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Remaining error sources in the nutation at the submilliarc second level

V. Dehant,1 M. Feissel-Vernier,2,3 O. de Viron,1 C. Ma,4 M. Yseboodt,1 and C. Bizouard2

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[1] Earth’s precession and nutations are mainly generated by the luni-solar tidal torque. Diurnal retrograde variations in the atmospheric and oceanic angular momenta in an Earth-fixed reference system induce some additional nutation motions. Observed precession and nutations are derived from very long baseline interferometry (VLBI) data, assuming that the direction of the observed quasars are fixed in space. In this study, we consider the effects of two possible causes for explaining discrepancies between the observed nutations and those modeled in MHB2000 (model adopted by the International Astronomical Union): (1) the time variations in the atmospheric (and potentially oceanic) forcing of the nutations, of the free core nutation (FCN), and of the free inner core nutation (FICN), and (2) the possible contamination of VLBI-derived nutation amplitudes by apparent changes in the directions of the extragalactic radio sources. The robustness of MHB2000 is tested by perturbing some of the parameters and assessing the validity of the resulting nutation amplitudes against realistic estimations. We show that even allowing for large discrepancies related to atmospheric forcing, the ranges of the possible changes in the FCN and FICN periods and damping factors are small.

INDEX TERMS:
1210 Geodesy and Gravity: Diurnal and subdiurnal rotational variations; 1213 Geodesy and Gravity: Earth’s interior—dynamics (8115, 8120); 1223 Geodesy and Gravity: Ocean/Earth/atmosphere interactions (3339); 1229 Geodesy and Gravity: Reference systems; KEYWORDS: Earth rotation, nutation, VLBI


1. Introduction

[2] The irregularities in the Earth’s rotation are described by the variations of the Earth rotation speed (associated with length-of-day fluctuations), polar motion, and variations in the direction of the rotation axis in space (precession and nutations). The polar motion indicates the position of the conventional reference axis relative to the Earth body, while the nutation angles together with the rotation angle express the orientation of the Earth in space. This paper deals with the nutations, which are mainly caused by the gravitational tidal attraction of the Sun, the Moon, and the planets. The Earth’s fluid layers, the atmosphere and the ocean, are also responsible for time-varying excitations of some of the nutations [Chao et al., 2000]. In addition, the fluid core and the solid inner core have free nutation modes that can amplify the Earth’s response and, if excited, induce additional components.

[3] Precession and nutations data are derived from very long baseline interferometry (VLBI) observations performed at stations whose positions are precisely known at the surface of the Earth. The VLBI data consist of delays between extragalactical radio signal arrival times at different stations. The classical way to analyze the data is to consider that the celestial objects used to realize the reference frame (International Celestial Reference Frame (ICRF) [Ma et al., 1998]) are fixed point sources in space, and to use this fixed reference frame to observe the fluctuations of the Earth orientation relative to the inertial space.

[4] As the Earth’s rotation changes with time, the orientation accumulates these changes. Nutation is an integrated phenomenon; therefore the lower the frequency, the larger its amplitude; this is the case for the 18.6-year nutation. This amplification of the low-frequency nutations enhances the geophysical contribution. Although these nutations are far away from the resonances, small geophysical effects associated with these resonances reach a level well above the observable level. Observations of these nutations thus provide unique geophysical insight. The series of available VLBI observations presently covers a period longer than 18.6 year, making it feasible to study this nutation and all those of shorter periods. The whole spectrum of the nutations should be analyzed to optimize the geophysical inferences from the data. Of particular interest are the annual nutations that are much affected by the free resonances of the outer core and inner core.

[5] In practice, the VLBI estimation of precession/nutation consists of two steps:
1. One pair of celestial pole offsets in longitude and obliquity ($d\psi$, $de$) referred to an a priori precession-nutation model is estimated for each 24-hour observation session. In this analysis, the source coordinates are kept fixed over the whole 20-year observation period. The other estimated parameters include global scale parameters like universal time and polar motion, and auxiliary parameters such as clock parameters and corrections for tropospheric effects. Keeping the source coordinates fixed and retaining the same analysis scheme over many years, a time series of ($d\psi$, $de$) is constructed, with one pair of values per session [Sovers et al., 1998].

2. The time series is then analyzed in searching for signatures at the nutation periods as well as linear trends. The trend in longitude is interpreted as a correction to the precession rate; the trend in obliquity is called the obliquity rate [Williams, 1994; Herring et al., 1986; Souchay et al., 1996]. The nutations are expressed then in terms of circular prograde and retrograde motions as explained by Defraigne et al. [1995].

3. Those VLBI-derived values are then compared with theoretical precession/nutation models and the differences are analyzed. The reference nutation model is that derived by Mathews et al. [2002] (hereinafter referred to as MHB2000) that was adopted as the international convention by the International Astronomical Union in 2000.

4. We study two possible causes for the observed discrepancies: (1) the variations in the atmospheric forcing (and potentially the oceanic forcing) derived from meteorological data [de Viron et al., 2001; Vseboodt et al., 2002], and (2) the possible contamination of VLBI-derived nutation amplitudes by apparent motions of the extragalactic radio sources [Feissel et al., 2000; Gontier et al., 2001].

5. In addition, we test the robustness of the theory underlying the MHB2000 model by perturbing the fitted geophysical parameters and investigating their stability. The tested parameters are the frequencies and damping factors of the free modes including the free core nutation (FCN) and of the free inner core nutation (FICN).

2. Modeling Precession and Nutation

1. Theoretical precession/nutation models are built in three steps:

2. The precession/nutation for a rigid, oblate Earth is computed from astronomical considerations; these values deduced from high precision celestial ephemerides are known to better than a few microarc seconds (Souchay and Kinoshita [1996, 1997], this model is called REN2000, and Roosbeek and Dehant [1998] and Bretagnon et al. [1998]).

3. Then the model is convolved with a transfer function accounting for the response of a realistic Earth; this transfer function involves physical properties of the Earth’s interior, and includes resonance phenomena. The precession is not affected by the nonrigidity of the Earth and consequently does not require such a convolution. It nevertheless involves the dynamical flattening of the whole Earth, which is fitted as a parameter in some models.

4. Additionally, the effects of the external global geophysical fluids (atmosphere, ocean, hydrosphere) should be accounted for; those effects include ocean tides, forcing by the atmosphere and hydrosphere, and the ocean response to the atmospheric forcing; only a part of these corrections has been considered in the current models.

5. The International Astronomical Union (IAU) has adopted the MHB2000 model of Mathews et al. [2002] based on REN2000 as rigid Earth nutation model (step 1), and on a semianalytical approach in which the parameters for the transfer function are fitted to the observations (step 2). For the computation of step 3 in MHB2000, ocean and atmosphere are supposed to contribute to the nutation at some given frequencies (such as the diurnal wave $S_1$ for the atmosphere and the tidal frequencies for the ocean). These effects are fitted or scaled to the observations together with Earth interior parameters [Mathews et al., 1991a, 1991b, 2002; Herring et al., 2002]. In particular, ocean tide contributions are computed by adjustment of the results of Chao et al. [1996]. The frequencies of these forcing are well known to be exactly the nutation frequencies. Note that the ocean responses to the atmosphere at the major tidal frequencies and to the tidal forcing are usually provided together in the ocean assimilated tide models, e.g., those derived from the observations of the space altimetry mission TOPEX/Poseidon [Tapley et al., 1994]. In addition, the $S_1$ atmospheric and oceanic influences are considered in terms of their fitted mean values over the total VLBI observations (1980–2000). Although they are not deduced from the atmospheric and oceanic angular momentum at that frequency, their magnitudes are compatible with the atmospheric angular momentum (AAM).

6. This model gives rise to amplitude discrepancies between theory and observation at the level of a few tens of microarc seconds ($\mu$as) in the frequency domain, while the statistical uncertainties of the VLBI-based parameters is of the order of 10 $\mu$as.

3. Atmospheric and Oceanic Nutation Contributions

3.1. Atmospheric Forcing

The atmospheric and oceanic excitation of the nutation results from forcing associated with diurnal variations mainly generated by the diurnal solar heating. As an example, the purely one-solar day forcing induces an effect on the nutation at the frequency corresponding to the frequency difference between one solar day and the Earth rotation period (one sidereal day), i.e., an annual period (prograde) in space. The angular momentum is conventionally divided into two parts [Munk and MacDonald, 1960]: the first one (pressure or matter term) is associated with the change in the inertia tensor of the fluid layer, and the second one (wind/current or motion term) corresponds to the relative angular momentum of the atmosphere/ocean with respect to the Earth.

The atmospheric angular momentum (AAM) series are computed from the analyses (or reanalyses) of the atmospheric data by the meteorological centers, in the framework of weather forecast [Salstein et al., 1993]. The meteorological models are built to simulate, as well as possible, the state of the atmosphere at the synoptical timescale (2–8 days); while their precision at the diurnal timescale is not perfect, they allow us to obtain a first order evaluation of the atmospheric effects on nutations.
Figure 1. Wavelet analysis around the 1-year period (prograde and retrograde) of the matter part of the celestial atmospheric angular momentum time series, provided by the NCEP/NCAR Reanalysis model for 40 years. (left) Skeleton of the wavelet transform (period along the wavelet spectrum ridge); (right) the amplitude along the ridge. (arbitrary unit).

[19] Provided by the International Earth Rotation Service (IERS) Special Bureau for the Atmosphere [see Chao et al., 2000] and computed from several atmospheric global circulation models, the AAM series can be used to compute the atmospheric contributions to nutations. The most important contributions are on the annual prograde and retrograde components [Bizouard et al., 1998; Yseboodt et al., 2002]. At the prograde annual frequency, the forcing corresponds to the diurnal solar thermal effect on the atmosphere. At the retrograde annual term it corresponds to a seasonal (semiannual) modulation of the diurnal effect of the Sun, amplified by the resonance of the FCN.

[20] Bizouard et al. [1998] have shown that the NCEP-NCAR AAM series was adequate to compute the effect of the atmosphere on the nutations as the order of magnitude of the amplitudes (at the level of some tens or hundreds of microarc second) agrees with the nutation residuals. Consequently, we will use these series for our study. Furthermore, this is the only series providing continuous 6-hourly data for the whole VLBI time span. Nevertheless, we have to keep in mind that Yseboodt et al. [2002] have shown that the corrections computed from different atmospheric models disagree with one another, at the level of some tens of microarc second as well.

[21] In section 5, we have used the amplitudes of the atmospheric effects from Yseboodt et al. [2002] for the NCEP-NCAR reanalysis model [see Kalnay et al., 1996] to perturb the MHB2000 model, which only accounts for a correction at the $S_1$ frequency, i.e., at the prograde annual nutation.

[22] We use the wavelet analysis for studying the atmospheric excitation of the annual prograde and retrograde nutations, from the matter term and the motion term of AAM in a celestial reference frame, as an extension to the study of Yseboodt et al. [2002]. As shown in that paper, the fluctuations with time of the excitation amplitude are as large as the atmospheric contribution itself. They would correspond to time variations of the mean nutation amplitudes at the level of some tens of microarc second, and even a fraction of milliarc second for the annual retrograde nutation. In Figure 1, we present complementary information: the skeleton of the wavelet transform of the angular momentum (matter term), analyzed over 50 years. The skeleton is the restriction of the wavelet transform on the ridge, the ridge being the curve of the local maxima of the wavelet transform. This provides instantaneous frequencies and amplitudes along the selected ridge. We only show the skeleton of the matter term of the AAM because it is the contribution which is the most amplified by the Earth’s response [Brzezinski, 1994]. Note that there is a gap in the curves when the frequency of the local maximum is not well defined due to low energy.

[23] Figure 1 also shows that the frequencies of the maximum excitation are not fixed at the periods corresponding to the $S_1$ (prograde annual nutation) and $\psi_1$ (retrograde annual nutation) frequencies but are changing with time. We found a range of about 30 days around the prograde annual frequency and about 100 days around the retrograde annual frequency. This figure shows also that the amplitudes along the ridges are more strongly varying with time in the case of the prograde annual nutation than of the retrograde annual nutation. In the former case, the estimates of the atmospheric excitation period (Figure 1) show on the other hand more stability with time than for the latter. Note that the scales of the amplitudes in Figure 1 are different.

[24] Those results show that it is undesirable to correct for the atmospheric effect as a global periodic contribution to the nutation corresponding to $S_1$, as done by many models. We would rather recommend to correct for the atmospheric effect on the Earth’s orientation by a direct convolution in the time domain; this would indeed account for all the components of the forcing, including the time dependent amplitude.

3.2. Nontidal Ocean Forcing

[25] The atmosphere does not only exchange angular momentum with the solid Earth, but it does so with the ocean that will, in turn, excite Earth’s nutations. The computation of the nontidal ocean contribution is not straightforward for historical and practical reasons. First, there has been no such an interest for oceanic modeling as for weather prediction and consequently no such data-based global circulation model development. Second, the large scale ocean dynamics is much more difficult to infer from the available observations. The limited angular momentum data available are thus obtained from hydrodynamical models [Ponte, 1997; Marcus et al., 1998; Johnson et al., 1999; Gross, 2000]. As ocean dynamics considered in those models is mainly at low frequency (periods longer than several days), the diurnal/semidiurnal motions involved in nutation forcing are not represented adequately. At present time, only barotropic ocean model such as Ponte’s [1993] model can be used for computing the oceanic effects on nutations as done by Petrov et al. [1998]. Our results are at the level of the model residuals (see Table 1). In what follows, we do not use them because the series is too short.
for properly evaluating the contributions to nutations. These
results are furthermore rather preliminary and other ocean
global circulation models are also being used for evaluating
this contribution [de Viron et al., 2002]. Note that at the tidal
frequency, the TOPEX/Poseidon data already include these
effects. The total effect of the atmosphere and ocean must be
compatible with the size of the nutation residuals.

4. VLBI Analyses of Precession and Nutation

[26] In this section, VLBI trends in the celestial pole
motion and main nutation terms are used to evaluate the
accuracy of the observations and to test the robustness of
the analysis results with respect to perturbations in the
celestial reference frame. The reference nutation model is
MHB2000.

[27] As the nutation modeling process considers only
permanent or mean forcing, the comparison of VLBI results
with the model is expected to exhibit time variable discrep-
ancies in nutation amplitudes at frequencies where the
atmosphere and/or ocean provide large excitations, and
where the Earth amplifies excitations e.g., at the annual
frequency and at free resonance frequencies, like the FCN
and the FICN with respective periods of 430 days (retro-
grade) and 1025 days (prograde) in MHB2000. Figures 2
and 3 show the observed signals at the annual and FCN
frequencies, respectively, as a function of time. The results
are plotted at 2-year intervals, each point being based on the
joint analysis of the annual, semiannual, 430-day and 1025-
day terms with a 6-year sliding window. The error bars
show the formal uncertainties of the least squares estimation
(one sigma). While the observed annual nutations are
consistent in the mean with MHB2000, they show signifi-
cant time variations around the mean. The time series of the
FCN results show a distinct change in amplitude and phase
in the late 1980s: the amplitudes drops from about 270 to
100 μas and less, and the phase changes by about one
month. It must be noted that, in their analysis for building
MHB2000, Herring et al. [2002] have also accounted for
2-year temporal variability of the free FCN amplitude and
phase.

[28] The classical VLBI analysis scheme is based on the
assumption that the directions of the observed radio sources
are fixed in space. However, the apparent direction of radio
sources has been shown to have variability [Feissel et al.,
2000]. Ma et al. [1998] raised two major causes for the
current limitation in accuracy of source positions:

[29] 1. The difficulty to account accurately for the prop-
gagation delay due to the wet component of the troposphere.
This may give rise to systematic errors in declination at the
level of a fraction of a milliarc second in the equatorial
zone. This effect is expected to be reduced by the use of the
so-called gradient correction [MacMillan and Ma, 1997]
that takes into account azimuthal variations of the tropo-
spheric delay.

Table 1. Nontidal Ocean Contributions to Nutationa

<table>
<thead>
<tr>
<th>Nutation Periods</th>
<th>Amplitude, mas</th>
<th>Phase, deg</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mass Term</td>
<td>Motion Term</td>
</tr>
<tr>
<td>Retrograde 365 days</td>
<td>15.9</td>
<td>0.5</td>
</tr>
<tr>
<td>Retrograde 183 days</td>
<td>1.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Prograde 365 days</td>
<td>5.2</td>
<td>2.0</td>
</tr>
</tbody>
</table>

*The phase is given with respect to the nutation forcing potential.

![Figure 2](image2.png)

**Figure 2.** Prograde and retrograde components of the annual nutation derived from VLBI observations. The MHB2000 model values are shown as the intersection of the straight lines. Unit is mas.

![Figure 3](image3.png)

**Figure 3.** VLBI-derived retrograde component at the FCN frequency (430 days). The origin of phases is J2000.0. Unit is mas.
Table 2. The VLBI Series of Celestial Pole Offsets Analyzed

<table>
<thead>
<tr>
<th>Institute</th>
<th>Series</th>
<th>Data Span</th>
<th>Points</th>
<th>Formal Uncertainty, mas</th>
<th>SD wrt MHB2000, mas</th>
<th>Sources</th>
<th>Tropo Delay</th>
<th>Tropo Grad</th>
<th>Elevation Cutoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>BKG</td>
<td>01 R 01</td>
<td>1984.0–2001.0</td>
<td>2273</td>
<td>0.17</td>
<td>0.19</td>
<td>578</td>
<td>1 est per hour</td>
<td>ye</td>
<td>-</td>
</tr>
<tr>
<td>GSFC</td>
<td>01 R 01</td>
<td>1980.0–2000.9</td>
<td>2696</td>
<td>0.19</td>
<td>0.18</td>
<td>552</td>
<td>3 est per hour</td>
<td>1 est per 6 hours</td>
<td>ye</td>
</tr>
<tr>
<td>IAA</td>
<td>01 R 01</td>
<td>1980.0–2001.2</td>
<td>2155</td>
<td>0.13</td>
<td>0.22</td>
<td>667</td>
<td>Kalman</td>
<td>no</td>
<td>-</td>
</tr>
<tr>
<td>SHA</td>
<td>01 R 01</td>
<td>1980.0–2001.2</td>
<td>2735</td>
<td>1.54</td>
<td>0.19</td>
<td>675</td>
<td>yes</td>
<td>yes</td>
<td>-</td>
</tr>
<tr>
<td>USNO</td>
<td>99 R 03</td>
<td>1980.0–2001.0</td>
<td>2489</td>
<td>0.28</td>
<td>0.20</td>
<td>652</td>
<td>1 est per hour</td>
<td>1 est per 3 hours</td>
<td>7°</td>
</tr>
</tbody>
</table>

*aThe formal uncertainties correspond to the time span 1990–2000. The standard deviation from MHB2000 is computed over the same time span, assuming a $-2.997$ mas/yr precession correction and a $-0.255$ mas/yr obliquity rate with respect to the IAU 1976–1980 model, and estimating the coefficients of a 430-day FCN oscillation. Also given is information on the treatment of atmospheric effects in the VLBI series; est, estimation; wrt, with respect to.

[30] 2. Despite the selection of quiet objects for astrogeneric work, some of the observed sources may exhibit changes in their emission structure that will make their apparent direction change with time. This phenomenon may propagate errors into the source positions at the level of a fraction of a milliarc second, particularly in the early years of VLBI observations, when the network geometry was still poor and the observed sources had to be brighter than could be allowed with the more recent, more efficient observing technology; variability is indeed a larger nuisance with brighter sources than with fainter ones.

[31] As a result of the source directions variability, the celestial reference pole in each observation session may be slightly shifted, raising the question of contamination of the derived nutation amplitudes.

[32] Another type of session analysis allows to determine source coordinates as “arc parameters”; that is, they are estimated independently for every session where the source is observed. Using time series of radio source coordinates determined for each observation session, one can compute their contribution to the celestial pole shift [Gontier et al., 2001].

[33] Finally, in order to evaluate the stability of VLBI nutation observations, we undertake a mixed analysis scheme, in which some sources coordinates are held fixed while the others are estimated for each session together with the celestial pole offsets.

[34] 1. The first set of VLBI results that we consider are time series of the celestial pole offsets obtained by five analysis centers in global analyses covering 17 to 21 years (Table 2) and submitted to IERS (see http://hpiers.obspm.fr/eop-pc). Four of them (BKG, GSFC, SHA, USNO) where obtained independently using the Calc-Solve software package [Ma et al., 1999], the fifth (IAA) by use of the OCCAM software package [Titov and Zarraoa, 1997; Shurikhina, 2000]. All analyses considered a fixed celestial reference frame defined by nearly 700 extragalactic radiosources. The exception of GSFC, where the coordinates of 138 very little observed sources were estimated as arc parameters, should have little effect in the present analysis. Diurnal and sub-diurnal variations in polar motion and universal time which, if ignored, would contaminate the estimated celestial pole offsets, were taken into account in the calculation of the five series. Table 2 also summarizes some details of the treatments, provided by their respective authors, concerning the correction of the effect of the atmosphere on signal transmission. The combination of these effects with the observing geometry results partially in errors on the source declinations, hence on the determination of the celestial pole. The time span on which the zenith troposphere corrections (including the troposphere gradients) are computed reflects the analyst’s assumption on the characteristic time variability of the signal perturbation induced by the troposphere. The elevation cutoff option avoids using low-elevation observations, for which the troposphere correction models are less reliable. Table 2 also gives the data span and number of individual values of the celestial pole offsets, together with their stated formal uncertainties (d$v$/sin(e), de), and their postfit residnal with respect to the MHB2000 precession-nutation model.

[35] The series of celestial pole offsets were analyzed under the same conditions for differences in the low-frequency terms (precession, obliquity rate and 18.6-year term) and interannual frequency band as well as variable medium-frequency terms (annual and semiannual forced nutation, 430-day FCN). While the observations in the five data sets are practically the same, their analysis differs somewhat in the modeling, estimation and editing schemes. One way to derive an estimation of the accuracy of the results is to use these differences. This agreement between the nutation amplitudes derived from the five solutions is at the level of 10 μas. In the following, we label “VLBI” the weighted mean of the results from the five analyses.

[36] 2. A second set of VLBI results used here are time series of (d$v$/sin(e), de) derived from the analysis of the complete observations data set (over 1980.0–2001.4) in two parallel ways:(1) similar to the GSFC line in Table 2, in particular with all source coordinates held fixed, and (2) by estimating for each session the coordinates of the 211 sources detected as unstable and listed as “arc source” in ICRF-Ext.1 [Ma et al., 1998; IERS, 1999]. We label “ΔVLBI” the difference of nutation results derived from the two different ways.

[37] 3. The third set of VLBI results that we consider are time series of source coordinates, estimated for each session [see Gontier et al., 2001]. The analysis strategy (T. M. Eubanks, 1999, personal communication to M. Feissel, 1999) aims at producing series of source coordinates as little perturbed as possible by known effects, such as biased a priori nutation-precession models and tropospheric corrections. This was accomplished by applying an atmospheric gradient model and fitting local diurnal corrections. The time variations of the right ascensions and declinations of 639 sources over 1980.0–1999.3 (150,000 individual coordinates) are analyzed by the least squares method to detect a possible contamination of the apparent celestial pole direction, either linear or with the main nutation periods. Should such signatures be found, we consider...
additive corrections $\Delta_{\text{CRF}}$ to the VLBI estimates of $(d\gamma, de)$ for the corresponding components.

The time series of celestial pole offsets and the source coordinates were analyzed to determine linear and periodic corrections, in parallel by the least squares method over the same time intervals and with the same lists of unknown parameters. The uncertainties listed are, in the case of the results labeled "VLBI", the standard error of the weighted means, and in the two other cases the formal standard error from the least squares solution. We analyze hereafter three different sets of nutation components: (1) the FCN, annual and semiannual forced nutations, (2) the long-period nutations, obliquity rate and precession correction, and (3) the interannual frequency band.

Table 3. VLBI-Derived Components and Their Perturbation at the FCN Frequency$^a$

<table>
<thead>
<tr>
<th>Span</th>
<th>Data</th>
<th>Prograde</th>
<th></th>
<th>Retrograde</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>In-Phase</td>
<td>Out of Phase</td>
<td>In-Phase</td>
<td>Out of Phase</td>
</tr>
<tr>
<td>1983.0–1989.0</td>
<td>VLBI</td>
<td>$-16 \pm 7$</td>
<td>$-62 \pm 10$</td>
<td>$140 \pm 12$</td>
<td>$-223 \pm 9$</td>
</tr>
<tr>
<td>1989.0–1995.0</td>
<td>VLBI</td>
<td>$-4 \pm 3$</td>
<td>$9 \pm 3$</td>
<td>$5 \pm 3$</td>
<td>$-147 \pm 2$</td>
</tr>
<tr>
<td>1995.0–2001.0</td>
<td>VLBI</td>
<td>$1 \pm 2$</td>
<td>$11 \pm 1$</td>
<td>$6 \pm 0.3$</td>
<td>$-47 \pm 2$</td>
</tr>
</tbody>
</table>

Estimated Perturbation Due to Source Selection

<table>
<thead>
<tr>
<th>Span</th>
<th>Data</th>
<th>Prograde</th>
<th></th>
<th>Retrograde</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>In-Phase</td>
<td>Out of Phase</td>
<td>In-Phase</td>
<td>Out of Phase</td>
</tr>
<tr>
<td>1983.0–1989.0</td>
<td>$\Delta$VLBI</td>
<td>$41 \pm 20$</td>
<td>$9 \pm 20$</td>
<td>$54 \pm 20$</td>
<td>$-15 \pm 20$</td>
</tr>
<tr>
<td>1989.0–1995.0</td>
<td>$\Delta$VLBI</td>
<td>$-6 \pm 6$</td>
<td>$2 \pm 6$</td>
<td>$-4 \pm 6$</td>
<td>$-2 \pm 6$</td>
</tr>
<tr>
<td>1995.0–2001.0</td>
<td>$\Delta$VLBI</td>
<td>$-4 \pm 4$</td>
<td>$-2 \pm 4$</td>
<td>$3 \pm 4$</td>
<td>$0 \pm 4$</td>
</tr>
</tbody>
</table>

$^a$The origin of phase is J2000.0. Unit is mas.

additive corrections $\Delta_{\text{CRF}}$ to the VLBI estimates of $(d\gamma, de)$ for the corresponding components.

The time series of celestial pole offsets and the source coordinates were analyzed to determine linear and periodic corrections, in parallel by the least squares method over the same time intervals and with the same lists of unknown parameters. The uncertainties listed are, in the case of the results labeled "VLBI", the standard error of the weighted means, and in the two other cases the formal standard error from the least squares solution. We analyze hereafter three different sets of nutation components: (1) the FCN, annual and semiannual forced nutations, (2) the long-period nutations, obliquity rate and precession correction, and (3) the interannual frequency band.

Table 3.

5. Studies of VLBI Residuals: Interpretation in Terms of Atmosphere and Source Stability

5.1. Free Core Nutation and Annual and Semiannual Forced Nutations

Table 3 lists the parameters of a free core nutation with a 430-day period and the estimated effect of source selection ($\Delta$VLBI) at these frequencies. The effects of the source selection cannot be directly compared with the VLBI residuals as the nutation parameters are fitted to minimize those residuals while ignoring the instability of the sources. Nevertheless, we conclude from this table that the source selection can hardly be invoked to explain the large observed variability of the FCN over the last 20 years (Figure 3).

Figure 4 summarizes results from the atmospheric excitation (section 3) and the VLBI analysis. Note that the points on the figure are computed from a sliding window over 6 years, shifted by 3-year steps, while the values in Table 3 are independent. The comparison of the period of the mostly atmosphere-excited retrograde term with the observed FCN parameters shows that the time interval when the FCN amplitude is enhanced corresponds roughly to maximum atmospheric excitation with periods around the FCN period. Similarly, the observed retrograde annual amplitude is enhanced when the maximum atmospheric excitation frequency is roughly annual, but nevertheless not as critical as for the FCN because there is no resonance at the annual frequency.

Tables 4 and 5 give VLBI and $\Delta$VLBI estimations for the annual and semiannual terms, together with the MHB2000 values and residuals. Our residuals agree in the mean with the MHB2000 values, while showing significant time variations as already mentioned (see Figure 2 for the annual components). The $\Delta$VLBI corrections indicate that the source estimation strategy in VLBI analysis may impact the observation of the variable seasonal nutations, at least between 1983 and 1989.

5.2. Precession, Obliquity Rate, and the 18.6-year Nutations

Table 6 gives the values of the precession correction and of the obliquity rate relative to the IAU1976 precession. The table also provides the difference of the VLBI results with these reference values, and two estimates of the effect of source selection ($\Delta$VLBI, $\Delta_{\text{CRF}}$). MHB2000 does not estimate the obliquity rate in the model, so no correspond-

Figure 4. Time variation of the FCN amplitude (violet), and period of maximum atmospheric annual retrograde excitation (dark red). The shape of the nonrigid Earth transfer function (as a function of the period on the right scale) is shown for reference (green); the closer to the resonance, the larger it is amplified.
### Table 4. Modeled and Estimated Values of the Annual Nutation Term

<table>
<thead>
<tr>
<th>Span</th>
<th>Data</th>
<th>Prograde</th>
<th>Retrograde</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>In Phase</td>
<td>Out of Phase</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MHB2000</td>
<td>25 645</td>
<td>131</td>
<td>−33 047</td>
</tr>
<tr>
<td>MHB2000 residual</td>
<td>0 ± 10</td>
<td>0 ± 10</td>
<td>8 ± 16</td>
</tr>
</tbody>
</table>

**VLBI Estimation** (Average of Results From the Five Time Series of Table 2)

- 1983.0–1989.0 VLBI-MHB2000: 4 ± 3 1 ± 4
- 1989.0–1995.0 VLBI-MHB2000: 0 ± 2

### Table 6. Modeled and Estimated Values of Trends in the Celestial Pole Motion

<table>
<thead>
<tr>
<th>Data</th>
<th>Precession Correction</th>
<th>Obliquity Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>MHB2000</td>
<td>−2 ± 997</td>
<td></td>
</tr>
<tr>
<td>MHB2000 residual</td>
<td>36 ± 18</td>
<td></td>
</tr>
<tr>
<td>VLBI-MHB2000</td>
<td>18 ± 5</td>
<td></td>
</tr>
</tbody>
</table>

**Estimated Perturbation Due to Source Selection**

- ΔVLBI: −1 ± 8  −31 ± 3
- ΔCRF:  9 ± 2  −8 ± 1

*Unit is μas/yr.

### Table 5. Modeled and Estimated Values of the Semianual Nutation Term

<table>
<thead>
<tr>
<th>Span</th>
<th>Data</th>
<th>Prograde</th>
<th>Retrograde</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>In Phase</td>
<td>Out of Phase</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MHB2000</td>
<td>−548 471</td>
<td>−502 −24 563</td>
<td>−43</td>
</tr>
<tr>
<td>MHB2000 residual</td>
<td>0 ± 8</td>
<td>3 ± 8</td>
<td>−5 ± 8</td>
</tr>
</tbody>
</table>

**VLBI Estimation** (Average of Results From the Five Time Series of Table 2)

- 1983.0–1989.0 VLBI-MHB2000: 20 ± 9  −37 ± 8  −12 ± 7  −16 ± 6
- 1989.0–1995.0 VLBI-MHB2000: 0 ± 2  −7 ± 2  1 ± 1  −3 ± 1
- 1995.0–2001.0 VLBI-MHB2000: 4 ± 3  1 ± 4  −4 ± 1  −18 ± 2

**Estimated Perturbation Due to Source Selection**

- 1983.0–1989.0 ΔVLBI: −2 ± 18  44 ± 18  15 ± 18  −17 ± 18
- 1989.0–1995.0 ΔVLBI:  −6 ± 6  17 ± 6  2 ± 6  12 ± 6
- 1995.0–2001.0 ΔVLBI:  4 ± 4  4 ± 4  2 ± 4  3 ± 4

*Unit is μas.

### 5.3. Interannual Frequency Band

Using spectral analysis, we now investigate the two series of $(d\psi, d\delta)$ estimated over the 1987–2001 time span (1) with respect to all VLBI-observed sources and (2) by restricting the set of reference sources to the most stable ones. We concentrate on periods longer than 500 days, after the components studied above (trend, 18.6 years, 430-day FCN, annual and semianual) have been filtered out by least squares estimation. Least squares periodograms are shown in Figure 6 for the prograde and retrograde nutations. The level of the low-frequency part (periods longer than five years) is much lower for the solution using the stabllest...
sources, which is consistent with the above remarks on the early years of the VLBI program. For periods between 1.4 and 5 years, the latter solution tends to show more signal than the solution based on all sources. The spectrum of the prograde component (top panel), where the theoretical FICN is expected to reside, shows a few significant peaks, but none at the MHB2000 FICN period of 1025 days. The spectrum of the retrograde component (bottom panel) has more energy than that of the prograde component. Because the Earth’s response to excitation, e.g., from fluid layers, is larger in the retrograde frequency band via the wings of the transfer function centered on the retrograde FCN (430 days), the fluid effects on nutation are a good candidate for explaining the nutation residuals at interannual time-scale. The spectral energy is the highest in the bands 500–600 days and 3–5 years.

[46] We computed the spectrum of nutations predicted from the AAM (see section 3) using a realistic transfer function (see section 6). The prefiltering of the time series of atmospheric-excited celestial pole motions is the same as that for the series of \(\Delta \varphi_\text{d}, \Delta \varphi_e\), except for the 430-day term, which is irrelevant in this context. The comparison of the spectra of the expected atmospheric-driven nutations and of the VLBI-observed nutations is done only on the prograde and retrograde components (see Figure 6). Of particular interest are frequency bands where the error bars of the atmospheric excitation spectrum are above the zero line and where there are maxima in the VLBI spectrum. For the graph associated with the selected sources, this occurs around 3–5 years and around the 500–600 day period band; the retrograde contribution to the 500–600 day frequency band of the spectrum has large error bars due to the FCN resonance amplification, but is compatible with energy in the atmosphere. In this graph, there is also a maximum at about 850 days in VLBI data which is not significant in the atmospheric data. This peak is a potential candidate for the FICN free mode. While it is possible, this could be difficult because we must keep in mind that the

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**Table 7. Modeled and Estimated Values of the 18.6-year and 9.3-year Nutation Terms**

<table>
<thead>
<tr>
<th>Span Data</th>
<th>Prograde</th>
<th>Retrograde</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.6-year</td>
<td>MHB2000</td>
<td>–8 024 775</td>
</tr>
<tr>
<td>residual</td>
<td>MHB2000</td>
<td>1 433</td>
</tr>
<tr>
<td>1980.0–2001.2</td>
<td>VLBI-MHB</td>
<td>–36 ± 8</td>
</tr>
</tbody>
</table>

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**Estimated Perturbation Due to Source Selection**

| 1980.0–2000.4 | ΔVLBI | 64 ± 10 | 71 ± 10 | 45 ± 10 | –39 ± 10 |
| 1980.0–1999.3 | ΔCRF | 27 ± 4 | –14 ± 7 | 29 ± 4 | 8 ± 7 |
| 1980–2000   | atmosphere | –1 ± 7 | 2 ± 7 | –21 ± 7 | 0 ± 7 |

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| residual      | MHB2000 residual | –14 ± 13 | 12 ± 12 | –28 ± 13 | 7 ± 13 |
| 1980.0–2000.4 | ΔVLBI | 33 ± 5 | –5 ± 5 | 25 ± 5 | 15 ± 5 |
| 1980.0–1999.3 | ΔCRF | –2 ± 3 | –9 ± 3 | 6 ± 3 | 14 ± 5 |
| 1980–2000 | atmosphere | 2 ± 6 | 4 ± 6 | –2 ± 7 | –5 ± 7 |

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*a*Unit is μas.

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**Figure 5.** Corrections to the MHB2000 18.6 years nutation term, obtained from analysis of the VLBI series (VLBI), corrected for the effect of arc sources ΔVLBI (+Arcs) or of celestial pole motion effect ΔCRF (+CRF), and then for the atmospheric excitation (+Atmo). Unit: mas.
resonance strength of the FICN is two orders of magnitude less than that of the FCN (but the $Q$ factor is different). In general, although the amplitudes of the AAM spectrum are only slightly significant, one may note the good agreement in level.

6. How Much Can One Perturb Theory and Still Keep It Self-Consistent?

In section 5, we have analyzed the nutations from the observational point of view to explain the discrepancy between the VLBI observation and the theory. In this section we conversely assess the range of possible changes in the model for explaining the nutations corrected for potential atmospheric effects.

The MHB2000 nutation theory is based on a physical model in which some geophysical parameters are fitted to the VLBI-derived nutation amplitudes and phases. This model is in reasonable agreement with the present knowledge of the Earth’s interior.

The fitted parameters appear directly in the transfer function (with one exception, the Earth’s global dynamic flattening). These parameters appear for instance in the resonance strengths and frequencies of the normal mode oscillations, i.e., the Chandler wobble (CW), the inner core wobble (ICW), the FCN, and the FICN. The transfer function takes the following form:

$$ T(\sigma) = \frac{\varepsilon_{\text{rigid}} - \sigma}{(1 + \varepsilon)} \varepsilon_{\text{rigid}} \left( 1 + (1 + \sigma) \left[ R_0 + \frac{R_{\text{CW}} \sigma_{\text{CW}} + R_{\text{FCN}} (1 + \sigma_{\text{FCN}})}{\sigma - \sigma_{\text{CW}}} + \frac{R_{\text{ICW}} \sigma_{\text{ICW}} + R_{\text{FICN}} (1 + \sigma_{\text{FICN}})}{\sigma - \sigma_{\text{ICW}}} \right] + \frac{R_{\text{FCN}} (1 + \sigma_{\text{FCN}})}{\sigma - \sigma_{\text{FCN}}} \right) $$

(1)

where $\varepsilon_{\text{rigid}}$ is the dynamical flattening of the Earth assumed in the construction of the rigid Earth nutation series, $\varepsilon$ is the dynamical flattening of the Earth, $\sigma$ is the frequency normalized by the sidereal day, $\sigma_{\text{CW}}$, $\sigma_{\text{ICW}}$, $\sigma_{\text{FCN}}$, and $\sigma_{\text{FICN}}$ are the normal mode frequencies for the CW, the ICW, the FCN and the FICN (referred to as the prograde free core nutation, PFCN, in MHB2000), and $R_0$, $R_{\text{CW}}$, $R_{\text{ICW}}$, $R_{\text{FCN}}$, and $R_{\text{FICN}}$ are constants determined in the MHB2000 analysis. Note that the MHB2000 modeling also considers theoretical corrections for the frequency dependence of ocean tide corrections and the mantle anelasticity. These ocean tide effects are derived from state-of-the-art ocean tide models, and an average atmospheric excitation at the $S_1$ frequency (prograde annual nutation) is considered. These corrections are taken into account in the test described hereafter. Nontidal oceanic excitation and time variations of the atmospheric excitation are not taken into account, while their effects are indeed present in the observations used in the fitting. This could give rise to errors in the values for some model parameters. The perturbations considered are relatively small; therefore they are expected to be observable only for the largest nutations, such as the prograde semiannual nutation or the 18.6-year nutations, and for the retrograde annual nutation, of which the effect is enhanced due to the FCN resonance.

To assess how much the normal mode parameters of the model may change if modified oceanic and atmospheric effects are considered in terms of discrepancies of the VLBI-derived nutation amplitudes with model predictions, we performed a perturbation test on some of the parameters involved in the transfer function. The test consists of two steps:

1. Decide on an acceptable range for the disagreement between observed and modeled amplitudes for selected astronomical nutations, based on our estimation of the fluid layer contributions (see section 3). The limits retained take also into account the uncertainty of the VLBI estimates (section 4) and the “observed-model” residual amplitudes resulting from the MHB2000 model implementation.

2. Implement the transfer function for a series of values of the real and imaginary parts of $\sigma_{\text{ICW}}$, $\sigma_{\text{FCN}}$ and $\sigma_{\text{FICN}}$ (which appear not only in the normal mode in the transfer function but also in the resonance strength) to determine the range of these parameters for which the modified nutation amplitudes stay within the limits set in step 1. The components considered are the 13.6-day, semiannual, annual and 18.6-year nutations.

Figures 7 and 8 show the regimes in which the FCN and FICN parameters can change, while still providing acceptable residuals for the considered nutations. The acceptance area lies between the two lines shown in a specific color for a specific nutation. The arrows indicate the part of the plane to be accepted. The acceptance interval for the FCN and FICN parameters is the intersections (hatched in red) of the bands so constructed.

Figure 7 shows the results for the FCN period and quality factor $Q$. The acceptance regime is very much constrained by the in-phase prograde semiannual nutation and the in-phase and out-of-phase retrograde annual nutations; the FCN period is mainly constrained by the in-phase retrograde annual nutation, and the FCN $Q$ is mainly con-
Figure 7. For each relevant nutation, intervals in which the theoretical values of the FCN period and quality factor \( Q \) have values in agreement with the observed nutation (allowing for errors in the atmospheric and oceanic corrections). The intersection (hatched in red) of all the intervals gives the locale for the period and \( Q \) of the FCN. Here the observations are assumed to be corrected for the atmospheric effects as computed from NCEP-NCAR reanalysis model AAM series by Yseboodt et al. [2001].

Figure 8. For each relevant nutation, intervals in which the theoretical values of the FICN period and \( Q \) have values in agreement with the observed nutation (allowing for errors in the atmospheric and oceanic corrections). The intersection (hatched in red) of all the intervals gives the locale for the period and \( Q \) of the FICN. Here the observations are assumed to be corrected for the atmospheric effects as computed from NCEP-NCAR reanalysis model AAM series by Yseboodt et al. [2001].
prograde semiannual nutation, as expected, but other nutations such as the 18.6-year nutations are also significant. [57] We also investigated the effect of instability of the celestial reference frame studied in section 4. The results were very similar except for a small increase of the ranges of the possible resonance periods. These periods still remain very close to the MHB2000 values. In fact, despite our attempts, the FCN and FICN frequencies and $Q$ remain quite robust.

[58] The corrections allowed are dictated by the precision and accuracy of the observation, the discrepancies between observation and theory, and the possible readjustments (within physical limits) of the fitted model. For the 18.6-year nutations for instance, it is possible to accept changes in the theory at the few tens of $\mu$as level. For the retrograde annual nutation, it is possible to accept changes in the amplitude of about 80 $\mu$as, and still get the same values of the Earth interior parameters (at the one day level for the FCN and a few days level for the FICN).

7. Conclusion

[59] In order to propose directions in which further progress can be made in the accuracy of precession-nutation modeling, we investigated several areas of possible perturbations in the observational and theoretical approaches. The study concentrated on the main forced terms (linear, 18.6-year, 9.3-year, annual and semiannual) and the free nutations in the observational and theoretical approaches. The progress can be made in the accuracy of precession-nutation predictions such as the 18.6-year nutations are also significant. The atmosphere and the ocean have large influences on the Earth, resides in the external geophysical fluid influences. The atmosphere and the ocean have large influences on the Earth, resides in the external geophysical fluid influences. The atmosphere and the ocean have large influences on the Earth, resides in the external geophysical fluid influences. The atmosphere and the ocean have large influences on the Earth, resides in the external geophysical fluid influences. The atmosphere and the ocean have large influences on the Earth, resides in the external geophysical fluid influences.

[60] Concerning the observations, we have determined precession and nutation corrections using VLBI time series of the celestial pole offsets provided by five analysis centers using two different software packages. The correction amplitudes found are consistent among the analysis centers and with the MHB2000 values and reach the level of a few tens of $\mu$as. We also investigated a possible perturbation due to instability of the radio sources. While this effect was found to become negligible at the seasonal and free nutation periods after 1990, for the linear terms, the 9.3-year nutations and the 18.6-year nutations, its amplitude (10 – 30 $\mu$as) is smaller than the observed discrepancies between VLBI results and the MHB2000 model that may reach nearly 100 $\mu$as. This result shows that the role of the celestial reference frame stability is worth analyzing in a rigorous way if further progress is sought in the measurements of the Earth’s precession and nutations.

[61] The model of nutation in terms of the interior of the Earth has been shown to be well constrained and robust (Figures 7 and 8). A part of the differences that cannot be explained by a priori improved modeling of the interior of the Earth, resides in the external geophysical fluid influences. The atmosphere and the ocean have large influences which are presently considered as constant with time. We have shown that significant time variations of these effects exist and are observable. The graph of the atmospheric forcing in space around the annual period (Figure 1) shows that the excitation of the atmosphere varies in amplitude and frequency. The maximum of the spectral energy of the AAM can even come very close to the FCN frequency, where the transfer function of the Earth gets amplified. This provides thus energy for explaining the changes in the FCN amplitude observed by VLBI (see Figure 3).

[62] The 500–600 day peak and the 2–3 year peak in the VLBI data could be related to the atmosphere. This spectral energy in the 500–600 day frequency band was long believed to be the FICN free mode because it was close to the theoretical FICN period determined without electromagnetic coupling at the inner core boundary [Dehant et al., 1993]. Our analysis shows that the 500–600 day spectral energy of the VLBI nutation residuals originates most probably from the AAM. Only the peak in the frequency band around 800 day is a plausible candidate for the free FICN mode. More careful analyses of the VLBI data corrected for the “instability” of the celestial reference frame are needed.

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