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► To cite this version:

Jennifer Dröse, Birgit Griese, Lena Wessel. Prospective teachers' diagnostic judgments on students' understanding of conditional probabilities. Twelfth Congress of the European Society for Research in Mathematics Education (CERME12), Feb 2022, Bozen-Bolzano, Italy. hal-03744585

HAL Id: hal-03744585

<https://hal.science/hal-03744585>

Submitted on 3 Aug 2022

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Prospective teachers' diagnostic judgments on students' understanding of conditional probabilities

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In recent years, different studies have emphasized the necessity to improve prospective teachers' professional knowledge and teaching competences, especially prospective teachers' diagnostic competence, which accounts for perception, interpretation, and decision-making concerning students' individual thinking processes and learning obstacles. In order to do so, this paper follows a content-related approach that focuses specifically on diagnostic judgments on conceptual understanding and procedural skills of students' understanding of conditional probabilities and their underlying prior knowledge elements. Prospective teachers are known to face challenges in adequately focusing on these aspects in their diagnostic judgments. Therefore, written diagnostic judgments of 26 prospective teachers on two transcript vignettes are investigated. The results indicate that prospective teachers show a high focus on prior knowledge and on procedures in their diagnostic judgments.

Keywords: Diagnostic judgments, conditional probabilities, prospective teachers, vignettes.

Introduction

As part of prospective teachers' professional preparation student-centered teaching plays an important role. The latter requires teachers to diagnose skillfully, i.e. to master the mental processes of perceiving, interpreting, and decision-making (Empson & Jacobs, 2008). In order to prepare prospective teachers for adequate diagnostic judgements, it is necessary to gain deeper insights into their diagnostic thinking processes, that can be inferred from their diagnostic judgements. Already existing studies line out that prospective teachers are known to struggle with addressing conceptual understanding and procedural skills in their diagnostic judgments (Bartell et al., 2013). This underlines the importance of focusing on the mathematical content, such as relevant knowledge elements of the current learning content as well as the underlying procedural skills and conceptual understanding in prospective teachers' diagnostic judgements (Prediger, 2020). Therefore, our research interest is to investigate which elements of the mathematical content teachers perceive and interpret (as processes of diagnostic thinking) in students' understanding of conditional probabilities in a one-to-one teacher-student discussion. This topic was chosen as it is a pivotal, but often challenging concept and holds relevance in vocational contexts (Binder et al., 2020).

Theoretical Background

Prospective teachers' content-related diagnostic judgments

Teachers' diagnostic competence has been found to be important for student-centered teaching (Empson & Jacobs, 2008). Synthesizing different approaches on diagnostic thinking and diagnostic competence, Loibl et al. (2020) provide a framework for locating different research approaches (see Figure 1). In addition, the framework displays the impact of the diagnostic thinking processes on diagnostic behaviors.

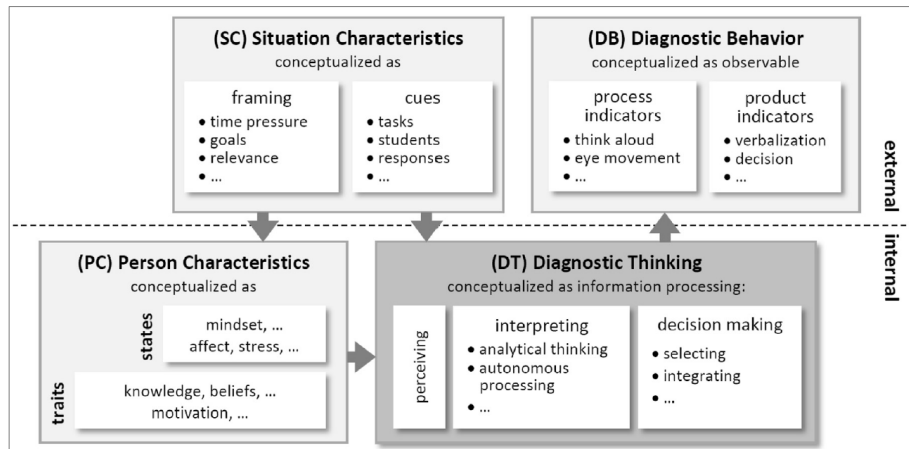


Figure 1: DiaCom framework of diagnostic thinking (Loibl et al., 2020, p. 3)

Therefore, those processes can be derived from diagnostic judgements, influenced by situation and person characteristics. Stahnke et al. (2016), with their systematic review of different studies and research approaches, show that although most of the studies investigating diagnostic competence use a particular mathematical content, only few studies explicitly integrate the content into the analysis of the diagnostic judgments. The study presented in this paper also focuses on content-related aspects in analyzing the diagnostic judgments, following Dröse and Prediger (submitted).

When taking this content-related approach, the principal distinction (as in Kilpatrick et al., 2001) is between conceptual understanding (as meaning of mathematical concepts, operations, and terms) and procedural skills (as procedures for algorithms and solution strategies). This allows for conceptualizing students' understanding as a network of the main mathematical knowledge elements, conceptual and procedural (Hiebert & Carpenter, 1992). The ensuing network contains knowledge elements of the current learning content as well as prior knowledge elements, i.e. conceptual understanding and procedural skills from previous years as a foundation (Prediger, 2020; Dröse & Prediger, submitted).

Prospective teachers in particular have been shown to focus (in the sense of perceiving and interpreting) more on general aspects of learning in their diagnostic judgments than on mathematics- and content-specific aspects (Jansen & Spitzer, 2009). Moreover, when focusing on content-related aspects, prospective teachers are more likely to perceive procedural obstacles – albeit not their cause (Cooper, 2009). What is more, they often misinterpret conceptual obstacles as procedural (Son, 2013). While obstacles are mostly interpreted by prospective teachers as lack of a procedural skill, when it comes to students' resources, prospective teachers often interpret students' statements as indicating conceptual understanding – even if the aspects they refer to are procedural skills (Bartell et al., 2013).

Against this background, it makes sense to first take a closer look at the conceptual understanding and procedural skills concerning conditional probabilities and the related students' obstacles.

Knowledge elements and students' understanding of conditional probabilities

For the *current mathematical content* of conditional probabilities, procedural skills as well as conceptual understanding are relevant learning goals. Conceptual understanding of conditional probabilities implies concepts concerning stochastic (in)dependence for describing the likelihood of an event under certain conditions or independent of conditions (Hoffrage et al., 2015). Students face

various obstacles in the area of conditional probabilities, e.g. distinguishing joint and conditional probabilities (Shaughnessy, 1992). For determining conditional probabilities and solving Bayesian problems, different visualizations and solution strategies can be used, e.g. tree diagrams, frequency grids, unit squares, or 2x2 tables, viewed as procedural skills (see overview in Binder et al., 2020).

These knowledge elements for conditional probabilities build on *prior mathematical content* knowledge, especially the part-whole relationship as well as the part-of-part determination as an important mental model for the multiplication of fractions. Both belong to the conceptual understanding of fractions (Post & Prediger, 2020; Prediger & Schink, 2009) and are known to present serious obstacles, e.g. in identifying the appropriate whole (Prediger & Schink, 2009). In this context, procedural skills (e.g. routine calculations with fractions, decimal numbers, and percentages) may also constitute a learning difficulty (e.g. Prediger & Schink, 2009).

In this paper, we will focus on the procedural skills and conceptual understanding of the current learning content, conditional probabilities, and its underlying prior knowledge elements, in connection to possible students' obstacles and individual mental models, found in the diagnostic judgments of prospective teachers. For designing adequate learning opportunities for prospective teachers, it is important to investigate the knowledge elements addressed in their diagnostic judgments.

Considering this design interest as well as the research areas and gaps, our research question reads: *Which concepts or procedures of the current / prior learning content do prospective teachers include in their diagnostic judgments on students' understanding of conditional probabilities?*

Methods

Data collection

The data was collected in a university mathematics education course for German prospective secondary school teachers. The sample consists of $n = 26$ prospective teachers, 81% studying for secondary and upper secondary school, and 19% for vocational schools. All students have reached the last year of their bachelor program, 69% after three and 31% after four years. They attended the first and second author's university course, which covered content knowledge as well as pedagogical content knowledge on conditional probabilities, e.g. students' errors, and related knowledge elements.

The prospective teachers' written diagnostic judgments were gained by analyzing a vignette as part of the weekly assignments (see Figure 2 for our vignette). Vignettes can be seen as an established instrument for investigating prospective teachers' competences (cf. overview in Buchbinder & Kuntze, 2018) and have been used to investigate content-related diagnostic judgements on procedural and conceptual knowledge elements in the mathematical content of arithmetic (Dröse & Prediger, submitted).

Our vignette consists of a task, two written student solutions and transcripts of subsequent dialogues between student and teacher. For the transcript, a real dialogue (based on transcripts in Post & Prediger, 2020; Post, in preparation) was chosen as basis, which was adapted taking theoretical considerations on conditional probabilities into account (current learning content from Hoffrage et al., 2015; Shaughnessy, 1992; Binder et al., 2020; prior learning content perspectives from Post & Prediger, 2020; Prediger & Schink, 2009).

Transcript vignette

For the following task you should put yourself in the teachers' position and react appropriately within the situation. Background information for the following scene: The class in a German upper secondary school has covered conditional probabilities and their calculation. In the following you will read two transcripts displaying excerpts from conversations between two students (Ole and Nazan) and their teacher, regarding the task displayed on the right.

Task: Exercising Teenagers

In a survey, 1200 teenagers were asked if they exercise regularly. 600 out of the 1200 teenagers are female. $\frac{1}{3}$ of the female teenagers do not exercise regularly. $\frac{3}{8}$ of the teenagers are male and exercise on a regular basis. What is the probability that a random male person exercises regularly?

(JIM study 2018)

	female (600)	male (600)
exercise regularly (850)	400	450
not exercise regularly (350)	200	150
total:	1200	

Part 1: Ole solves the task. He writes down the following solution.

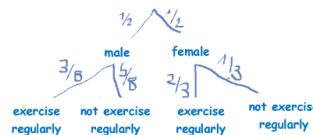
$$\frac{3}{8} = 0.375 = 0.375\%, \text{ the probability is } 0.375\%.$$

The following interaction with the teacher evolves:

- 1 T: Ole, how did you solve the task?
- 2 Ole: Actually, there is not much to calculate. The text says that $\frac{3}{8}$ of all teenager are male and do sports' [reads from the text]. And then I just have to convert this into percentages and that is 0.375%.
- 3 T: Aha. This share is very small. Hm. Perhaps it might help you, if we had a look at the unit square below the task again? [Points at the unit square that is printed below the task.]
- 4 Ole: So, look at this numbers here, 600 male [points at the labelling "male (600)"] and 850 exercising. The 600 are male and the 450 are the ones who exercise, in addition. So, these are the 450 [points to the area with the number 450].
- 5 T: And what's the size the share that is sought in the task?
- 6 Ole: Ehm, the share is 450 of the whole, 1,200? Ehm [reads the question again] no, in the question there is just this group here [points at the area with the labelling "male (600)"] so this, these are the males, but these here [points at the two areas on the left], those are not considered in the denominator, and the counter would be 450 [points to the area with 450]. Or in other words: This is the whole group, these are the males and this is the share of them exercising regularly.
- 7 T: Good, so you have the fraction $\frac{450}{600}$. But what about the probability or... ehm the share that you calculated before, so the $\frac{3}{8}$?
- 8 Ole: Perhaps it can be cancelled, and then it is equal.

Part 2: Nazan also solves the task. She writes down the following solution.

$$\frac{3}{8} \cdot \frac{1}{2} = \frac{3}{8} \cdot \frac{4}{8} = \frac{12}{64} = 0,1875 = 18.75 \%$$



The following interaction with the teacher evolves:

- 1 T: Nazan, how did you solve the task?
- 2 Nazan: So at first I calculated that the probability for boys is 50%, that is $\frac{1}{2}$. Then I can write this into the tree diagram [points at her tree diagram] and then I have to calculate $\frac{1}{2}$ times $\frac{3}{8}$. So you convert them to the same denominator, and $\frac{1}{2}$ is equal to $\frac{2}{4}$ [points to her written calculation] and multiplied this is $\frac{12}{64}$. And this is 18.75 percent. So the probability is 18.75%.
- 3 T: Let's have a look at the unit square below the task to be sure. Which parts do we have to look at? [Points at the unit square below the task.]
- 4 Nazan: So for the numbers we have 600 here [points at the label "male (600)"] and 850 exercising [points at the label "exercising (850)"]. Yes, and then this is $\frac{600}{1200}$ times $\frac{850}{1200}$, because ehm those are the important issues, male and exercising.
- 5 T: And how did you transfer that into your tree diagram?
- 6 Nazan: So, the $\frac{600}{1200}$, that is $\frac{1}{2}$ in my tree diagramm, the probability for boys. $\frac{600}{1200} = \frac{1}{2}$, cancelled out. Are $\frac{3}{8}$ the same as $\frac{850}{1200}$, if you cancel? Somehow this has to be, because in the text says teenager, who are male and exercise regularly, and that is the same as in the question. Just the "and" here, that has changed in the question [points at the "and" in the text and then at the question].

Task: Analyze the two transcripts:

- (1) Describe which prior knowledge and resources (conceptual knowledge, procedural knowledge, representations, etc.) Ole and Nazan draw on.
- (2) Describe which obstacles Ole and Nazan display. Explain the possible causes of these obstacles. Give transcript lines for (1) and (2) that underpin where you locate the aspect within the transcript or the notes.

Figure 2: Transcript vignette with task for prospective teachers

The transcript vignette therefore provides sample insights into students' understanding of conditional probabilities and the underlying prior knowledge elements, and thus sufficient possibilities for

diagnostic judgements on conceptual understanding and procedural skills as well as knowledge elements of the current learning content and prior knowledge elements. Cues for the different knowledge elements are presented in equal numbers.

Data analysis

The 26 written diagnostic judgments were coded with respect to the knowledge elements for the current or prior learning content in the two dimensions of procedures and concepts. The coding scheme was deduced from the theoretical analysis of the knowledge elements and inductively enriched by the knowledge elements named by the prospective teachers. Two raters following the coding scheme yielded an interrater reliability of Cohen’s $\kappa = 0.86$, which is almost perfect. Table 1 displays the codes used and excerpts of exemplary diagnostic judgements.

Table 1: Excerpt of a written diagnostic judgement and knowledge elements assigned to them

	Conceptual understanding	Procedural skills
Current learning content	Knowledge element: understanding conditional and joint probabilities Excerpt: “Does not know the difference between $P(A \cap B)$ and $P(B A)$.”	Knowledge element: calculating and solution strategies for conditional probabilities Excerpt: “Difficulties in calculating conditional probabilities.”
Prior learning content	Knowledge element: understanding of fractions (part-of-part), unit square Excerpt: “Ole has an understanding of part-of-parts”	Knowledge element: calculating with fractions, decimal numbers and percentages Excerpt: “Ole can transform fractions into decimal numbers.”

Empirical findings on prospective teachers’ diagnostic judgments

In total, 327 codes were set for the statements of the 26 prospective teachers. Figure 3 displays the coded knowledge elements in prospective teachers’ diagnostic judgments. Table 1 provides first excerpts of written diagnostic judgements. In the following the relationships between the coded knowledge elements are described and enriched by excerpts of the written diagnostic judgements.

Comparing the knowledge elements, the results indicate that the prospective teachers tend to focus more on the *prior learning content* in their diagnostic judgments than on knowledge elements of the *current learning content* (Figure 3, first line). In Addition, their statements address a higher amount of *procedural knowledge elements* than of *conceptual knowledge elements* (Figure 3, second line). For the procedural elements there seems to be a higher number of obstacles addressed, while for conceptual knowledge elements a higher amount of statements is related to *resources*.

For the *current learning content* (Figure 3, third line), *procedural* and *conceptual* elements seem to be addressed equally. While the conceptual elements are equally described as *resources* and *obstacles* (e.g. conceptual resource: “has a concept of probabilities”, conceptual obstacle: “doesn’t know the difference between joint and conditional probabilities”), only few statements concerning

procedural skills address them as resources (e.g. procedural resource: “knows how to calculate joint probabilities”, procedural obstacle: “has difficulties in calculating conditional probabilities”).

Most of the statements for the *prior learning content* as well as most of the statements overall concern *procedural* skills, e.g. operating with fractions, decimal numbers, and percentages (Figure 3, third line). These elements are equally addressed as *resources* and as *obstacles* (e.g. procedural resource: “can shorten fractions”, procedural obstacles: “cannot convert fractions into decimal numbers”). The addressed elements of *conceptual* understanding are expressed more often as *resources* than as *obstacles* (e.g. conceptual resource: “can interpret parts in the unit square”, conceptual obstacle: “cannot derive part-of-part relations from the unit square”).

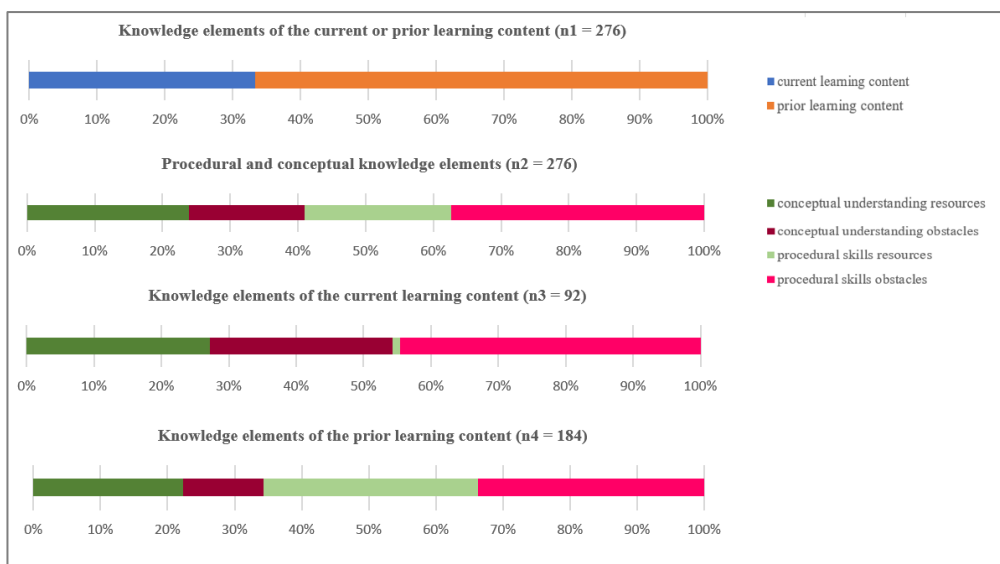


Figure 3: Knowledge elements in diagnostic judgments

Discussion and outlook

Referring to the research question (*Which concepts or procedures of the current / prior learning content do prospective teachers include in their diagnostic judgments on students’ understanding of conditional probabilities?*), our content-related approach revealed the following findings:

In general, the prospective teachers in our study focus more extensively on *procedural skills* than on *conceptual understanding*. These findings are in line with previous research (Cooper, 2009; Son, 2013; Bartell et al., 2013). For the current learning content, however, statements on conceptual understanding are dominant. Further qualitative investigations (e.g. interviews) are needed to interpret this result, which is divergent to previous research. By distinguishing resources and obstacles, we saw that procedural skills are far more often addressed as obstacles than as resources overall, although they are categorized equally as obstacles and resources for the prior learning content. As Cooper (2009) indicates, prospective teachers are more likely to perceive procedural obstacles in students’ utterances but not their origins which might be conceptual. Future studies could ask prospective teachers to classify their judgements as referring to conceptual or to procedural knowledge elements, as Son (2013) revealed that prospective teachers misinterpret conceptual obstacles as procedural. However, the current study can provide no insights here. Yet, it might present a problem if prospective

teachers (in terms of the DiaCom framework) mostly perceive and identify procedural obstacles in students' utterances, as that might influence their decision-making as active teachers, hindering them to address conceptual learning adequately (see Loibl et al., 2020, for details). On the basis of these findings, starting points for redesigned learning opportunities can be identified, e.g. using authentic tasks for diagnosis and discussing with prospective teachers' options for student activities that explicitly promote conceptual understanding.

In addition, by distinguishing *prior and current learning content*, we found that prospective teachers are indeed able to describe different content elements building upon each other. This is relevant, as an interconnected network of mathematical knowledge is paramount for the sustainable learning of mathematics. In particular, low-performing students have been shown to lack sufficient prior content knowledge for keeping up with the current learning content (Prediger, 2020). Therefore, our analysis reveals possible potentials of prospective teachers' diagnostic judgements that have not yet been investigated in depth and can provide first starting-points for offering further learning opportunities and developing teaching-learning arrangements, e.g. combining and connecting university courses concerning CK and PCK more deeply.

Due to the aforementioned aspects it would be beneficial to compare the prospective teachers' diagnostic judgements and the identified knowledge elements to the diagnostic judgments of experienced in-service teachers or teacher educators. By doing so, it might be possible to identify further aspects that could in turn be integrated as learning content in prospective teachers' courses at university.

Our research is limited due to the following aspects: (a) Our sample comprises only 26 prospective teachers all from the same university. Future research is planned to extend the sample size and the sample itself to in-service teachers. (b) Our research addresses a specific content. As diagnostic judgments might vary between contents, other content should also be investigated. (c) The use of a transcript vignette meant that it could have been read several times. It is possible that the diagnostic judgments would focus on less aspects in a different format, e.g. video vignettes (Buchbinder & Kuntze, 2018). (d) The vignette displays a teacher-student one-to-one interaction and not a whole group classroom discussion. Where the vignette can be extended to in the future. Future research should also clarify and investigate these limitations. In addition, it should be explored how transcript vignettes can be used to foster prospective teachers' diagnostic judgments.

Acknowledgment

We thank Sarah Dohle and Lukas Bartsch for their support in the data analysis as part of their bachelor and master theses, and our colleagues and experts for their collaboration.

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