# Obtaining an optimal compression spring design directly from a user specification

Manuel Paredes, Marc Sartor, C Masclet

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M. PAREDES, M. SARTOR and C. MASCLET

Département de Génie Mécanique, INSA, 31077 Toulouse, France, manuel.paredes@insa-tlse.fr

**Abstract :** Common mechanical components can be designed manually but this involves problem simplifications. A computer can help to develop a suitable approach for optimizing design. Industrial software for spring design are comprehensive tools for validation only. To improve design assistance, to the designer, this paper presents a software which is not only comprehensive, but can also serve as a dimensional synthesis tool for compression spring design. It exploits an optimization process for use even in the early design steps as specifications are set with interval values. It includes a large set of constraints which can express not only the designer specifications but also the standards and the technical capability limits of spring manufacturers. This process has been successfully tested by a spring manufacturer, who used it to solve real industrial problems for two years. One case study is presented.

**Key words** : optimization, interval analysis, compression springs, industrial application.

## NOTATION (based on DIN [1] standards)

$A$ : Least compressed sate of the spring

$B$ : Most compressed sate of the spring

$D_o$ : outside diameter [mm]

$D$ : mean diameter [mm]

$D_i$ : inside diameter [mm]

$d$ : wire diameter [mm]

$E$ : elastic modulus of the material [N/mm$^2$]

$F_1$, $F_2$ : operating loads [N]

$f_e$ : natural frequency of surge waves [Hz]

$G$ : torsion modulus of the material [N/mm$^2$]

$k$ : stress correction factor

$L_0$ : free length [mm]

$L_1$, $L_2$ : operating lengths [mm]

$L_c$ : solid length [mm]

$L_K$ : buckling length [mm]

$L_n$ : minimal operating length to respect the minimum space between coils [mm]

$L_r$ : minimal operating length to respect the maximum allowable stress [mm]

$M$ : spring mass [g]

$m$ : pitch of the spring [mm]

$N$ : number of cycles

$n$ : number of active coils

$n_i$ : number of inactive coils

$R$ : spring rate [N/mm]

$R_m$ : Ultimate Tensile Strength of the material [N/mm$^2$]

$S_a$ : minimum space between coils [mm]

$S_h$ : spring travel [mm]

$V_0$ : overall space taken up at $L_0$ [cm$^3$]

$V_2$ : overall space taken up at $L_2$ [cm$^3$]

$W_h$ : internal energy stored during operating travel [N.mm]

$w$ : spring index

$\alpha_c$ : pourcentage of $R_m$ at $L_c$

$\alpha_2$ : pourcentage of $R_m$ at $L_2$

$\alpha_F$ : fatigue life factor

$\gamma$ : helix angle [degrees]

$\nu$ : end fixation factor

$\rho$ : density of the material [kg/m$^3$]

$\tau_d$ : fatigue limit of the material [N/mm$^2$]

$\tau_m$ : mean shear stress [N/mm$^2$]

$\tau_a$ : alternate shear stress [N/mm$^2$]

**Left superscripts**

S : from the specification sheet

M : from the manufacturer constraints or form standards

**Right superscripts**

U : Upper limit

L : Lower limit

# 1. INTRODUCTION

The design of machines imposes the dimensioning of numbers of common mechanical components (gears, cams, shafts, springs). These components have their own dimensioning rules and require specific manufacturing knowledge. The problem of designing a mechanical component is often solved by using tables and charts for certain pre-selected specifications and certain pre-selected objectives. These calculations can be carried out manually, but without computer assistance, designers are often obliged to oversimplify the procedures, e.g. by assigning a value to certain parameters in order to reduce the number of problem variables to only 2 or 3 [2,3]. They are thus unable to exploit all the specification possibilities and consequently to optimize design.

Progress in computer assisted design should lead to successful solutions to this kind of problem. In theory, it should be possible, for a given component, to define the problem globally and find the appropriate resolution method that would provide the optimal solution whatever the specifications.

This paper deals with optimal spring design. There are many kinds of springs, for example helical springs with circular wire [4], helical springs with non-circular wire [5] or tape springs [6]. This paper focuses on helical compression springs with circular wire which are the most common in use today. Unfortunately, the main industrial software available to a designer during the spring definition work ("Compression Spring Software" from IST® [7], "FED1©" from Hexagon© [8], "Spring Design Software" from SMI [9]) use standard calculations solely to make a full check of the proposed spring's compatibility with the given specifications. "Spring Design Software" can carry out minimum mass optimization to adjust one design parameter but three other design parameters have to be known beforehand. Another software, "Spring Master FEA" [10], uses a helical finite element computational engine to make a stress analysis and visualize 3D color stress contours. It can run a minimum stress and wire length optimization but fails to take into account fatigue life and the constraints imposed by

standards. As far as we are aware, the following capability is not commonly presented : suggesting the most suitable spring design from global specifications.

This paper proposes an approach which intends to satisfy this need. An algorithm is given to determine the optimal spring design from a specification sheet where data is set with interval values. It can be implemented using any standard optimization code or a spreadsheet endowed with an optimization solver. In this study, only steel and stainless steel helical compression springs with closed ends ($n_i = 2$) and closed and ground ends ($n_i = 3$) are considered.

# 2. DEFINITIONS

## *2.1. Spring design parameters*

The main parameters which define spring geometry are: $D_o$ , $D$ , $D_i$ , $d$ , $R$ , $L_0$ , $L_c$ , $n$ , $\gamma$ , $m$. Fig. 1 illustrates these parameters which characterize the intrinsic properties of the spring.

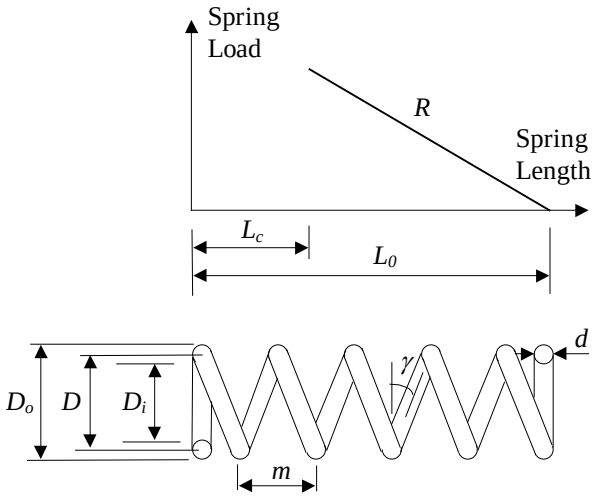


Fig. 1 Design parameters

Four independent parameters have to be known to calculate the six others.

## *2.2. Spring operating parameters*

A spring works traditionally between two configurations, one corresponding to the least compressed state *A*, the other corresponding to the most compressed state *B*. The parameters which define the use of a spring are: $F_1$ , $F_2$ , $L_1$ , $L_2$ and $S_h$ (see Fig. 2).

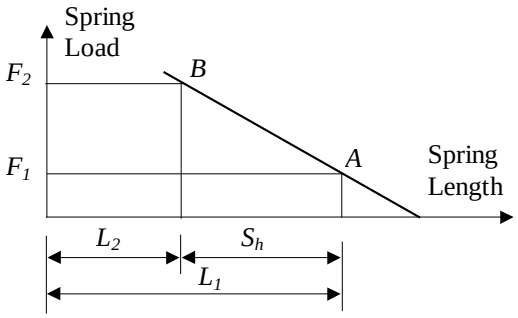


Fig. 2 Operating parameters

When the design parameters are known, two independent operating parameters (to be taken among $F_1$ , $F_2$ , $L_1$ , $L_2$ and $S_h$) are necessary to determine the two points *A* and *B*.

### *2.3. Specification sheet*

In the early design stages, there are always many parameters which have not been already fixed. Thus it is difficult to give fixed values for a spring and it is more convenient to define parameters through their possible lower and upper limits. For this reason, a specification sheet has been defined with interval values. This specification sheet is used as a simple graphical interface as shown in Fig. 3. Design and operating parameters can be set by giving their bounds (lower and/or upper limits: ${}^{S}L_0{}^{L}$ , ${}^{S}L_0{}^{U}$ ,… ${}^{S}F_1{}^{L}$ , ${}^{S}F_1{}^{U}$ ,...). To fix the value of a parameter, it must be placed in both the lower and upper limits defining that parameter in the specification sheet (see Fig 3).

Moreover, many other characteristics to be respected can be defined (with interval values):

- Natural frequency of surge waves ($f_e$)
- Internal energy stored during operating travel ($W_h$)
- Spring mass ($M$)
- Overall space taken up at free length ($V_0$)
- Overall space taken up at minimal operating length ($V_2$)

Additional data can also be provided to calculate other characteristics:

- The number of cycles $N$ to calculate the fatigue life factor $\alpha_F$
- The end fixation factor $\nu$ to calculate the buckling length without a guide or housing $L_K$
- The allowable percentage of the Ultimate Tensile Strength $R_m$ at $L_c$, called ${}^{S}\alpha_c{}^{U}$. The elastic limit of steel is about 50% of the $R_m$ for steel. This is the default value of ${}^{S}\alpha_c{}^{U}$ but when it is certain that the spring will not be compressed to its solid length $L_c$, higher $R_m$ percentages can be accepted.

The designer can also specify the material and the spring ends required. Finally, to be able to calculate the best spring, the objective function F (minimum mass, maximum fatigue life...) is to be given.

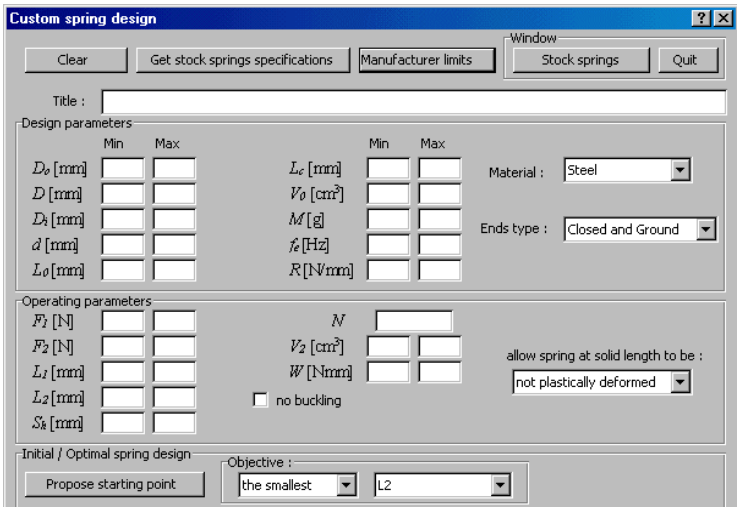


Fig. 3 Specification sheet

# 3. THE OPTIMIZATION PROBLEM

## *3.1. Defining the problem*

The discovery of the spring design offering the best possible value of the objective function is an optimization problem with six variables. Four design parameters ($D_o$ , $d$ , $L_0$ , $R$) and two operating parameters ($L_1$ and $L_2$) are stored in the « vector of design variables » $\boldsymbol{X}$, so $\boldsymbol{X} = [D_o , d , L_0 , R , L_1 , L_2]^T$.

The objective function is expressed using the conventional form :

Minimise or Maximise F($\boldsymbol{X}$)

where F($\boldsymbol{X}$) is either the fatigue life factor, or anyone of the parameters that could be set in the specification sheet : $D_o$ , $D$ , $D_i$ , $d$ , $R$ , $L_0$ , $L_1$ , $L_2$ , $L_c$ , $S_h$ , $F_1$ , $F_2$ , $M$ , $f_e$ , $V_0$ , $V_2$ , $W$.

In addition to this objective, a large set of constraints is considered to enable the resolution of most real-life industrial problems. Table 1 enumerates the 42 selected constraints which can express not only the designer specifications but also the standards and the technical capability limits of the spring manufacturer. All the necessary formulae for coding these

equations are given in the Appendix. An optimization system with 6 variables and a maximum of 42 constraints is thus obtained and has to be resolved.

Each set of data not defined in the user specification sheet is automatically set to a default value to be sure that they will not influence the optimization process : 0 for a lower limit and $10^7$ for an upper limit. A standardized wire range exists, but a wire can be manufactured to any diameter. When using standardized wire**,** each wire diameter ($d$) considered implies placing ${}^{S}d^{L} = {}^{S}d^{U} = d$ in the specification sheet.

Table 1 Constraints

| Constraint about : | Upper limit | Lower limit |
|---|---|---|
| $D_o$ | g1(X): $D_o$ - Min (${}^{S}D_o{}^{U}$, ${}^{M}D_o{}^{U}$) ≤ 0 | g2(X): Max (${}^{S}D_o$, ${}^{M}D_o{}^{L}$) - $D_o$ ≤ 0 |
| $d$ | g3(X): $d$ - Min(${}^{S}d^{U}$, ${}^{M}d^{U}$ ) ≤ 0 | g4(X): Max(${}^{S}d^{L}$, ${}^{M}d^{L}$) - $d$ ≤ 0 |
| $L_0$ | g5(X): $L_0$ - ${}^{S}L_0{}^{U}$ ≤ 0 | g6(X): ${}^{S}L_0{}^{L}$ - $L_0$ ≤ 0 |
| $R$ | g7(X): $R$ - ${}^{S}R^{U}$ ≤ 0 | g8(X): ${}^{S}R^{L}$ - $R$ ≤ 0 |
| $L_1$ | g9(X): $L_1$ - ${}^{S}L_1{}^{U}$ ≤ 0 | g10(X): ${}^{S}L_1{}^{L}$ - $L_1$ ≤ 0 |
| $L_2$ | g11(X): $L_2$ - ${}^{S}L_2{}^{U}$ ≤ 0 | g12(X): ${}^{S}L_2{}^{L}$ - $L_2$ ≤ 0 |
| $D$ | g13(X): $D$ - ${}^{S}D^{U}$ ≤ 0 | g14(X): ${}^{S}D^{L}$ - $D$ ≤ 0 |
| $D_i$ | g15(X): $D_i$ - ${}^{S}D_i{}^{U}$ ≤ 0 | g16(X): ${}^{S}D_i{}^{L}$ - $D_i$ ≤ 0 |
| $n$ | g17(X): $n$ - ${}^{M}n^{U}$ ≤ 0 | g18(X): ${}^{M}n^{L}$ - $n$ ≤ 0 |
| $\gamma$ | g19(X): $\gamma$ - ${}^{M}\gamma^{U}$ ≤ 0 | ******* |
| $F_1$ | g20(X): $F_1$ - ${}^{S}F_1{}^{U}$ ≤ 0 | g21(X): ${}^{S}F_1{}^{L}$ - $F_1$ ≤ 0 |
| $F_2$ | g22(X): $F_2$ - ${}^{S}F_2{}^{U}$ ≤ 0 | g23(X): ${}^{S}F_2{}^{L}$ - $F_2$ ≤ 0 |
| $S_h$ | g24(X): $S_h$ - ${}^{S}S_h{}^{U}$ ≤ 0 | g25(X): ${}^{S}S_h{}^{L}$ - $S_h$ ≤ 0 |
| $w$ | g26(X): $w$ - ${}^{M}w^{U}$ ≤ 0 | g27(X): ${}^{M}w^{L}$ - $w$ ≤ 0 |
| $L_n$ | ******* | g28(X): $L_n$ - $L_2$ ≤ 0 |
| $L_K$ | ******* | g29(X): $L_K$ - $L_2$ ≤ 0 |
| $f_e$ | g30(X): $f_e$ - ${}^{S}f_e{}^{U}$ ≤ 0 | g31(X): ${}^{S}f_e{}^{L}$ - $f_e$ ≤ 0 |
| $W_h$ | g32(X): $W_h$ - ${}^{S}W_h{}^{U}$ ≤ 0 | g33(X): ${}^{S}W_h{}^{L}$ - $W_h$ ≤ 0 |
| $M$ | g34(X): $M$ - ${}^{S}M^{U}$ ≤ 0 | g35(X): ${}^{S}M^{L}$ - $M$ ≤ 0 |
| $V_0$ | g36(X): $V_0$ - ${}^{S}V_0{}^{U}$ ≤ 0 | g37(X): ${}^{S}V_0{}^{L}$ - $V_0$ ≤ 0 |
| $V_2$ | g38(X): $V_2$ - ${}^{S}V_2{}^{U}$ ≤ 0 | g39(X): ${}^{S}V_2{}^{L}$ - $V_2$ ≤ 0 |
| $\alpha_2$ | g40(X): $\alpha_2$ - $\alpha_2{}^{U}$ ≤ 0 | ******* |
| $\alpha_c$ | g41(X): $\alpha_c$ - ${}^{S}\alpha_c{}^{U}$ ≤ 0 | ******* |
| $\alpha_F$ | ******* | g42(X): 1 - $\alpha_F$ ≤ 0 |

### *3.2. Problem resolution*

The goal here is to find a fast, reliable and complete method for solving our problem. Moreover, as we wish to avoid asking the user to give more data, this method has to be carried out automatically.

Artificial intelligence has been applied to optimal spring design. MOTZ [11] has used an expert system combining conventional design formulae with a finite element analysis, but it is time-consuming. LIN [12] describes an expert system with an external optimization program using the simulated annealing algorithm. This is a theoretically global optimization algorithm [13] but the result is critically dependent on the annealing schedule and on the search diversification features.

From another point of view, the spring optimization problem is often used to illustrate mathematical methods (genetic algorithms, nonlinear programming in integer and discrete variables...). As the examples dealt with are theoretical illustrations, the presented problems are simplified and only manage the specific constraints of the examples. Thus, they generally do not take into account all the constraints of a real specification sheet. YOKOTA et al. [14, 15] only apply genetic algorithms in their spring problem to constraints related to the spring rate $R$ (g7, g8) and $\alpha_2$ that represents the stress at $L_2$ (g40). A complete problem resolution using this approach would require a large amount of calculations. DEB and GOYAL [16], KANNAN and KRAMER [17] and SANDGREN [18] impose operating parameters $F_1$, $F_2$ and $S_h$ which moreover, only take into account the limits on $D_o$ (g1), $d$ (g4), $L_0$ (g5), $w$ (g27), $L_n$ (g28) and $\alpha_2$ (g40). They use springs to illustrate their optimization methods in discrete variables by imposing a discrete number of coils, but spring makers can manufacture the number of coils as a continuous parameter. Thus, in our calculations, the number of coils remains continuous. QIAN [19] also envisaged an optimization problem where the operating points are entirely given ($F_1$, $F_2$, $L_1$, $L_2$ fixed). This method takes into account only 7 constraints on $D_o$ (g1), $D_i$ (g16), $\gamma$ (g19), $w$ (g26, g27), $L_n$ (g28) and $\alpha_2$ (g40) but can easily be extended to the whole of the problem constraints. However, the obligation to define completely the operating points limits its practical use.

None of these methods can thus be applied directly. For our problem, we have chosen to implement a direct method [20] because each step of the resolution process is based on a displacement inside the solution area. This kind of property is useful for us as, if the resolution process is stopped before the end (for example if the calculation time is too great), the tool will be more likely to provide an acceptable if non optimal solution to the designer. Moreover, our optimisation problem contains inequality constraints that could sometimes be equivalent to equality constraints. Indeed, a designer may wish to obtain a specific value for a parameter by placing in the specification sheet the expected value in both the lower and upper limits related to this parameter. For example, on Fig. 8, the required value for $F_2$ is 127 N as ${}^{S}F_2^{L} = {}^{S}F_2^{U} = 127$ N. Thus, the two associated inequality constraints of our optimisation problem, g22(X): $F_2 - 127 \leq 0$ and g23(X): $127 - F_2 \leq 0$ are equivalent to one equality constraint. Most optimisation algorithms do not give good results to such a problem. That is why we have used an algorithm based on the Generalised Reduced Gradient solution process, as it is a direct method that can manage equality constraints [20]. Two options are offered in this algorithm : the conjugate gradient method or a quasi-Newton approach (BFGS). Taking into account the low number of variables, the BFGS method has been used [20].

Direct methods require a starting point close to the solution area. Indeed, the closer this is to the final solution, the more likely is the algorithm to converge towards the optimal solution. This is particularly true here in viewing the high number of

constraints. For that reason, we have developed another algorithm that automatically determines the calculation starting point by analysing the main parameters of the specification sheet, so as to respect the most possible constraints.

# 4. AUTOMATIC DETERMINATION OF THE CALCULATION STARTING POINT

To determine the calculation starting point, the most usual data ($D_o$, $D$, $D_i$, $d$, $L_0$, $R$, $F_1$, $F_2$, $L_1$, $L_2$, $S_h$ ) are analyzed in order to define an initial spring which respects the majority of the problem constraints. Fig 4 presents the different steps of the proposed method.

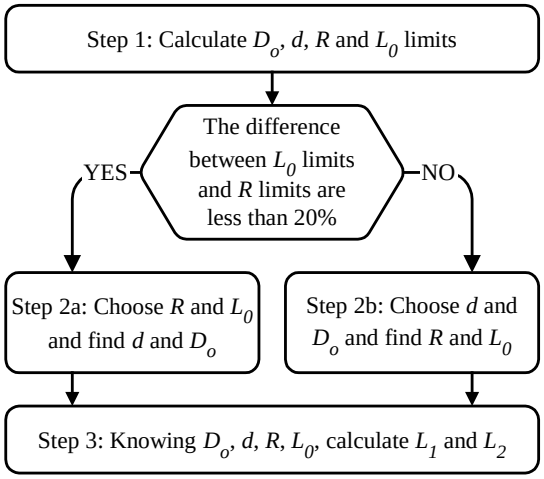


Fig. 4 Automatic determination of the starting point

## *4.1. Step 1 : Calculation of $D_o$ , $d$ , $L_0$ and $R$ limits*

The upper and lower limits of the four design parameters $D_o$ , $d$ , $L_0$ , $R$ are calculated using interval arithmetic [21]. The calculation proceeds in two steps:

- Step 1.1 - Initially, the limits of $d$ and $D_o$ are determined;

- Step 1.2 - Then the limits of $R$ and $L_0$ are calculated by taking into account ${}^{S}R$ , ${}^{S}L_0$ and the operating parameters (${}^{S}F_1$ , ${}^{S}F_2$ , ${}^{S}L_1$ , ${}^{S}L_2$, ${}^{S}S_h$).

- Step 1.1 Calculation of the limits of $D_o$ and $d$ considering the limits of ${}^{S}d$ , ${}^{S}D_o$ , ${}^{S}D_i$ , ${}^{S}D$ and ${}^{M}w$.

The appendix gives three basic relations linking the considered parameters concerning spring diameters :

$D = D_o - d$ ; $D_i = D_o - 2\,d$ ; $w = D / d$.

Then the following different relations available to calculate $d$ can be deduced :

$d = D_o - D = D - D_i = (D_o - D_i) / 2 = D_o / (w + 1) = D / w = D_i / (w\text{-}1)$.

All the limits of the parameters have positive values. Thus interval arithmetic is easily used to calculate $d$ limits :

$d^U = \text{Min} ( {}^S D_o^U - {}^S D^L ; {}^S D^U - {}^S D_i^L ; [{}^S D_o^U - {}^S D_i^L] / 2; {}^S D_o^U / [{}^M w^L + 1]; {}^S D^U / {}^M w^L; {}^S D_i^U / [{}^M w^L - 1]; {}^S d^U)$

$d^L = \text{Max} ( {}^S D_o^L - {}^S D^U ; {}^S D^L - {}^S D_i^U ; [{}^S D_o^L - {}^S D_i^U] / 2; {}^S D_o^L / [{}^M w^U + 1]; {}^S D^L / {}^M w^U; {}^S D_i^L / [{}^M w^U - 1]; {}^S d^L)$

In the same way :

$D_o^U = \text{Min}( {}^S D^U + {}^S d^U ; {}^S D_i^U + 2\,{}^S d^U ; 2\,{}^S D^U - {}^S D_i^L ; {}^S D^U / [ 1 + 1 / {}^M w^L ]; {}^S D_i^U / [1+2 / ( {}^M w^L - 1)]; {}^S d^U [{}^M w^U + 1]; {}^S D_o^U)$

$D_o^L = \text{Max} ( {}^S D^L + {}^S d^L ; {}^S D_i^L + 2\,{}^S d^L ; 2\,{}^S D^L - {}^S D_i^U ; {}^S D^L / [ 1 + 1 / {}^M w^U ]; {}^S D_i^L / [1+2 / ( {}^M w^U - 1)]; {}^S d^L [{}^M w^L + 1]; {}^S D_o^L)$

Now the operating parameters are taken into account to determine the limits of $R$ and $L_0$.

- Step 1.2 Calculation of the operating limits of $R$ and $L_0$ ($R^U$ , $R^L$ and $L_0^U$ , $L_0^L$), considering the parameters ${}^S F_1$, ${}^S F_2$, ${}^S L_1$, ${}^S L_2$, ${}^S S_h$ , ${}^S R$, ${}^S L_0$.

The following basic relation links the operating lengths $L_1$, $L_2$ and the spring travel $S_h$ (see Fig. 2):

$S_h = L_1 - L_2$.

This relation is exploited to calculate the limits of $L_1$, $L_2$ and $S_h$ from the limits given in the specification sheet.

$L_1^U = \text{Min} ({}^S L_1^U ; {}^S L_2^U + {}^S S_h^U)$

$L_1^L = \text{Max} ({}^S L_1^L ; {}^S L_2^L + {}^S S_h^L)$

$L_2^U = \text{Min} ({}^S L_2^U ; {}^S L_1^U - {}^S S_h^L)$

$L_2^L = \text{Max} ({}^S L_2^L ; {}^S L_1^L - {}^S S_h^U)$

$S_h^U = \text{Min} ({}^S S_h^U ; {}^S L_1^U - {}^S L_2^L)$

$S_h^L = \text{Max} ({}^S S_h^L ; {}^S L_1^L - {}^S L_2^U, \varepsilon)$

where ε represents a small number greater than zero (to avoid numerical problems).

Then the limits of $R$ and $L_0$ due to the operating parameters can be calculated.

The two basic relations linking the considered parameters are known (see appendix) :

$F_1 = R\,(L_0 - L_1)$

$F_2 = R\,(L_0 - L_2)$

From these relations, $R$ and $L_0$ limits are calculated :

$R^U = \text{Min} ({}^S R^U; [ {}^S F_2^U - {}^S F_1^L ] / S_h^L ; {}^S F_1^U / [ {}^S L_0^L - L_1^U ] ; {}^S F_2^U / [ {}^S L_0^L - L_2^U ] )$

$R^L = \text{Max} ({}^S R^L; [ {}^S F_2^L - {}^S F_1^U ] / S_h^U ; {}^S F_1^L / [ {}^S L_0^U - L_1^L ] ; {}^S F_2^L / [ {}^S L_0^U - L_2^L ] )$

$L_0^U = \text{Min} ({}^S L_0^U ; {}^S F_1^U / R^L + L_1^U ; {}^S F_2^U / R^L + L_2^U)$

$L_0^L = \text{Max} ({}^S L_0^L ; {}^S F_1^L / R^U + L_1^L ; {}^S F_2^L / R^U + L_2^L)$

### *4.2. Step 2 : determine D, d, R, $L_0$ values*

If the difference between $L_0$ limits and the difference between $R$ limits are both less than 20% then step2a is performed. If not, step2b is performed.

- Step 2a : choose $R$ and $L_0$ then find $D_o$ and $d$

In this step, the operating parameters are quite well defined, so that the limits of $R$ and $L_0$ are close to each other. Thus $R$ and $L_0$ are chosen as the mean value of their limits. Fortunately, as the parameters $D_o$ and d have finite limits (the maximum limits are those of the manufacturer), it is possible to adapt the algorithm suggested by QIAN [19] to determine $D_o$ and $d$. This algorithm only tests several possible values of $d$, calculates $D_o$ limits for each one and retains the first allowable solution (see Fig. 5).

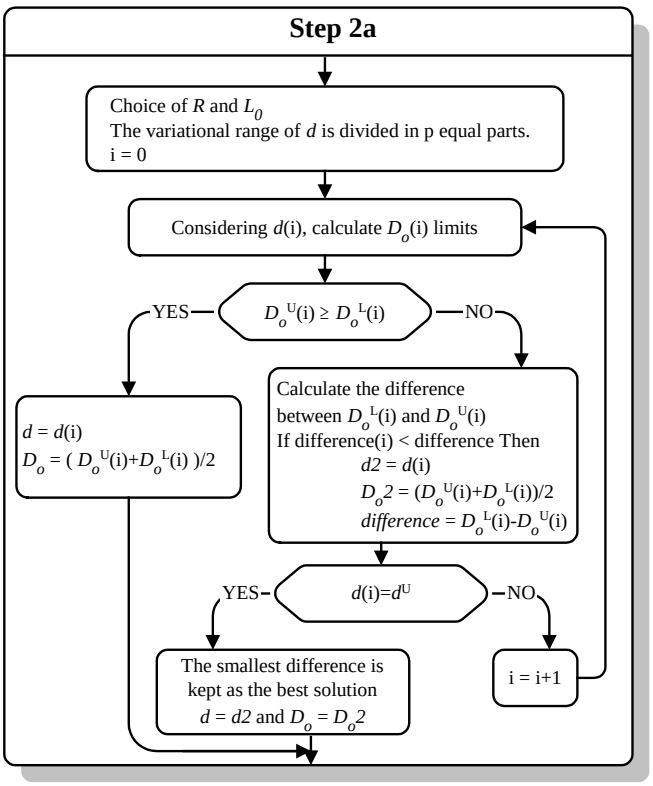


Fig. 5 Step 2a : Choose $R$ and $L_0$ and find $d$ and $D_o$

- Step 2b : choose $D_o$ and $d$ and find $R$ and $L_0$

In this part, the operating points have great range values. Some limits of $R$ and $D_o$ can be equal to the default values, for example when no data related to the operating parameters is given. That prevents from using the preceding

method. Thus $d$ is fixed to its medium allowable value and then $D_o$ is fixed so as to obtain a spring index ($w$) close to 8. Then $R$ and $L_0$ are calculated so as to have the greatest allowable pitch ($w$) as shown in Fig. 6.

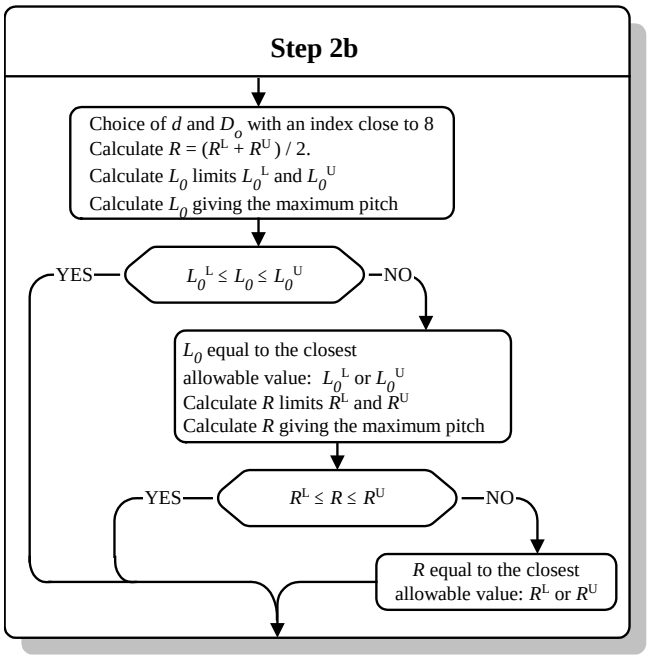


Fig. 6 Step 2b : choose $d$ and $D_o$ and find $R$ and $L_0$

### *4.3. Step 3 : Calculation of $L_1$ and $L_2$*

Now that the design parameters are known, the calculation of the operating points is obtained to find the maximum fatigue life, which implies finding the maximum value of $L_2$ and then the minimum value of $L_1$ in order to reduce maximum shear stress and alternate stress. To choose $L_1$ and $L_2$ , the method proposed by Paredes [22] for finding the optimal operating lengths of stock springs is used.

First the new $L_1$ and $L_2$ limits are calculated :

$$L_1^U = \text{Min}\ ({}^S L_1^U\ ;\ L_0 - {}^S F_1^L / R)$$

$$L_1^L = \text{Max}\ ({}^S L_1^L\ ;\ L_0 - {}^S F_1^U / R)$$

$$L_2^U = \text{Min}\ ({}^S L_2^U\ ;\ L_0 - {}^S F_2^L / R)$$

$$L_2^L = \text{Max}\ ({}^S L_2^L\ ;\ L_0 - {}^S F_2^U / R\ ;\ L_K\ ;\ L_n\ ;\ L_r)$$

Then $L_1$ and $L_2$ values are defined using the algorithm detailed in Fig. 7.

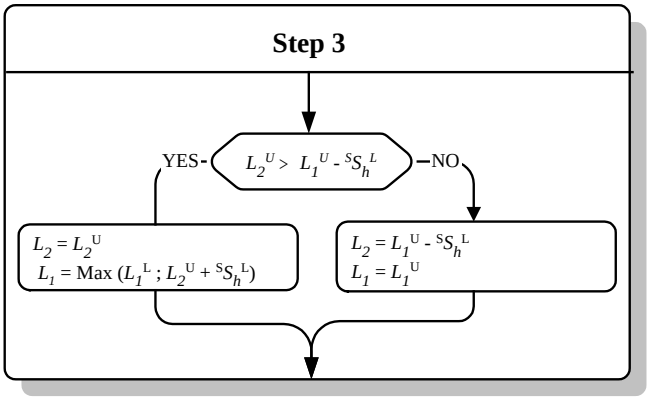


Fig. 7 Step 3: choose $L_1$ and $L_2$

# 5. CASE STUDY : A SPRING FOR A HYDRAULIC COMPONENT

A spring maker has successfully tested the proposed tools on real-life industrial problems over a period of two years. The case study presented here has been made in collaboration with a designer of hydraulic components in which helical springs are often used. All the calculations below have been obtained by using the initialisation and resolution algorithms presented previously. Maximum calculation time on a personal computer (300Mhz) is about 6 seconds. All results have been confirmed by using several starting points and other optimization algorithms.

In the following problem, the goal is to obtain the smallest value of $L_2$ in order to reduce the axial length of the system.

**Specification sheet :** the spring has to be of steel with closed and ground ends. Spring travel $S_h$ is constant (4.5 mm) and the study of the system defines the maximum spring load $F_2$ (127 N) as well as the value of the spring rate $R$ (5.56 N/mm). The spring must have a minimum life of $10^7$ cycles. To avoid any problem of wear due to friction, the spring should not buckle, the supports are parallel and guided giving an end fixation factor of 0.5 [1]. Finally the compressed spring must be included in the housing: its outside diameter $D_o$ is lower than 21.5 mm. As it is not possible to control the conditions of assembly and carriage strictly, care must be taken to avoid plastic deformation of the spring at its solid length. In this design stage there is no other geometrical constraint, the final design of the contiguous parts will be adjusted to fit with the optimal spring design.

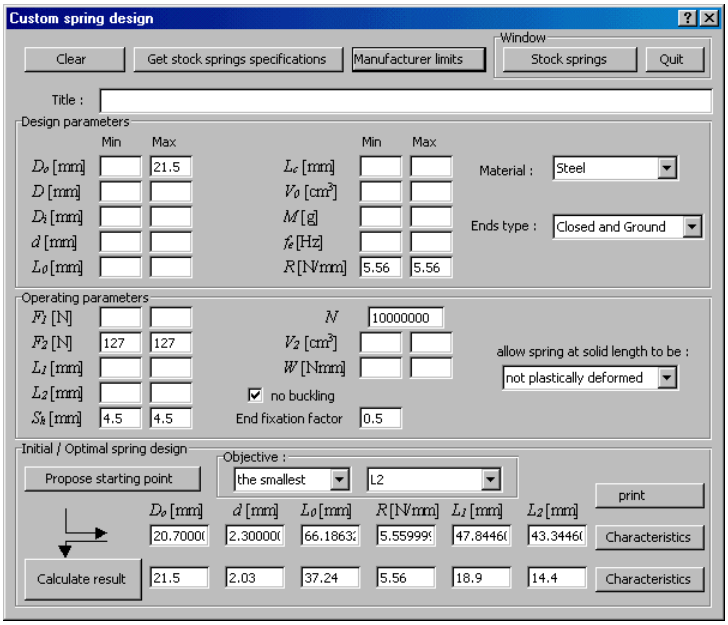


Fig. 8 An optimal spring design

**Result of research :** problem resolution in continuous variables provides the following result (Fig. 8):

$D_o$ = 21.5 mm, $d$ = 2.03 mm, $L_0$ = 37.24 mm, $R$ = 5.56 N/mm, $L_1$ = 18.9 mm, $L_2$ = 14.4 mm.

For that solution, the active constraints are g2, g7, g8, g19, g22, g23, g24, g25, g28.

The variations of the objective function during the iterative resolution process of the solver are presented in Fig. 9. This demonstrates the efficiency of both the initialisation algorithm and the resolution algorithm. The latter has quickly converged to the optimum point.

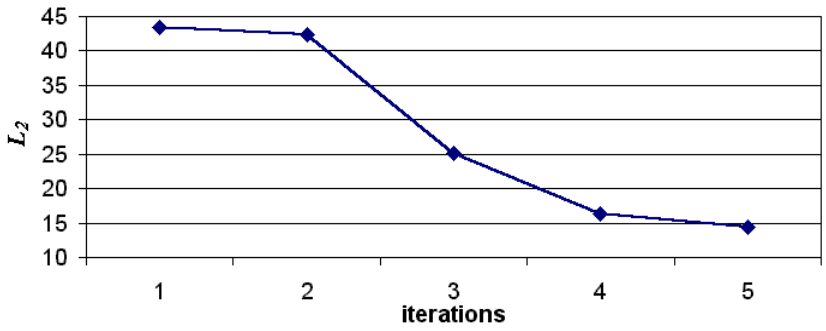


Fig. 9 Variations of $L_2$ during the resolution process

The proposed tool can also be used to search for optimal springs made from standardized wires. A quick investigation on the minimum and on the maximum allowable wire diameters has been manually performed by setting the objective to minimize $d$ and then to maximize $d$. The associated results (1.62 mm and 3.35 mm) show that the closest allowable standardized wire diameters form the optimal solution are : 1.8 / 2.0 / 2.2 mm. It is sufficient to add to the first the

specification sheet : ${}^{S}d^{L} = {}^{S}d^{U} = 1.8$ mm to get the best design related to this wire diameter, the objective been to minimize $L_2$. Tests with the standardized wire diameters provide the following results (Table 2):

Table 2 Smallest $L_2$ with standardized wires

| $d$ [mm] | Smallest $L_2$ [mm] |
|---|---|
| 1.8 | 19.56 |
| **2.0** | **14.54** |
| 2.2 | 20.1 |

The solution adopted with a standardized wire is therefore the one with a wire diameter of 2.0 mm :

$D_o = 20.9$ mm, $d = 2.0$ mm, $L_0 = 37.38$ mm, $R = 5.56$ N/mm, $L_1 = 19.04$ mm, $L_2 = 14.54$ mm.

As the optimal spring design with a continuous wire diameter offers only 1% less length than the best spring design with a standard wire diameter, the solution with $d = 2.0$ mm has been retained for the final design (Fig. 10).

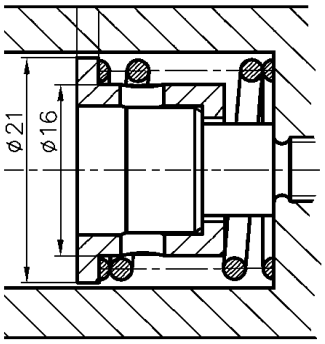


Fig. 10 Partial view of the final design of the hydraulic component

# 6. CONCLUSION

A comprehensive dimensional synthesis tool for compression spring design is presented. Its main advantage is to exploit an optimisation process in order to propose a solution starting directly from the specification sheet. Furthermore the specification sheet can accept data tolerances in order to be used in the early design stages. The short calculation time allows the designer to test several configurations and determine which is the best (optimal spring design or optimal spring design with a standardized wire diameter). This study shows that mechanical springs can now be designed to solve most real-life industrial problems.

Spring manufacturers or designers can easily implement this process using any standard optimization code, or a spreadsheet endowed with an optimization solver, to find the optimal solution of spring design problems.

## Acknowledgements

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# Appendix

Details of all the formulae used to define the problem constraints.

**Constraints related to variables**

◊ Specifications of the spring manufacturer upon $D_o$ and d ($^{M}D_o^{U}$, $^{M}D_o^{L}$, $^{M}d^{U}$, $^{M}d^{L}$) have to be taken into account.

Here are the values used in our study :

For steel : $^{M}D_o^{U}$ = 252 mm, $^{M}D_o^{L}$ = 3.2 mm, $^{M}d^{U}$ = 12 mm , $^{M}d^{L}$ = 0.3 mm

For stainless steel : $^{M}D_o^{U}$ = 315 mm, $^{M}D_o^{L}$ = 1.6 mm, $^{M}d^{U}$ = 15 mm , $^{M}d^{L}$ = 0.15 mm

**Constraints related to design parameters**

◊ Mean diameter : $D = D_o - d$

◊ Inside diameter : $D_i = D_o - 2\,d$

◊ Number of active coils : $n = G\,d^4 / (8\,R\,D^3)$

DIN standards explain that all the formulas used are available if the number of coils lies between ${}^{M}d^{L} = 1$ and ${}^{M}d^{U} = 100$ :

◊ Helix angle : $\gamma = 180\ \text{atan}(m / \pi / D) / \pi$ with $m = (L_0 - n_i\, d) / n$

The helix angle has to be lower than ${}^{M}\gamma^{U} = 7{,}5°$ so that the calculations remain valid.

◊ Spring index : $w = D / d$. The standard imposes the spring index lies between ${}^{M}w^{L} = 4$ and ${}^{M}w^{U} = 20$

**Constraints related to operating parameters**

◊ Minimum spring load : $F_1 = R\,(L_0 - L_1)$

◊ Maximum spring load : $F_2 = R\,(L_0 - L_2)$

◊ Spring travel : $S_h = L_1 - L_2$

◊ The minimum operating length must respect the minimum space between coils : $S_a = n\,(0.0015\,D^2 / d + 0.1d)$.

Thus, according to DIN [1] standards : $L_n = d\,(n + ni) + S_a$

◊ It is also necessary to take into account, if required in the specification sheet, the constraint related to buckling according to the end fixation factor ν [1] :

If $\nu\, L_0 / D < \pi \sqrt{\dfrac{2\mu + 1}{\mu + 2}}$ then $L_K = 0$ (there is no risk of buckling)

else $L_K = L_0 \left\{ 1 - \dfrac{\mu + 1}{2\,\mu + 1} \left[ 1 - \sqrt{1 - \dfrac{2\,\mu + 1}{\mu + 2} \left( \dfrac{\pi\, D}{\nu\, L_0} \right)^2} \right] \right\}$

with $\mu = \dfrac{E}{2G} - 1$

◊ Natural Frequency of surge waves : $f_e = \sqrt{\dfrac{R}{\pi^2\, \rho\, n\, D\, d^2}}$

◊ Internal energy : $W = 0.5\, R\, S_h^2$

◊ Total mass : $M = \pi^2\, \rho\, D\, d^2\, (n + 1.5) / 4$

◊ Free overall space taken up : $V_0 = \pi\, L_0\, D_o^2 / 4$

◊ Operating overall space taken up : $V_2 = \pi\, L_2\, D_o^2 / 4$

◊ Minimum static safety under operating conditions. The spring is automatically designed to ensure operating length $L_2$ achieves a maximum $\alpha_2^U$ % of $R_m$.

$L_r = L_0 - (\pi\, \alpha_2^U\, R_m\, d^3) / (800\, D\, R\, k)$

$\alpha_2 = L_2 / L_r$

with $k = 1.6 / (w)^{0.14}$ : stress correction factor (many other formulations of this factor exist, all with equivalent results [4,23]

In the calculation $\alpha_2^U = 50$ for steel and $\alpha_2^U = 48$ for stainless steel.

◊ Allowable percentage of $R_m$ at $L_c$.

$\alpha_c = 800\ D\ R\ k\ (L_0 - L_c) / (\pi\ R_m\ d^3)$ with :

$L_c = d\ (n + n_i)$

◊ Fatigue life factor [4]

$$\alpha_F = \text{Min}\left\{ \frac{\tau_{d(N)}\ (2\ \alpha_2^U\ R_m - \tau_{d(N)})}{\tau_a\ (2\ \alpha_2^U\ R_m - \tau_{d(N)}) + \tau_m\ \tau_{d(N)}} ; \frac{\alpha_2^U\ R_m}{\tau_a + \tau_m} \right\}$$ [22] with :

$\tau_m = 4\ D\ R\ (2\ L0 - L_1 - L_2) / (\pi\ d^3)$

$\tau_a = 4\ D\ R\ (L_1 - L_2) / (\pi\ d^3)$

$$\tau_{d(N)} = \begin{cases} \bullet\ \tau_d \quad \text{if} \quad N \geq 10^7 \text{(fatigue limit)} \\ \bullet\ \left[ \left( \tau_d - \alpha_2^U R_m \right) \ln N / \ln 10 + 7\ \alpha_2^U R_m - 4\ \tau_d \right] / 3 \quad \text{if} \quad 10^4 < N < 10^7 \\ \bullet\ \alpha_2^U R_m \quad \text{if} \quad N \leq 10^4 \text{(static limit)} \end{cases}$$