicted and how some of the same KQ codes might reveal different reasoning patterns pertaining to different teacher perspectives. As mentioned earlier, Alin had created a task (PIP 1a) for her students to make sense of the graph of the quadratics functions,  $f(x) = ax^2 + bx + c$  in terms of the meanings of coefficients  $a$ ,  $b$ , and  $c$ . During the pre-interview, when asked how the lesson plan would possibly promote the learning goal Alin had in mind for her students, Alin stated the following:

Researcher: Why do you think this [lesson] plan will promote your students' learning?

Alin: I'm starting with the amount of change in  $y = x^2$ ; therefore, students need to recognize the arms of the graph gets open and there is a decrease, I mean, there is a change in slope... Starting always with  $y = x^2$ , how this change is formed and how this change affects the graph, so thinking this point... I mean, my activity provides quantitative operations by playing with something existing in their mind that they know.

Researcher: You said playing with something they know, what do they know? Like could you explain one more time what is quantitative operations?

Alin: They know what  $y = x^2$  is, what its roots are, how the change occurs in  $y = x^2$ , I mean, how the slope is changing and how it looks in the graph. However, they don't have any idea about what happens to the graph when " $a$ " changes, because they don't observe  $ax^2 + bx + c$  for changing " $a$ " values. Therefore, the quantitative operations formed in their mind when they changed " $a$ ", I mean the thing they know in their mind, is like how the slope in  $y = x^2$  is, how the amount of increase is, and drawing the graph.... we need to observe the change of " $a$ " one by one, and keep " $b$ " and " $c$ " constant so that we can only be aware of the change in " $a$ "... Let me say the amount of change in  $y$  in terms of  $x$ , rather than amount of increase, because " $a$ " can be negative too. It is necessary for students to observe how the amount of change in  $y$  is changing. When " $a$ " changed and  $x$  changed as one unit, they can compare the amount of change corresponding to  $y$ , so that they can have an idea about the shape of the graph, I mean the arms (referring to the parts of the parabola)... Actually, what I am learning is to compare the amount of change in  $y$  with respect to change in  $x$  as one unit for different " $a$ " values.

In terms of MKT, data showed that Alin had an awareness of purpose for her teaching (KQ 1b). She also effectively analysed which mental operations students need to engage in to make sense of the effect of the coefficient " $a$ " on the graph, in her own words: "compare the amount of change in  $y$  with respect to change in  $x$  as one unit for different " $a$ " values". Her analyses revealed that she has a strong subject knowledge and theoretical background about coefficient " $a$ " for quadratic functions (KQ 1a and 1d). Also, she anticipated the complexity of the concept: She planned her lesson in such a way that by keeping " $b$ " and " $c$ " constant, students' examining the change of " $a$ " would be more

efficient (KQ 3c). In addition, her consideration of graphical demonstrations such as starting with the graph of  $y = x^2$  while students were examining the coefficient “ $a$ ” together with the tables showing  $x$  and  $y$  values showed that Alin had knowledge about different representations (KQ 2c) and she could integrate these representations into her lesson with relations to each other (KQ 2d). Regarding the perspectives, data showed that Alin’s statement about comparing the amount of change in the dependent variable with respect to the one unit change in the independent variable suggested that for Alin students might create an idea about the graph of the quadratic functions and its structure through their mental operations (covariation). This suggested that Alin considered mathematics as constructed on the learner’s mind. Alin’s planning her lesson hypothetically depending on her students’ mental operations and actions (PIP 1a1) also suggested that she considered mathematics dependent on the knower. In fact, further data in her written lesson plan pointed to more evidence for this claim: She articulated how students might possibly reason on the questions in the task sequence for different values of  $a$  such as  $a=1, 2, 4$  and  $10$ . Alin wrote: “By giving “ $a$ ” different values and obtaining related  $y$  values, this time students compare the respective rate of changes in  $y$  values for different values of “ $a$ ”. For the same change in  $x$  values, the rate of change in  $y$  gets bigger as absolute value of “ $a$ ” increases. Meanwhile, in students’ minds legs of the parabola gets steeper and hence the width of the opening of the parabola gets narrower.” Data suggested that she was hypothetically envisioning how students would reason given different values of “ $a$ ”. In other words, she would expect students to go through the mental activity, the simultaneous comparison of the change in the variables  $x$  and  $y$ , so that they would know the reason behind the effects of the changes in the values of “ $a$ ” on the graph (PIP 1a1). In addition, her choice of example  $y = x^2$  to start the lesson indicated that she has chosen this example as a conceptual anchor to activate what they already know for the intended learning to take place (PIP1b). Alin stated “It is because  $y = x^2$ , I mean it will be easier for them to understand, to start with, they can start from what they can compare, like for that reason I did not include  $bx+c$  first, so that they don’t get confused. This way, because  $y = x^2$  stays on the symmetry axis, like this is what they already know, they do not have to deal with finding the roots, they can focus on the changes on “ $a$ ” more easily”. On the contrary, regarding the same interview question, Meryem stated:

Researcher: Why do you think this [lesson] plan will promote your students’ learning?

Meryem: I will be asking “could you explain for each of the graph?” I mean, after they have written for the first graph, the second graph, the third graph, I think they will realize the sets of domain and range will differ, like for different intervals we will be writing them. I think they will realize this. I mean I will ask again after they work on the examples, “if anything gets your attention”, “if you see anything similar in those examples?” If they can see, they can say for different intervals of domain corresponds to different intervals of range, then I will ask them how they can name it. Like I will ask for their guess, like they can say this or that. If they like they cannot say anything then I will ask them to write down domain and range sets for each example...If like noting comes from them, then I will tell them if we can call these functions as piecewise, “do you think this makes sense?” Then I will ask if they can support my explanation by different examples. I think they can say that. I mean I do not expect them to write it but they can see it in the graphs like they can explain that the graphs start and end at some points and then start again at the same point. I think they can give examples. Then I will summarize the lesson.

In fact, Meryem had planned to provide three problem situations to the students based on which she expected them to draw the graphs. Regarding MKT, Meryem had a sense of awareness of purpose

such that she had modified the task based on her learning goal for the students (KQ1b). Similarly, once asked how she determined the learning goal, Meryem stated that she had read about a research article on students' understanding of piecewise functions. The article was about how students made sense of domain and range of different functions. Such analyses also revealed that she had a strong subject knowledge and theoretical background about piecewise functions (KQ1a and 1d). She further stated "If I have enough time, like I will ask them to draw graphs of linear, quadratic and constant piecewise functions". Her choice of examples (KQ 2d) in her lesson plan together with her further planning of including different piecewise functions also indicated that she had sequenced examples supporting students' deductions (KQ 3d). In terms of the perspectives, though, Meryem expected her students to see a similarity among the graphs they would construct based on the problem scenarios. That is, after their examination of the graphs visually students would realize that the domain and range of those functions would differ. This suggested that Meryem had expected her students to see the mathematics obvious to her in those representations (PBP 1b). Her use of problem scenarios having similar characteristic suggested that she wanted to create a learning trajectory for students so that those problems would make the mathematical relationships as apparent to them as possible (PBP 1a). In contrast to Alin's planning of her lesson on her students' background knowledge, Meryem's focus was on mathematics of other students. That is, albeit important, Meryem's rationale for how she has chosen the learning goal for her students was based on some other students' understanding of the domain and range of different functions. This also suggested that she might have believed that mathematics as part of objective reality existed in those representations ready for students' perception through engaging in tasks that would allow them to 'see' and connect the intended ideas (PBP 1c).

## **Conclusion**

This study investigated prospective teachers' mathematical knowledge for teaching and their perspectives. In particular, data from two prospective teachers having a PIP and a PBP revealed that prospective teachers having both PBP and PIP depicted all the codes in Knowledge Quartet; indicating that they held mathematical knowledge for teaching (MKT). Data from the pre-interviews both from Alin and Meryem had shown that they did not adhere to the textbooks, rather they focused on students' prior knowledge while they were deciding and arranging the lesson activities, they had a sense of choice of examples and sequence of ideas within the lesson. Similarly, they had strong subject matter knowledge and theoretical background about the topics they have taught. Yet, their reasons behind such knowledge were different: Alin's focus on students' mind activities and their abstracting the relationships between the covarying quantities (the dependent and independent variables) suggested that she thought of mathematics as dependent on the knower. Similarly, this suggested that Alin viewed mathematics learning occurring through students' own activities. On the other hand, Meryem had different reasoning behind such mathematical knowledge that she viewed mathematics as depicted in the graphs and examples she expected students to perceive. These results are consistent with Weston (2013) results, who found that although different prospective teachers demonstrated the same codes in KQ, the nature of such demonstration differed from one prospective teacher to the other in terms of how much of such knowledge they had. Results suggest that teachers with different teacher perspectives might depict a strong MKT albeit different reasons behind such knowledge. This therefore suggests the need to pay attention to teachers' mathematical knowledge



together with their rationale behind it. Yet we acknowledge that the data came from only two prospective teachers. Therefore, we suggest further examination of MKT of prospective and in-service teachers having different perspectives. Also, since the results suggest that prospective teachers having PIP might have a strong rationale behind their MKT we suggest to promote at least the development of a PIP on the part of prospective and in-service teachers during professional development studies.

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