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# Prospective elementary teachers' selection of mathematical tasks

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*Mathematical tasks are powerful tools to develop mathematical ideas in classrooms and are useful in teacher education linking teacher decision-making, student work, and curricular themes. One of the core elements of teacher development is to be able to choose and use “good” mathematical tasks. Understanding task perceptions of prospective teachers can help predict their eventual modification and appropriation for classroom use. The current study investigated the features that prospective elementary teachers (PT) attend while selecting “good” mathematical tasks. Employing a modified repertory grid technique, we identified attributes of desirable tasks as viewed by PTs. We then present interpretation of findings for mathematical task design and for teacher education.*

*Keywords: Documentational approach to didactics, teacher education, mathematical tasks, task design.*

## Introduction

Due to recent reform efforts in mathematics education, many innovative mathematical tasks for classroom use have been available from commercial resources, professional organizations, or released public accountability tests such as NAEP (e.g., Mohr et al., 2019). This necessitated sharper skills for in-service and prospective teachers for selecting “good” mathematical tasks.

A mathematical task is a problem devoted to developing or assessing a particular mathematical idea (Stein et al., 1996). They are effective tools for communicating curricular visions of mathematics education. Therefore, designing, analyzing, appropriating tasks, and talking about leading classrooms to effectively implement them have become common practice in teacher education programs (Tekkumru-Kisa et al., 2020).

Given the potential role mathematical tasks can play in mathematics classrooms, considerable research has been done on the topic. Stein et al., for example, (1996) looked into the role of mathematical tasks in realizing curricular goals. They observed that mathematical tasks typically move through stages from *tasks as published* in the original curricular resources, to *tasks as set up* by the teacher, and to *tasks as implemented* in classrooms. It was possible to engage students with higher forms of mathematical thinking when assigned mathematical tasks with high cognitive demand but assigning students these tasks did not ensure their high levels of reasoning. Tasks often decline in elicited student thinking due to *inappropriateness* of the task or other factors such as prevalent norms in classrooms, issues of classroom management, or use of time (Stein et al., 1996; Tekkumru-Kisa et al., 2020). We believe that a core skill to be developed in teacher education should be to understand the factors that underlie a better match of tasks with student characteristics and develop the ability to modify the tasks to better match with the needs of students.

## Theory

Taking a resource perspective, we consider mathematical tasks as a type of curriculum resource. Hence, we chose *documentational approach to didactics* (DAD) as a framework in this study

(Gueudet & Trouche, 2009). The framework provided a perspective to understand how teachers choose and interact with mathematical tasks. Pepin (2018) for example, proposed that looking into how teachers (and PTs) relate with mathematical tasks “as is” is not very informative, but we need to explore how they actually work with these tasks. The first step to use a task is to select them. By inference from the DAD framework, we think that PTs actively construct meanings and learn from choosing, transforming, and revising mathematical tasks. Further, as an extension of DAD perspective, the dialectic process of *instrumentation* (how a mathematical task influences a PTs didactic activity) and *instrumentalization* (how the PTs shape mathematical tasks for possible use in classroom) can capture the PTs’ interaction with mathematical tasks (Gueudet & Trouche, 2009). Before instrumentation and instrumentalization take place, a PT needs to choose a mathematical task from among alternatives and consider using it in lesson planning. Then the question is, what attributes do PTs attend to choose a mathematical task from among alternatives? Answering this question is important to understand teacher-task interaction *vis-a-vis* mathematical tasks. Only the tasks that are selected or fall below the perceptual sieve can be modified and eventually used. It would be reasonable to expect that didactical affordances of a task (i.e., *instrumentation*) or modifications needed to be made on a task once it is selected (that is, *instrumentalization*) should be shaped by the PTs perception of the original task. So, we need to know more about how a (prospective) teacher think while selecting tasks before the teacher enters into a serious relationship with the task. DAD offers powerful insights into what happens after a task falls into the “basket” of use for a teacher. The goal of the present study is however to uncover what happens before it is selected. So, we hope that our findings will complement and further the insights offered by the DAD about teacher reasoning in the context of task use for instruction.

## Methodology

To explore the task features PTs attend while they select tasks for classroom use, we employed a modified repertory grid technique for data collection. The technique is based on the assumption that a person’s thinking is directed by the way in which s/he interprets events or objects through his/her personal constructs (Pope & Denicolo, 2016).

We compiled a set of six mathematical tasks organized into groups of two triads. The tasks in each triad were selected to address a common key concept at the same grade level. The tasks were selected from published sources (see Table 1) and they were adapted to the study.

To make the comparisons meaningful, we formed the task triads addressing the same key mathematical concept. However, we wanted to include tasks that represented variation in context; with abstract, illustrative, and applied contexts. This was to see whether PTs prefer tasks with certain contextual structures. However, we did not want to vary the tasks in cognitive demand level, as this may bias participant’s selection towards tasks with higher cognitive level. So, all of our tasks had high cognitive demand (either “procedures with connections” or “doing mathematics”) according to Smith and Stein’s (1998) scheme. This was ensured by independent assessment of the tasks by two mathematics educators prior to the study.

The participants of the study were 14 PTs enrolled in a course “problem-based mathematics education” at master’s level. Student teachers are given weekly problems to solve in small groups in

**Table 1: Mathematical tasks used in data collection and their key mathematical concepts**

Triad	Name of task	Key concept	Grade level	Source
1	The pocket problem	Reading and	4	NCES (1993, p. 126)
	The siblings problem	interpreting data		Small (2020, p. 186)
	The mystery graphs	from graphs		Zawojewski (1996, p. 376)
2	The matchsticks pr.	Representing	8	Rivera & Becker (2008, p. 66)
	The string task	linear functions		Posamentier & Krulik (2009, p. 79)
	The sumo wrestler			(Please see footnote below) <sup>1</sup>

this course that exemplify an inquiry vision of teaching and learning mathematics, analyze them for the curricular ideas and problem-solving strategies embedded, and offer and discuss classroom strategies for lessons centered around these problems. The course was offered in the fourth year of a combined 5-years teacher education program for elementary teachers at a Norwegian university.

In line with the repertory grid technique, data collection was completed in two steps. The first step required comparison of tasks in triads and the second involved individual surveys. The set of tasks were given in print to participants during class time. They were asked to discuss in small groups and solve the tasks. Then for each triad, the groups were invited to write any similarities or differences they could think of between the pairwise combinations of the three tasks (the constructs) from the perspective of their possible use in classroom. Once they complete writing the constructs, they were asked to indicate three tasks that “they like best for classroom use” from among the six. Following the session, the researcher prepared an all-inclusive but non-repeating list of the constructs elicited from the groups before the next class<sup>2</sup>.

After the first data collection session, the researcher organized the constructs into rows of a table and the names of tasks into columns. In this session of data collection, individuals were given a print copy of the table (similar to table 3) and were asked if the constructs in the table were clear. The purpose was to make sure that all the constructs were understandable to participants. Then they were invited to rate each task for the constructs using a Likert-type scale from 1 to 5; 1 standing for “not true at all” and 5 “very true” and the values in between varying accordingly. Ratings were completed individually and took approximately 30 minutes.

<sup>1</sup> Link for this task: <https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:forms-of-linear-equations/x2f8bb11595b61c86:writing-slope-intercept-equations/e/constructing-linear-functions-word-problems>

<sup>2</sup> Repertory grid technique typically requires construct elicitation and ratings from individuals. Here, for the purpose of eliciting richer constructs, we asked the PTs to work in small groups to compare the tasks in step 1. But they rated the tasks in step 2 individually. Hence, we called our version of the data collection a “modified” repertory grid technique.

## Results

Table 2 gives the tasks “liked” by the participants from among the given set and the number of participants who selected these tasks. The “string task” was liked by all PTs. The “matchsticks” and the “sibling” problems were the next two most popular tasks and were selected by nine and six participants respectively. The “mystery graphs” problem was least preferred with only 3 participants.

**Table 2: The tasks liked by the participants**

Tasks	# of participants who selected the task
1. The pocket problem	5
2. The sibling problem	6
3. The mystery graphs	3
4. The matchsticks problem	9
5. The string task	14
6. The sumo wrestler	5

To understand the attributes that might have led the PTs choice of the tasks, list of constructs and the average ratings of the tasks for the 14 participants are computed and given below in table 3. For ease of comparison, ratings were given separately for the top three tasks based on PTs choices and the remaining three tasks. We present the constructs based on which the 6 tasks received high ( $M > 3.5$ ) and low ( $M < 2.5$ ) average ratings. High ratings are color coded in yellow, and low ratings are coded in brown for ease of seeing patterns in ratings. Because some constructs were worded negatively, we wanted to examine the tasks that received low and high average ratings so that we can identify both the negatively and positively viewed task features for the preferred tasks. Consequently, a yellow rating in table 3 does not always mean a desirable task attribute.

In our interpretation of findings, we give reference to constructs with construct-numbers presented in table 3. Below, we report by the common themes we think constructs are related to, for task attributes.

### Use of visual elements

Using visual elements effectively seems to be a common feature among the favored tasks (see construct 1), with the most favored tasks having the highest rating based on this criterion.

### Mathematical power

The task’s potential to create a deep mathematical discussion in class (construct 3) and to require deep understanding and reflection (construct 18) were two factors that seemed to underlie favorable tasks. The only exception was the “sibling problem” to this observation. This task was assessed to

**Table 3: Elicited constructs for comparison with average ratings of tasks**

The constructs / task number in table 2	5	4	2	1	6	3
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1	This task uses visual elements effectively.	4.6	4.2	3.9	3.3	1.7	3.1
2	The context is interesting in this task.	3.2	3.1	4.1	4.0	3.6	4.2
3	This task can create a deeper discussion in class.	4.5	4.2	3.8	3.9	3.5	3.9
4	This task requires developing a formula to solve it.	4.7	5.0	1.0	1.0	4.9	1.1
5	This task is structured in steps and is easier to follow.	4.3	2.3	3.3	3.2	2.4	2.3
6	This is an easy and clear task to do for pupils.	3.4	2.7	3.9	3.2	3.0	1.5
7	You need to interpret textual information to solve this task correctly.	4.1	2.6	3.3	4.8	4.5	5.0
8	This would be a good task to introduce a topic.	2.9	3.0	3.8	2.6	2.6	1.9
9	This task is more demanding in thinking and reasoning.	4.2	3.9	1.9	3.4	3.4	4.6
10	The story may be overwhelming for the kids in this task.	2.8	1.9	1.3	2.4	2.8	4.4
11	This task may be misinterpreted by the pupils.	2.5	2.7	2.0	3.5	3.4	4.4
12	This task may be difficult for individual pupils and hence better for a class discussion.	3.9	3.3	2.4	3.9	3.5	4.1
13	The wording is a bit difficult in this task.	2.2	1.9	1.6	2.0	2.2	4.5
14	This task requires just reading off information and is not a real problem-solving task.	1.6	1.4	4.8	2.4	2.4	1.9
15	There is too much text here.	1.6	1.1	1.3	1.5	1.8	4.4
16	This is more like a "pure mathematics" task.	3.2	4.4	3.4	2.5	3.9	2.1
17	This task is not very original, I have seen tasks like this one before.	2.8	3.4	4.2	2.4	3.1	1.9
18	This task requires deep understanding and reflection.	3.8	3.8	1.4	2.6	2.8	3.5
19	This task has a realistic and applied context.	2.6	3.2	4.3	4.1	3.9	4.1
20	There is a simplicity that I like in this task.	3.8	3.7	3.9	3.1	3.9	1.9

have the potential to create good mathematical discussion in class, but not requiring deep understanding or reflection by itself. PTs might have thought that a deep discussion opportunity can potentially be created by teachers while using this task.

### Simplicity

Simplicity was another prominent attribute for preferred tasks (construct 20). Preferred tasks did not have too much text to read and interpret for solution (construct 15). The story of the task being

overwhelming seemed also to be a distinct feature of the *least* preferred task (line 10) which may also lead to difficulty and misinterpretation in reading the text (constructs 11 and 13). Another aspect of complexity in textual information, namely a high level of need for interpretation (and hence requiring higher reading ability) was not seen favorably among PTs in mathematical tasks (construct 7).

### **Level of challenge**

A high level of challenge and demand in mathematical reasoning seem to be important but not sufficient (and in one case, not even necessary) for the tasks to be seen favorably (constructs 9, 12 and 14). The highest cognitive challenge was seen in the “mystery graphs” task, but it did not receive high ratings for preference (for probably lack of simplicity). However, the two most favored tasks (the “string task” and the “matchsticks problem”) had the next two highest ratings in demand for mathematical thinking and reasoning. Conversely, easiness of a task was not a common theme in the desired or not-so-desired tasks (construct 6). It looks like the *right level* of cognitive challenge (not too much, not too little) seems to be an attribute deemed desirable by PTs.

### **Other features of the tasks**

Other features of tasks did not seem to be strongly associated with task preference. Among these are the features related to the context of tasks being interesting, applied or pure (constructs 2, 16 and 19), and tasks being structured in steps (construct 5).

### **Conclusions and discussion**

In our view, the results paint a positive picture of prospective elementary teachers’ ability to distinguish among tasks. In particular, PTs seem to have a sophisticated view of mathematical tasks. This may be due to prior course experiences on task design and analysis. However, this needs to be further investigated, perhaps with a bigger sample size. Effective use of visual elements was common to all highly rated tasks. Mathematical power embedded in the task that can support a good classroom discussion, nurture deep understanding of mathematical ideas, and provide opportunity for pupil reflection were also considered highly for their perception. However, high mathematical power did not necessarily mean complicated design, too many words, or a cluttered story line in their eyes. Hence, simplicity was another prominent feature of desirable tasks according to PTs. In other words, tasks should have not only the power, but also the economy and clarity and hence simplicity in set up, story line and wording. An optimum level of difficulty or challenge was considered highly for desirable tasks. A high level of difficulty or easiness by itself were not associated with “good” tasks. Selected tasks included both not so demanding as well as reasonably demanding tasks which pointed to an optimum level of challenge.

How do we interpret these findings from the DAD perspective? As PTs look for mathematical tasks for instructional planning, their *instrumentation* process could be guided by such powerful notions as mathematical power, visual elements, simplicity and level of challenge for pupils. These same notions can guide the *instrumentalization* (that is, task modification) for actual classroom use as well. When PTs are considering a task, they may be already thinking about how they will use it (see for example construct 8 in Table 3). Our study provides new insights into DAD and how teachers’ perception of a task might be guiding task appropriation before use.

We also find it interesting that mathematical power and simplicity came out as prominent task features. In fact, their presence together reminded us Birkoff's (1956) model of aesthetics in mathematical products (as quoted in Dreyfus & Eisenberg, 1986). Birkoff offered a formulaic model of aesthetics;  $M = O/C$ , where  $M$  stands for *aesthetics*,  $O$ , a measure of *order* or mathematical structure, and  $C$  for *complexity*<sup>3</sup>. The formula suggests that aesthetic value of a mathematical object is a *proportion*, counterbalanced by two factors, *order* and *complexity*. In other words, aesthetics may be directly proportional to the mathematical structure embedded in it, while complexity being inversely proportional, and aesthetics may change with differing values of order and complexity. It looks like, even though the PTs did not use the term "aesthetics" in this study, their search for power and simplicity at the same time, might imply a desire for aesthetics in mathematical tasks. We thought this would connect with Sinclair and Crespo's (2006) point on aesthetics in a new light in mathematical task design.

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<sup>3</sup> We propose that Birkhoff's (1956) notion of *mathematical order* resembles the idea of *mathematical power* as suggested by the participants in this study as both refer to a central mathematical idea or concept in a mathematical product.



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