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# Ice particle impact on solid walls: size modeling of reemitted fragments

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## Abstract

The present work deals with ice particle fragmentation resulting from impact on a solid wall. First, a semi-empirical model to predict the size of the largest reemitted fragment is presented. It is based on the energy-horizon theory of fragmentation developed by Grady (*J. Mech. Phys. Sol.*, 36(3),353-384, 1988) in combination with a strain rate scaling based on the indentation radius formed upon impact. Model predictions are in good agreement with experimental data from six different sources.

In addition, an empirical fit to the ice fragment volume distribution is sought. Different candidate fits, namely power law, Weibull and lognormal are proposed and evaluated both qualitatively and quantitatively. The fragment volume distributions appear to exhibit different trends for impact conditions representative of ice crystals and hailstones. For this reason, a less

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accurate yet more robust power law fit is proposed to model the available fragment volume distribution data.

*Keywords:* ice crystal icing, impact, fragmentation, energy-horizon theory, maximum fragment diameter, fragment volume distribution

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## 1. Introduction

Ice crystal icing of civil aircrafts generally occurs in the vicinity of deep convective clouds at cruise altitudes [1]. Ice crystals, whose diameters range between  $5\ \mu\text{m}$  and  $2\ \text{mm}$  approximately, may adhere to the surfaces they impinge on when the latter are heated or when the crystals themselves are partially melted [2]. Thus, ice crystal icing necessarily occurs within the engine, where significant fragmentation occurs as impact velocities are comprised between  $50\ \text{m s}^{-1}$  to  $250\ \text{m s}^{-1}$  approximately. The associated size reduction enhances melting, enabling the sticking of ice fragments on engine walls and favoring ice accretion. The latter may cause aerodynamic blockage, mechanical damage due to ice block detachment or even flameout. For this reason, ice crystal icing still represents a serious threat to aviation safety [3, 4, 5] and the characterization of reemitted fragment properties is a necessary ingredient to improve the predictive capabilities of numerical icing tools [6, 7].

Several experiments reproducing impact conditions representative of ice crystal icing were recently conducted. Hauk *et al.* [8] characterized the impacts of both spherical and nonspherical ice particles within a large size ( $30\ \mu\text{m}$  -  $3.5\ \text{mm}$ ) and a moderate normal impact velocity range ( $5$  -  $70\ \text{m s}^{-1}$ ). Considering the portions of the crystal volumes shattered into fragments as well as the distinct disintegration patterns in their experiments, Hauk *et*

21 *al.* [8] proposed to classify fragmentation into minor, major and catastrophic  
22 regimes.

23 Vargas *et al.* [9] carried out spherical ice particle impacts on a flat glass  
24 plate. The authors characterized ice particle diameters and velocities before  
25 impact. In addition, they placed a high resolution camera under the glass  
26 plate to capture the fragment contours and reported that the resulting frag-  
27 ment number distributions could be well approximated by lognormal laws.

28 Experiments dedicated to the characterization of hailstone fragmenta-  
29 tion [10, 11] consider normal impact velocities up to  $200 \text{ m s}^{-1}$  and ice parti-  
30 cle diameters ranging from 5 to 27.5 mm. In order to determine the validity  
31 limits of the models proposed in the current work, measurements from hail-  
32 stone impact experiments are added to the validation database. Pan and  
33 Render [10] examined the fragmentation of hailstone sized ice particles on a  
34 flat plate. Among others, they derived empirical correlations to determine  
35 the fragments' maximum diameter and proposed to model the reemitted frag-  
36 ment volume distribution with a Weibull law. Guégan *et al.* [11] performed  
37 impact experiments of hailstone sized ice particles on a glass plate. They  
38 found the ratio of the average radial fragment velocity to the normal impact  
39 velocity to be approximately constant. From the same experiments, Guégan  
40 *et al.* [11] also provided unpublished size distribution data to the authors [12],  
41 which will be used for validation purposes.

42 Tippman *et al.* [13] performed detailed numerical simulations of the im-  
43 pact of a hailstone sized ice particle on a rigid surface to reproduce their  
44 own experimental measurements for the force history resulting from such  
45 impact. They propose to distinguish between two main impact phases:

46 elastic-dominated structural response followed by cracking/ fragmentation.  
47 Performing numerical simulations of the impact of brittle spheres using the  
48 Discrete Element Method (DEM), Carmona *et al.* [14] reported that cracks  
49 formed within the particle in the region above a compressive cone delimited  
50 by the indentation radius formed upon impact. They also found the largest  
51 radial and circumferential stresses to occur in a ring shaped area delimiting  
52 the outer contact zone between particle and substrate, leading to the prop-  
53 agation of meridional cracks. Hauk *et al.* [8] also conclude that meridional  
54 cracks drive major and catastrophic fragmentation, while they assume lateral  
55 crack formation to dominate at lower impact velocities.

56 Despite the numerous insights gained from these research efforts, current  
57 models predicting the properties of the reemitted fragments remain essentially  
58 empirical. Villedieu *et al.* [15] proposed an empirical model yielding size and  
59 velocity properties of the reemitted fragments based on regression fits to var-  
60 ious experimental data. Guégan *et al.* [11] provide correlations for the radial  
61 and normal velocities of the reemitted fragments while Pan and Render [10]  
62 propose a Weibull fit for the reemitted fragment volume distribution. On  
63 the contrary, Weiss [16] reports that ice fragment size distributions follow  
64 power-laws.

65 The present work aims at deriving a model for the size of the largest  
66 reemitted fragment based on a simple energy principle. First, a semi-empirical  
67 model relying on the energy-horizon theory of fragmentation [17, 18] is used  
68 to predict the maximum fragment size. The current model is closed via  
69 a strain rate scaling from the literature [19, 20] based on the indentation  
70 radius formed upon impact. The consistency of the model predictions is

71 demonstrated by comparison with a large experimental database. Then, the  
72 available fragment volume distribution data is examined. Different fits to the  
73 fragment volume distribution are proposed, namely power, lognormal and  
74 Weibull law. These fits are compared both qualitatively and quantitatively.  
75 The main findings and outlooks are summarized in the conclusion.

## 76 **2. Maximum fragment size modeling**

### 77 *2.1. Energy based fragmentation model*

78 The main assumptions governing the choice of the current model, which  
79 will be further justified below, were those of a dynamic and brittle frag-  
80 mentation process. Furthermore, a simple and tractable model suitable for  
81 application purposes, hence based on a limited set of global impact param-  
82 eters, was sought. In particular, the current model for the maximum reemitted  
83 fragment size relies on an energy conservation principle between the initial  
84 and end states of fragmentation [17, 18]. It states that the energy con-  
85 tributions fuelling fragmentation, namely elastic and kinetic energies, must  
86 exceed the energy required to create new fracture surface at the local scale.  
87 The modeling framework is completed using two additional hypotheses:

- 88 • first, a correlation distance set by the product of the crack propagation  
89 speed, taken as the speed of sound, and the characteristic time scale  
90 of fragmentation is defined. This correlation distance sets the upper  
91 bound on the fragment size.
  
- 92 • second, a uniform dilatational expansion within the body under con-  
93 sideration is assumed

94 The resulting equation system then writes:

$$s \leq 2ct \quad (1)$$

$$\frac{1}{2} \frac{P_s^2}{\rho c^2} + \frac{1}{120} \rho \dot{\epsilon}^2 s^2 \geq \frac{6G_c}{s} \quad (2)$$

$$P_s = \rho c^2 \dot{\epsilon} t \quad (3)$$

In eq. 1,  $s$  denotes the characteristic correlation distance/ fragment size,  $t$  the time variable, while the crack propagation velocity is given by the speed of sound  $c$ . The latter is evaluated as:

$$c = \sqrt{\frac{E}{\rho}} \quad (4)$$

95 with  $E = 10.5 \times 10^9$  Pa the Young modulus and  $\rho = 917$  kg m<sup>-3</sup> the mate-  
 96 rial density of ice<sup>1</sup>, which may both be assumed constant [21]. The resulting  
 97 crack propagation speed appears to exceed the value reported by Tippman *et*  
 98 *al.* [13], i.e. approximately 2000 m s<sup>-1</sup>, by almost 70% and should thus rather  
 99 be considered as providing the correct order of magnitude. Equation 2 rep-  
 100 resents the energy balance between the contributions fuelling fragmentation,  
 101 namely the elastic and kinetic energies on the left-hand side, which must  
 102 exceed the energy required to create new fracture surface on the right-hand  
 103 side. In eq. 2,  $P_s$  denotes the tensile pressure,  $\dot{\epsilon}$  the strain rate and  $G_c$   
 104 the energy per unit area required to create new fracture surface. While the  
 105 latter is assumed constant in the standard formulation of the model, with  
 106  $G_c = G_{c,0} = 0.12$  J m<sup>-2</sup> [22], a relation allowing to account for its temper-  
 107 ature dependency is provided in Appendix A. Finally, eq. 3 describes the  
 108 time history of tensile pressure leading to failure.

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<sup>1</sup>all physical parameters/ constants are written in SI units

Writing eqs. 1 and 2 as equalities, the characteristic fragment size follows a power law on the strain rate when combining eqs. 1-3:

$$s = K\dot{\epsilon}^\alpha \quad (5)$$

109 with  $K = (45G_c/\rho)^{1/3}$  and  $\alpha = -2/3$ , a result characteristic of highly dy-  
 110 namic fragmentation [23]. In addition, the lower bound  $\alpha = -1$  may be  
 111 derived for quasi-static loadings [18].

112 Following Glenn and Chudnovsky [23], two normalization parameters are  
 113 defined for the fragment size and the strain rate:

$$s_0 = 12 EG_c P_{s,0}^{-2} \quad (6)$$

$$\dot{\epsilon}_0 = \frac{1}{6} P_{s,0}^3 c^{-3} \rho^{-2} G_c^{-1} \quad (7)$$

with the constant reference tensile pressure set to  $P_{s,0} = 1 \times 10^6$  Pa. Using these expressions, eq. 5 may be rewritten in the following dimensionless form:

$$\frac{s}{s_0} = C_f \left( \frac{\dot{\epsilon}}{\dot{\epsilon}_0} \right)^\alpha \quad (8)$$

114 where  $C_f$  is a dimensionless parameter to be adjusted to available experi-  
 115 mental data. The characteristic strain rate required for closure is presented  
 116 in the following.

## 117 2.2. Strain rate estimate

118 A sketch of the envisioned fragmentation mechanism is shown in fig. 1.  
 119 It is assumed that the early instants of impact are characterized by a rigid  
 120 body motion of the particle while the latter is plastically deformed/ crushed  
 121 in the contact area with the solid surface (top view of fig. 1). The assumption



122 of plastic deformation within the contact area is in line with the constitutive  
 123 laws used to model the mechanical behavior of ice [24] and numerical sim-  
 124 ulations of ice particle impact [13, 25]. Furthermore, it is assumed that the  
 125 strong compressive deformation of the crystal remains restricted to the imme-  
 126 diate vicinity of the contact area with the solid wall, whereas the remaining  
 127 part of the crystal is subject to significant strain, eventually leading to its  
 128 fragmentation via crack propagation (bottom view of fig. 1). This assump-  
 129 tion is consistent with the respective impact and crack propagation speeds,  
 130 the latter corresponding to the speed of sound and thus being an order of  
 131 magnitude larger than the former. It also seems in line with the numerical  
 132 simulations of brittle sphere impacts reported by Carmona *et al.* [14] as they  
 133 observe crack initiation due to increasing tensile radial and circumferential  
 134 stresses in a ring shaped region above the strongly compressive contact area.  
 135 Contrary to compression, the tensile failure of ice shows brittle behavior and  
 136 a negligible influence of strain rate [21], in agreement with the interpretation  
 137 of the fragmentation process outlined in section 2.1.

Following Hutchings [19], the characteristic strain driving fragmentation  
 is assumed to scale as the ratio of the indentation radius  $a$  to the initial  
 particle diameter  $D_0$ :

$$\varepsilon \sim \frac{a}{D_0} \quad (9)$$

Assuming plastic deformation in the contact area, Roisman and Tropea [20] found the indentation radius  $a$  to scale as:

$$a \sim U_{n,0}^{1/2} D_0 \rho^{1/4} Y_c^{-1/4} \quad (10)$$

with  $U_{n,0}$  being the normal impact velocity and  $Y_c$  the compressive yield  
 strength of ice. Hutchings [19] obtained the same result as in eq. 10 but used

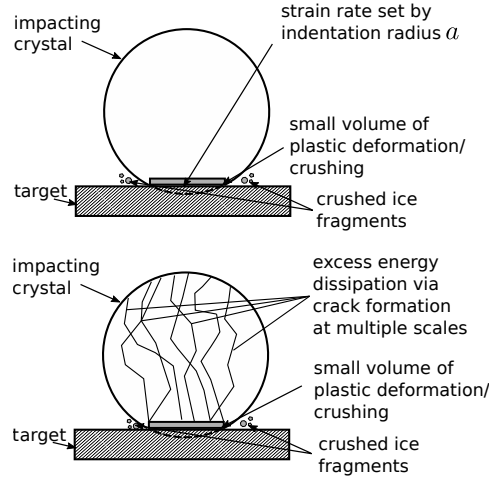


Figure 1: Sketch of the envisioned two main fragmentation steps. Above: early instants of impact, where the area of contact between the crystal and the surface is plastically deformed /crushed. Below: the tensile strain level within the crystal is determined by the indentation radius, leading to crack formation, propagation and eventually fragmentation.

a quasi-static indentation hardness to characterize the material's compressive resistance. The strain scaling in eq. 9 is then divided by the time necessary to form the indentation [20]:

$$t_i \sim D_0 \rho^{1/2} Y_c^{-1/2} \quad (11)$$

Combining eqs. 9-11 finally yields:

$$\dot{\epsilon} \sim U_{n,0}^{1/2} D_0^{-1} \rho^{-1/4} Y_c^{1/4} \quad (12)$$

138 In the standard formulation, a constant quasi-static value is used for the  
 139 compressive yield strength,  $Y_c = Y_{c,0} = 5.2 \times 10^6$  Pa. Relations allowing to  
 140 account for the strain rate and temperature dependency of this parameter  
 141 are provided in Appendix A.

142 *2.3. Validation*

143 The model predicting the size of the largest fragment given by eq. 8  
 144 in conjunction with eq. 12 is evaluated using a large experimental dataset  
 145 compiling data of Karpen *et al.* [26], Reitter *et al.* [27], Hauk *et al.* [8], Vargas  
 146 *et al.* [9], Pan and Render [10] and previously unpublished data of Guégan  
 147 *et al.* [12]. For the experiments of Hauk *et al.* [8] and Karpen *et al.* [28],  
 148 data characterizing impacts without fragmentation were discarded. Also,  
 149 size measurement data is only available for a subset of the fragment velocity  
 150 data published by Guégan *et al.* [11]. Finally, the raw experimental data  
 151 was not accessible for the experiments of Pan and Render [10] and was hence  
 152 retrieved by digitizing the curves from their article.

The resulting ranges for the normal impact velocities, initial diameters and initial particle temperatures of each experiment are summarized in table 1. In addition, the ranges spanned by two nondimensional numbers commonly used to characterize ice particle fragmentation thresholds in the literature are also provided. The nondimensional parameter  $\xi$  was introduced by Hauk *et al.* [8] to estimate the probability of fragmentation:

$$\xi = \frac{U_{n,0} D_0^{2/3}}{\beta} \quad (13)$$

with  $\beta = 0.14 \text{ m}^{5/3} \text{ s}^{-1}$ . The probability of fragmentation approaches zero for  $\xi \rightarrow 0$  while the probability of catastrophic fragmentation tends to one for  $\xi > 1.1$  [8]. Vidaurre and Hallet [22] define a nondimensional impact parameter relating the kinetic energy upon impact to the surface energy:

$$\mathcal{L} = \frac{1}{12} \frac{\rho U_{n,0}^2 D_0}{G_c} \quad (14)$$

Variable Units	$U_{n,0}$ (m s <sup>-1</sup> )	$D_0$ (mm)	$T_0$ (K)	$\xi$ (-)	$\mathcal{L} \times 10^3$ (-)
Karpen <i>et al.</i> [26]	[5-75]	[0.02-0.45]	255	[0.3-2.2]	[0.06-0.8]
Reitter <i>et al.</i> [27]	[7-74]	[1.3-4.5]	[258-263]	[0.9-9.3]	[0.07-6.0]
Hauk <i>et al.</i> [8]	[15-70]	[0.08-3]	248	[0.5-3.5]	[0.08-2.9]
Vargas <i>et al.</i> [9]	[22-65]	[2.3-2.6]	266	[2.8-8.7]	[2.3-21.4]
Pan and Render [10]	[60-150]	[12.7-19]	233	[23.3-76.3]	[48.9-457]
Guégan <i>et al.</i> [12]	[40-203]	6.2	255	[9.7-31.6]	[7.7-124.1]

Table 1: Main impact parameter ranges of the experimental data used for validation. For different reasons exposed in the present section, the data used for validation consists of a subset of the original data for the experiments of Karpen *et al.* [26], Hauk *et al.* [8] and Guégan *et al.* [12] (previously unpublished data for the latter).

153 The threshold for major/ catastrophic fragmentation is generally set to  $\mathcal{L}_2 =$   
154 90 [15]. Considering the orders of magnitude of  $\xi$  and  $\mathcal{L}$  in table 1, all  
155 impact experiments performed by Vargas *et al.* [9], Pan and Render [10]  
156 and Guégan *et al.* [12] lie well within the catastrophic fragmentation regime.  
157 On the contrary, impact outcome classifications performed by Karpen *et*  
158 *al.* [28] and Hauk *et al.* [8] confirm that their experiments also comprise  
159 minor and major fragmentation impacts. Thus, the present experimental  
160 data covers all fragmentation regimes according to the classification of Hauk  
161 *et al.* [8].

162 First, the correlation of the maximum fragment diameter with the pro-  
163 posed strain rate estimate is examined. For this purpose, the scaling in eq. 12  
164 is interpreted as an equality with a unitary constant. In order to validate the  
165 assumption that the maximum fragment diameter and strain rate are linked  
166 by a power law relation, the data is plotted on a log-log scale. Because they

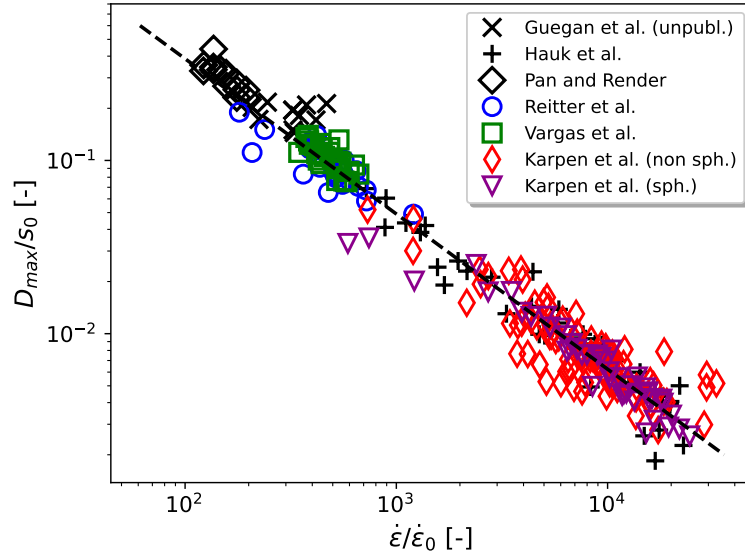


Figure 2: Experimental normalized maximum fragment diameter over the normalized strain rate for the experimental data of Karpen *et al.* [26], Reitter *et al.* [27], Hauk *et al.* [8], Guégan *et al.* [12] (previously unpublished data) and Pan and Render [10] on a log-log scale. For the experimental dataset of Karpen *et al.* [26], the impact data was split into initially spherical ("sph.") and non-spherical ("non sph."). The regression curve to the entire dataset ( $\ln(D_{max}/s_0) \approx -0.896 \ln(\dot{\epsilon}/\dot{\epsilon}_0) + 3.175$ ) is displayed as a black dashed line.

167 contain significantly more datapoints, only one out of respectively six and  
168 ten datapoints is plotted for the experiments of Reitter *et al.* [27] and Karpen  
169 *et al.* [26]. The result is displayed in fig. 2 and illustrates an excellent cor-  
170 relation as all experimental datapoints approximately collapse on the linear  
171 regression curve. From a regression to the entire experimental dataset, the  
172 exponent  $\alpha$  and the multiplicative constant  $C_f$  of the semi-empirical model  
173 are obtained as:

$$\alpha \approx -0.896 \quad (15)$$

$$C_f \approx 23.936 \quad (16)$$

174 Note that these model parameters are obtained assuming both the fracture  
175 surface energy per unit volume and the compressive yield strength to be con-  
176 stant, respectively  $G_c = G_{c,0} = 0.12 \text{ J m}^{-2}$  and  $Y_c = Y_{c,0} = 5.2 \times 10^6 \text{ Pa}$  .  
177 Modeling the variations of  $G_c$  with temperature and of  $Y_c$  with both temper-  
178 ature and strain rate, see Appendix A, yields:

$$\alpha_v \approx -0.885 \quad (17)$$

$$C_{f,v} \approx 42.824 \quad (18)$$

Thus, variations of  $G_c$  and  $Y_c$  mostly affect the value of the multiplicative constant of the model, but not its exponent. Further analysis shows that the temperature dependencies of these material properties have a negligible influence on the increase of  $C_{f,v}$  with respect to  $C_f$ . Thus, this significant raise is mostly due to the nonlinear dependency of the compressive yield strength  $Y_c$  on the strain rate, see eqs. A.2 and A.3, as well as the choice of a quasi-static reference value for the latter in the model formulation based on constant

material properties. While the value for the exponent  $\alpha$  lies within the expected bounds, i.e.  $-1$  (quasi-static loading) and  $-2/3$  (dynamic loading), see section 2.1, it was expected to lie somewhat closer to the upper bound characteristic of highly dynamic loadings. Despite this fact, the quantitative agreement with the different experimental data appears very satisfactory, as may be seen in table 2, which provides the average modeling errors for each dataset. As for the plot in fig. 2, the experimental data of Karpen *et al.* [26] was split into initially spherical and non spherical particles to assess the influence of the initial shape on modeling accuracy. In order to further assess the accuracy of the present model, average errors are also computed for the models of Villedieu *et al.* [15] and Pan and Render [10]. The model of Villedieu *et al.* [15] expresses the maximum fragment diameter as a power of the Vidaurre number (eq. 14) to its threshold value for fragmentation with a constant exponent:

$$D_{max}^v/D_0 = \left( \frac{\mathcal{L}_2}{\mathcal{L}} \right)^{2/11} \quad (19)$$

The model of Pan and Render [10] reads:

$$D_{max}^{pr}/D_0 = 0.437 - 0.922 \left( \frac{U_{n,0}}{a_L} \right) \quad (20)$$

179 with a nondimensionalization constant taken as the speed of sound in air at  
 180 ambient temperature, i.e.  $a_L = 340 \text{ m s}^{-1}$ . Since eq. 20 is only meaningful  
 181 for  $U_{n,0} < 161 \text{ m s}^{-1}$ , it becomes invalid for the largest impact velocity ex-  
 182 periments of Guégan *et al.* [12]. Finally, the average errors obtained with  
 183 varying fracture surface energy and compressive yield strength are given in  
 184 parenthesis. The values of the latter indicate that uncertainties on material  
 185 properties do not significantly affect the precision of the present model.

186 The accuracy of the present model appears good for all datasets, the  
187 lowest average error amounting to 12.6 % (both Pan and Render [10] and  
188 Vargas *et al.* [9]) and the largest to 26.3 % for the nonspherical impact data  
189 of Karpen *et al.* [26]. Error levels between the present model and the model  
190 of Villedieu *et al.* are comparable for the datasets of Hauk *et al.* [8] and  
191 Guégan *et al.* [12]. On the contrary, the present model performs better than  
192 the model of Villedieu *et al.* [15] for the datasets of Karpen *et al.* [26], Vargas  
193 *et al.* [9] and Pan and Render [10], with error reductions ranging between  
194 5 and 11 %. As expected, the model of Pan and Render [10] performs best  
195 for their own experimental data and its error levels with respect to the other  
196 models remain similar for the data of Reitter *et al.* [27] and Guégan *et al.* [12].  
197 For the remaining data, the model of Pan and Render [10] displays much  
198 larger average error levels. For the current model, the average errors do not  
199 seem to correlate with the main impact parameters as it performs similarly  
200 well for ice crystal and hailstone sized particles. On the contrary, it displays  
201 a significant increase in error levels when considering impacts of nonspherical  
202 particles. Such behavior could be expected as impacts of irregular particles  
203 are more difficult to model due to contact area variations and associated  
204 rotation effects. A more visual illustration of the present model accuracy  
205 is provided in fig. 3, where the modeled maximum fragment diameters are  
206 directly compared to experimental data in a scatterplot. While the model  
207 appears accurate on average, significant errors may be noted for single impact  
208 events.



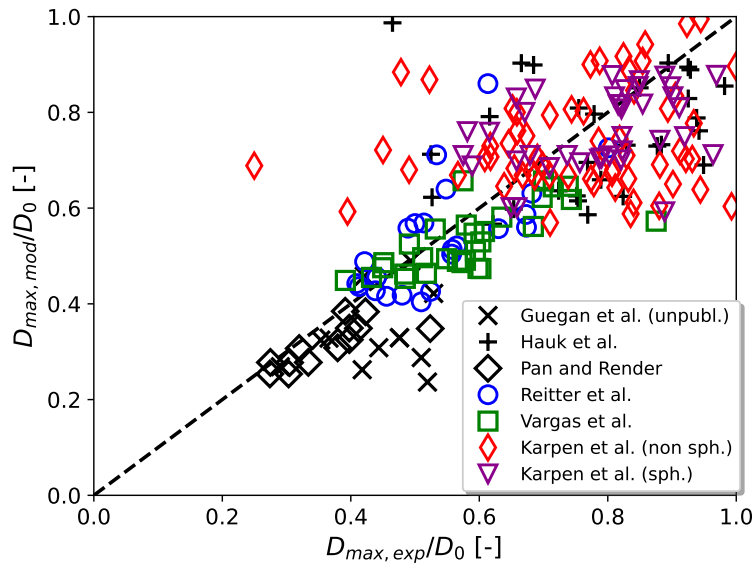


Figure 3: Experimental maximum fragment diameter normalized by the initial diameter plotted over the current model values for the different experimental data. For the experimental dataset of Karpen *et al.* [26], the impact data was split into initially spherical ("sph.") and non-spherical ("non sph.") particles.

Experiment	Particle shape	Current model avg. error [%]	Villedieu <i>et al.</i> [15] avg. error [%]	Pan and Render [10] avg. error [%]
Karpen <i>et al.</i> [26]	sph.	17.1 (17.2)	28.2	44.7
Karpen <i>et al.</i> [26]	non sph.	26.3 (26.4)	31.6	46.4
Reitter <i>et al.</i> [27]	sph.	16.1 (16.1)	14.7	19.2
Hauk <i>et al.</i> [8]	mixed	23.0 (23.0)	26.5	45.7
Vargas <i>et al.</i> [9]	sph.	12.6 (12.8)	20.9	27.0
Pan and Render [10]	sph.	12.6 (7.9)	21.3	7.8
Guégan <i>et al.</i> [12]	sph.	23.6 (23.1)	24.1	24.2

Table 2: Average errors of the maximum fragment diameter obtained with the current model (the numbers in parentheses correspond to the average errors obtained with varying material parameters, see Appendix A), the model of Villedieu *et al.* [15] and the model of Pan and Render [10], each compared with data from the different experimental datasets.

### 209 **3. Fragment size distribution**

210 Fracture in brittle solids is weakly dissipative and may thus proceed on  
211 successively finer scales to dissipate the excess energy [29]. If the fragmenta-  
212 tion process is further considered scale invariant, it may be described using  
213 fractal theory. Size distributions resulting from fractal fragmentation pro-  
214 cesses follow power laws [30], also known as Dinger-Funk distributions [31],  
215 and the latter were reported to accurately reproduce fragment size distribu-  
216 tions in many experiments [16, 32].

217 Considering ice fragmentation more specifically, size distributions indicat-  
218 ing fractal behavior were reported for uniaxial compression creep, crushing  
219 and indentation experiments [16]. On the other hand, the numerical simula-  
220 tions of brittle sphere impacts of Carmona *et al.* [14] indicate that only the  
221 smaller fragments follows a power law distribution. Thus, the authors chose a  
222 Weibull law to model their numerical fragment mass distributions, as do Pan  
223 and Render [10] for their hailstone impact experiments. Finally, Vargas *et*  
224 *al.* [9] report that the fragment number distributions are well approximated  
225 by lognormal laws in their ice particle impact experiments, indicating that  
226 the fragmentation process could be interpreted as the production of large  
227 fragments successively and randomly split into smaller fragments [33]. From  
228 the above discussion, it appears that the best candidate fit to the present  
229 size distribution data is unknown *a priori*.

230 Following a brief description of the available experimental size distribu-  
231 tion data, the method used to derive power, Weibull and lognormal fits to  
232 the latter is presented. The predictions of these fits are then compared qual-  
233 itatively and quantitatively.

### 234 3.1. *Experimental data*

235 Within the present dataset, three experiments provide information on the  
236 fragment size distribution resulting from impact, namely Reitter *et al.* [27],  
237 Vargas *et al.* [9] and previously unpublished size fragment data [12] from the  
238 experiments of Guégan *et al.* [11]. Vargas *et al.* [9] and Guégan *et al.* [12]  
239 obtain their size distribution measurements via high resolution images of the  
240 fragments once they have come to rest on the impinged surface. Reitter  
241 *et al.* [27, 34] detect the contour of the reemitted fragments in motion and  
242 track each fragment separately. The raw experimental data is not available  
243 for the experiments of Guégan *et al.* [12], but only the number and volume  
244 size distributions discretized over 80 bins. For the experiments of Reitter  
245 *et al.* [27] and Vargas *et al.* [9], the raw data consists of approximately 200  
246 and 2000 fragment size measurements per impact experiment. The impact  
247 parameter ranges are identical to those reported in table 1 in section 2.3.  
248 Hence, the data of Vargas *et al.* [9] and Guégan *et al.* [12] entirely lie in the  
249 catastrophic fragmentation regime. While the experimental data of Reitter  
250 *et al.* [27] also cover minor and major fragmentation, only a few data of  
251 their data points lie in the latter regimes, which is insufficient to warrant  
252 meaningful conclusions. Thus, the analysis presented in the following only  
253 applies to the catastrophic fragmentation regime.

### 254 3.2. *Candidate fitting distributions*

255 When considering ice accretion, impact dynamics are driven by particle  
256 inertia as larger particles are more likely to impinge on solid walls within the  
257 engine. Further assuming broad size distributions, smaller particles may be  
258 expected to carry a negligible amount of mass and thus not to significantly

259 contribute to the accretion process. For this reason, a fit to the fragment  
 260 volume distribution is sought.

The definitions of the fragment diameter moments in the literature are generally based on the number distribution  $p_n$ :

$$D_{ij}^n = \left( \frac{\int_0^\infty p_n(D) D^i dD}{\int_0^\infty p_n(D) D^j dD} \right)^{\frac{1}{i-j}} \quad (21)$$

261 with  $i$  and  $j$  integer numbers. As the focus lies on statistical quantities  
 262 characteristic of the largest fragments, a fit accurately matching the Sauter  
 263 mean diameter  $D_{32}^n$  and the mean volume diameter  $D_{43}^n$  is sought. In addition,  
 264 the volume median diameter  $D_{50}^v$  separating 50% of the fragment volume from  
 265 the 50% remaining, is used. Similarly, the  $D_{10}^v$  and  $D_{90}^v$  diameters are also  
 266 employed.

267 Power, Weibull and lognormal laws are examined as candidate fits to  
 268 the present fragment volume distribution data in the following. They are  
 269 respectively defined as:

$$p_{p,v}(D) = \frac{\gamma + 1}{D_{max}^{\gamma+1} - D_{min}^{\gamma+1}} D^\gamma \quad (22)$$

$$p_{ln,v}(D) = \frac{1}{\sqrt{2\pi} D \sigma_{ln}} \exp\left(-\frac{(\ln(D) - \mu_{ln})^2}{2\sigma_{ln}^2}\right) \quad (23)$$

$$p_{wb,v}(D) = \frac{\beta}{\lambda} \left(\frac{D}{\lambda}\right)^{\beta-1} \exp\left(-\left(\frac{D}{\lambda}\right)^\beta\right) \quad (24)$$

270 with  $\gamma$ ,  $D_{min}$ ,  $D_{max}$  respectively representing the exponent, lower and upper  
 271 bound of the power law in eq. 22, where the upper bound  $D_{max}$  is deduced  
 272 from the model presented in section 2. In eq. 23,  $\mu_{ln}$  and  $\sigma_{ln}$  denote the log-  
 273 arithmic mean and variance of the lognormal law while  $\lambda$  and  $\beta$  respectively  
 274 represent the scale and shape parameters of the Weibull law in eq. 24.

275 The power law, Weibull and lognormal laws have analytical expressions  
 276 for their (volume based) moments:

$$D_{ij,p}^v = \left[ \frac{\gamma + j + 1}{\gamma + i + 1} \frac{(D_{max}^{\gamma+i+1} - D_{min}^{\gamma+i+1})}{(D_{max}^{\gamma+j+1} - D_{min}^{\gamma+j+1})} \right]^{\frac{1}{i-j}} \quad (25)$$

$$D_{ij,ln}^v = \exp(\mu_{ln}) \left[ \exp\left(\frac{\sigma_{ln}^2}{2}(i+j)\right) \right]^{\frac{1}{i-j}} \quad (26)$$

$$D_{ij,wb}^v = \lambda \left[ \left( \frac{\Gamma(1 + \frac{i}{k})}{\Gamma(1 + \frac{j}{k})} \right) \right]^{\frac{1}{i-j}} \quad (27)$$

Noting that volume and number distributions are linked as  $p_v(D)dD \sim p_n(D)D^3dD$ , their respective diameter moments may be related as:

$$D_{i-3j-3}^v = D_{ij}^n \quad (28)$$

### 277 3.3. Parameter evaluation of the candidate fitting distributions

278 The most accurate fits to the present fragment volume distribution data  
 279 were obtained when using the current strain rate estimate to model the Sauter  
 280 mean and mean volume diameters of the fragments first and then computing  
 281 the associated distribution parameters. This is because no clear correlation  
 282 of the parameters driving the shapes of the distributions, namely  $\gamma$ ,  $\sigma_{ln}$  and  
 283  $\beta$ , could be found with respect to the impact parameters. On the contrary,  
 284 the Sauter mean and mean volume diameters of the fragments appear to  
 285 correlate rather well with the current strain estimate (based on constant  
 286 material properties), as may be seen from figs. 4 and 5.

287 While the regression curves indicate a somewhat poorer correlation com-  
 288 pared to the maximum diameter, reasonably accurate fits to these quantities  
 289 may be proposed for the data of Reitter *et al.* [27] and Vargas *et al.* [9]. On

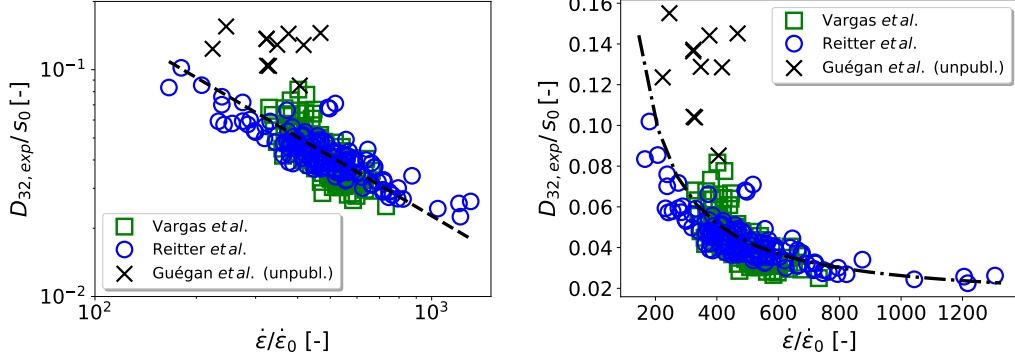


Figure 4: Left: regression curve ( $\ln(D_{32}^n/s_0) \approx -0.872 \ln(\dot{\epsilon}/\dot{\epsilon}_0) + 2.237$ ) displayed as a dashed black line on a log-log scale of the Sauter mean diameter over the strain rate estimate given by eq. 12. Right: Evolution of the Sauter mean diameter over the strain rate estimate and proposed fit (dash-dotted black line).

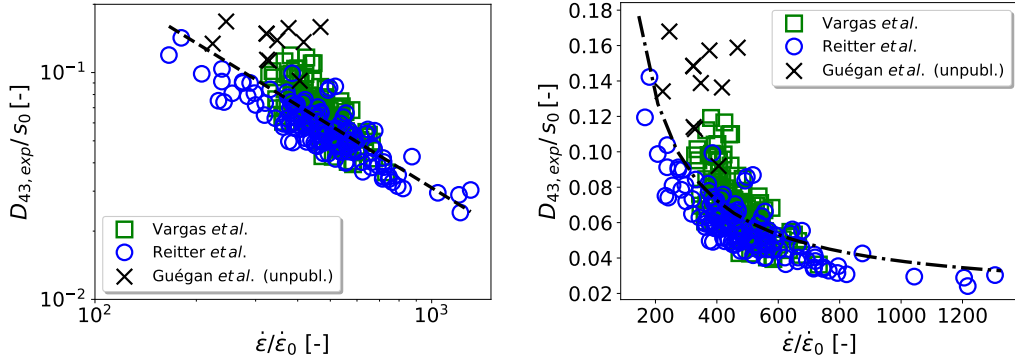


Figure 5: Left: regression curve ( $\ln(D_{43}^n/s_0) \approx -0.912 \ln(\dot{\epsilon}/\dot{\epsilon}_0) + 2.831$ ) displayed as a dashed black line on a log-log scale of the volume mean diameter over the strain rate estimate given by eq. 12. Right: Evolution of the volume mean diameter over the strain rate estimate and proposed fit (dash-dotted black line).

Experiments	$D_{32}^n$ average fitting error [%]	$D_{43}^n$ average fitting error [%]
Reitter <i>et al.</i> [27]	13.87	20.42
Vargas <i>et al.</i> [9]	14.89	14.54
Guégan <i>et al.</i> [12]	35.09	48.17

Table 3: Average errors on the Sauter mean diameter  $D_{32}$  and volume weighted mean diameter  $D_{43}$  for the different experiments using the fits given in eqs. 29 and 30

290 the contrary, poor agreement is observed for the data of Guégan *et al.* [12]  
291 which displays significantly larger values for both the Sauter mean diameter  
292 and mean volume diameter in comparison to the data of Reitter *et al.* [27]  
293 and Vargas *et al.* [9] for comparable strain rates. It is interesting to note  
294 that the regression slopes obtained for these quantities, respectively  $-0.872$   
295 for the Sauter mean diameter and  $-0.912$  for the mean volume diameter,  
296 are close to the value obtained for the maximum diameter, i.e.  $\alpha \approx -0.896$   
297 for constant material properties. In addition, an asymptotic behavior seems  
298 visible for these quantities at the largest strain rates, such that a constant is  
299 added to the current fits. In order to further minimize the average error of  
300 the fits, it is chosen not to constrain their exponents, yielding:

$$\frac{D_{32}^n}{s_0} = C_{f,32} \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^{\alpha_{32}} + a_{0,32} \quad (29)$$

$$\frac{D_{43}^n}{s_0} = C_{f,43} \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^{\alpha_{43}} + a_{0,43} \quad (30)$$

301 with respectively,  $C_{f,32} \approx 58.682$ ,  $\alpha_{32} \approx -1.266$ ,  $a_{0,32} \approx 0.020$  and  $C_{f,43} =$   
302  $29.997$ ,  $\alpha_{43} = -1.051$ ,  $a_{0,43} \approx 0.171$ . The average error levels with respect to  
303 the experimental values of these parameters are provided in table 3.

304 The parameters of the lognormal and Weibull distributions are separable



305 in eqs. 26 and 27 and may thus be directly determined from relations 29 and  
 306 30. On the contrary, the parameters of the power law fit  $D_{min}$  and  $\gamma$  are not  
 307 separable in eq. 25 and the dedicated method used to determine the latter is  
 308 presented below. Note that all distributions are renormalized when necessary  
 309 to prevent the prediction of fragment volume beyond the maximum diameter  
 310 estimate presented in section 2.

### 311 3.4. Determination of the power law exponent

Regression fits on a log-log scale may produce inaccurate estimates of  
 power law exponents. For this reason, Clauset *et al.* [35] proposed the fol-  
 lowing maximum likelihood estimator to determine both the exponent  $\gamma$  and  
 minimum diameter  $D_{min}$  of powerlaws more accurately:

$$\gamma = 1 + n \left[ \sum_{i=1}^n \ln \left( \frac{D_i}{D_{min}} \right) \right]^{-1} \quad (31)$$

312 where  $D_i$  and  $n$  represent the diameter of the  $i$ -th fragment and the total  
 313 number of fragments, respectively. The value of  $D_{min}$  is chosen so as to mini-  
 314 mize the Kolmogorov Smirnov statistic with respect to the input dataset [35].  
 315 In the current work, the dedicated python package [36] *power law* is used to  
 316 evaluate eq. 31.

317 Unfortunately, the number of fragment size samples provided by Reitter  
 318 *et al.* [27] for each of their impact experiments (about 200) appears insuffi-  
 319 cient to apply the methodology described above. On the contrary, consistent  
 320 results could be obtained for the experiments of Vargas *et al.* [9] as their data  
 321 comprises about 2000 samples per impact. Finally, the use of the statisti-  
 322 cal tools of Clauset *et al.* [35] requires access to the raw fragment size data,  
 323 which is not available for the experiments of Guégan *et al.* [12]. Therefore,

324 this method may only be applied for the experiments of Vargas *et al.* [9],  
325 while a less precise regression approach must be employed for the datasets  
326 of Reitter *et al.* [27] and Guégan *et al.* [12].

327 The uncertainty on power law exponent estimates obtained from regres-  
328 sions on a log-log scale may be reduced by fitting cumulated volume dis-  
329 tributions as they generally display lower statistical fluctuations [35, 36]. In  
330 addition, a regression method adapted to the presence of outliers and im-  
331 plemented in python’s scikit-learn library and known as ‘RANdom SAMple  
332 Consensus’ (RANSAC) is used [37]. The use of such method appears nec-  
333 essary as the fragment volume data does not follow a power law for the  
334 smallest fragment sizes, possibly due to resolution limitations inherent to  
335 such experiments.

336 The exponent of the cumulated volume distribution function  $\delta = \gamma + 1$   
337 deduced for the different experiments is shown in fig 6. Considering the  
338 data of Vargas *et al.* [9], it appears that either estimates of the exponent,  
339 namely the RANSAC regressor [37] or the maximum likelihood estimator of  
340 Clauset *et al.* [35], yield very similar results. Since each impact experiment  
341 was repeated about 10 times in the experiments of Vargas *et al.* [9], an  
342 average over the repetitions could be performed to evaluate the exponent  $\gamma$ .  
343 This probably explains the low dispersion for the data of Vargas *et al.* [9] as  
344 the exponents cluster within the range [1.1 – 1.4]. More generally, no clear  
345 trend regarding the evolution of the exponent with normal impact velocity  
346 seems visible for any of the data sets and the same remark holds for the  
347 impact diameter (not shown). Therefore, an average value is used for the  
348 power law exponent, yielding  $\delta \approx 1.27$ , close to the value reported for glass

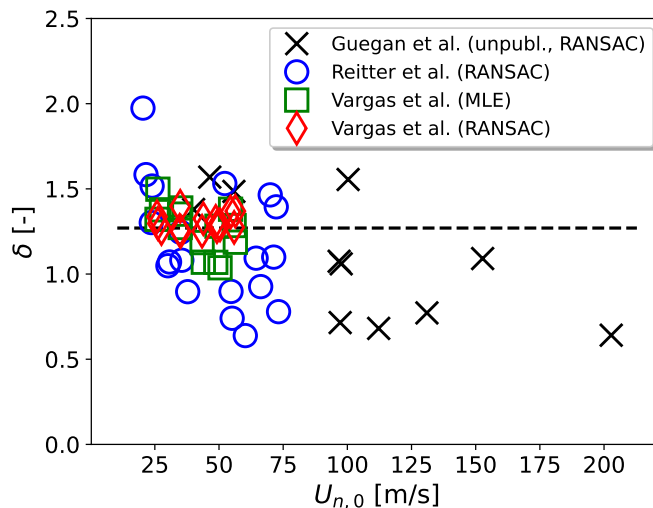


Figure 6: Exponent of the cumulated volume size distribution as a function of the normal impact velocity for the different experimental datasets. For the experimental data of Vargas *et al.* [9], results are computed with both the maximum likelihood estimator of Clauset *et al.* [35] and the RANSAC regressor [37].

349 fragmentation in Kolsky torsion bar experiments [38]. Similarly, the ratio  
 350 between the minimum diameter and the initial diameter is approximately  
 351 constant on average,  $D_{min}/D_0 \approx 0.052$ .

### 352 3.5. Fit evaluation

353 The quality of the proposed fits is illustrated by comparison to experi-  
 354 mental cumulated volume distributions for moderate, intermediate and large  
 355 impact velocities in figs. 7, 8 and 9. The data respectively correspond to  
 356 distributions obtained from the experiments of Reitter *et al.* [27] (moder-  
 357 ate impact velocity), Vargas *et al.* [9] (intermediate) and Guégan *et al.* [12]  
 358 (large). In each case, distributions obtained for two almost identical impact

359 conditions are shown to illustrate their variability. Finally, the cumulated  
 360 volume distributions predicted by the model of Pan and Render [10] are  
 361 shown for comparison. Pan and Render [10] assume the fragment volume  
 362 distribution to follow a Weibull distribution, see eq. 24. They express the  
 363 parameters of their Weibull distribution as:

$$\frac{\lambda^{pr}}{D_0} = 0.3 - \left(\frac{U_{n,0}}{a_L}\right)^2 \quad (32)$$

$$\beta^{pr} = 3.512 - 4 \left(\frac{U_{n,0}}{a_L}\right) \quad (33)$$

364 with  $a_L = 340 \text{ m s}^{-1}$  a reference value for the speed of sound. The strict  
 365 positivity constraint on these parameters limits the validity of relations 32-  
 366 33 to  $U_{n,0} < 186 \text{ m s}^{-1}$  approximately.

367 The cumulated volume distributions are either found to be "s-shaped",  
 368 see fig. 7a, or "upward concave", see fig. 7b. The "s-shaped" may best be  
 369 reproduced with the lognormal law for the present fits. On the contrary, they  
 370 are not well reproduced by the current Weibull fit because of the truncation  
 371 performed at the predicted maximum fragment diameter. The fit proposed  
 372 by Pan and Render [10] predicts comparatively larger shape parameters and  
 373 results in cumulated volume distributions that are not truncated, hence "s-  
 374 shaped". Finally, the current power law fit may only reproduce "upward  
 375 concave" cumulated volume distributions. It appears from all figures that  
 376 there is significant variability in the distributions, apparently resulting in  
 377 different best fit candidates, even for almost identical conditions, see for  
 378 instance figs. 7a and 7b. Since the current fits are all normalized by the  
 379 current maximum fragment diameter estimate, they are inherently sensitive  
 380 to errors of the latter, as may be seen from fig. 9a.

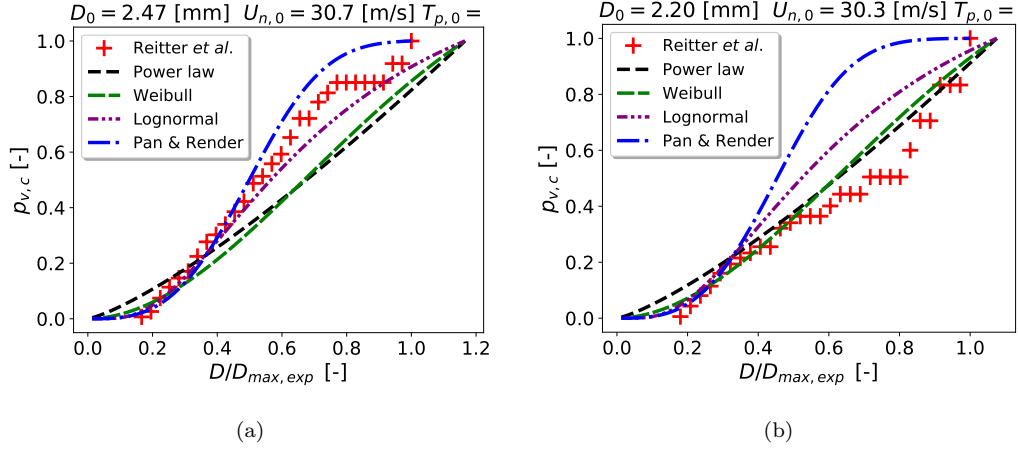


Figure 7: Experimental and model cumulated volume distributions from the experiments of Reitter *et al.* [27], comprising the power law, Weibull and lognormal fits of the present work as well as the Weibull law proposed by Pan and Render [10]. The impact parameters are provided above each plot.

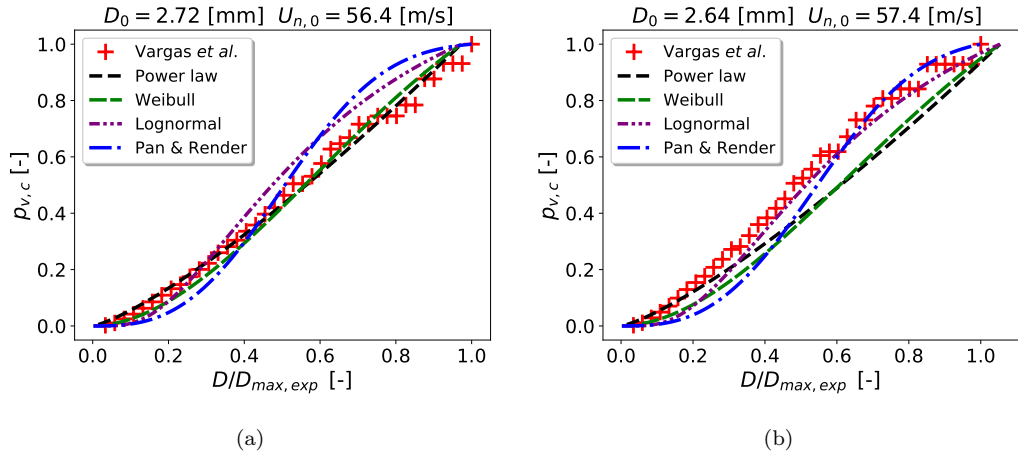


Figure 8: Experimental and model cumulated volume distributions from the experiments of Vargas *et al.* [9], comprising the power law, Weibull and lognormal fits of the present work as well as the Weibull law proposed by Pan and Render [10]. The impact parameters are provided above each plot.

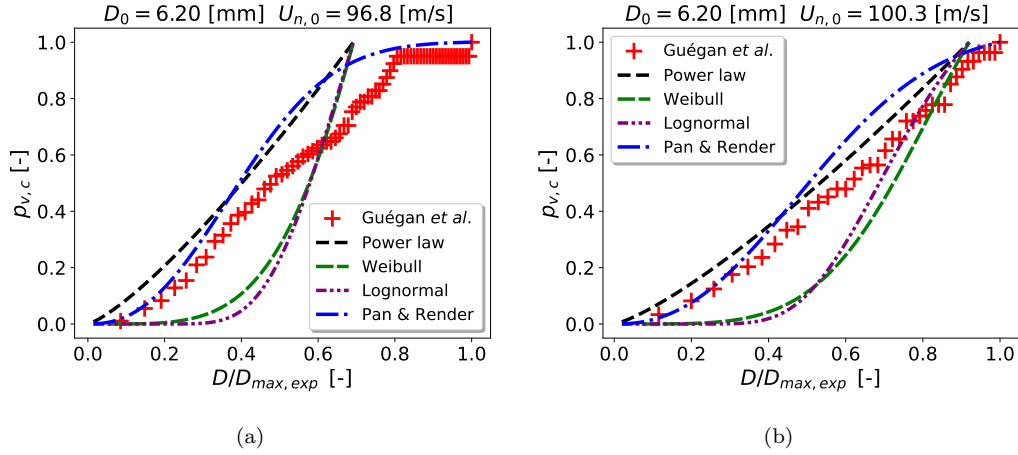


Figure 9: Experimental and model cumulated volume distributions from the experiments of Guégan *et al.* [12] (previously unpublished data), comprising the power law, Weibull and lognormal fits of the present work as well as the Weibull law proposed by Pan and Render [10]. The impact parameters are provided above each plot.

381 In order to provide a more quantitative analysis, the average errors for  
382 the Sauter mean diameter  $D_{32}^n$ , volume mean diameter  $D_{43}^n$  as well as the  $D_{10}^v$ ,  
383  $D_{50}^v$  and  $D_{90}^v$  diameters are given in tables 4, 5 and 6, respectively for the  
384 data of Reitter *et al.* [27], Vargas *et al.* [9] and Guégan *et al.* [12]. The trends  
385 are somewhat similar for the data of Reitter *et al.* [27], Vargas *et al.* [9],  
386 both in terms of error levels and candidate fits. For these experiments, the  
387 current Weibull fit seems the best approximation to the fragment volume  
388 data. While this behavior is expected for the  $D_{32}^n$  and  $D_{43}^n$  diameters given  
389 the fitting strategy exposed in section 3.2, the low error levels observed on  
390 the  $D_{10}^v$ ,  $D_{50}^v$  and  $D_{90}^v$  diameters confirm its ability to reproduce the experi-  
391 mental distributions with good accuracy. These conclusions do not apply for  
392 the data of Guégan *et al.* [12]. First, the fitting strategy proposed for the  
393 lognormal and Weibull laws suffers from the inability to accurately predict

394 the experimental  $D_{32}^n$  and  $D_{43}^n$  diameters for the data of Guégan *et al.* [12]  
395 using relations 29 and 30. The associated error levels may then be either mit-  
396 igated or further enhanced given the maximum diameter estimate used for  
397 normalization. The lognormal and Weibull laws predict excessively "upward  
398 concave" shapes for the data of Guégan *et al.* [12], resulting in huge errors on  
399 the  $D_{10}^v$ , while recovering satisfactory error levels for the remaining quanti-  
400 ties. More generally, no clear trend in favor of any distribution seems visible  
401 for the data of Guégan *et al.* [12]. Because of a much more simple design,  
402 namely a maximum diameter estimate that seems accurate on average and  
403 a constant exponent obtained via averaging the entire size distribution data,  
404 the power law appears as the most robust distribution over the three exper-  
405 imental datasets, at the cost of larger error levels, in particular on the  $D_{32}^n$   
406 diameter. While the Weibull fit of Pan and Render [10] also shows remarkable  
407 accuracy for the data of Reitter *et al.* [27] and Vargas *et al.* [9], it may not  
408 be extrapolated towards the largest normal impact velocities representative  
409 of engine ice crystal icing.

Experiments of Reitter <i>et al.</i> [27]	Average errors [%]				
	$D_{32}^n$	$D_{43}^n$	$D_{10}^v$	$D_{50}^v$	$D_{90}^v$
Pan and Render [10], Weibull law	16.07	19.71	22.52	18.61	20.95
Current, power law	30.78	24.95	26.49	17.45	17.03
Current, Weibull law	<b>12.62</b>	<b>12.14</b>	19.84	17.08	16.31
Current, lognormal law	14.05	13.39	<b>19.22</b>	<b>13.27</b>	<b>14.38</b>

Table 4: Average errors on the Sauter mean diameter  $D_{32}^n$ , volume weighted mean diameter  $D_{43}^n$ , the  $D_{10}^v$ ,  $D_{50}^v$  and  $D_{90}^v$  diameters for the model of Pan and Render [10] as well as the present power, Weibull and lognormal laws for the experimental data of Reitter *et al.* [27]. The minimum average error in each column is displayed in bold.

Experiments of Vargas <i>et al.</i> [9]	Average errors [%]				
	$D_{32}^n$	$D_{43}^n$	$D_{10}^v$	$D_{50}^v$	$D_{90}^v$
Pan and Render [10], Weibull law	15.68	28.45	37.02	24.37	28.48
Current, power law	32.06	<b>15.03</b>	<b>16.89</b>	13.93	<b>15.07</b>
Current, Weibull law	<b>12.22</b>	17.02	22.46	<b>13.91</b>	15.59
Current, lognormal law	13.16	23.67	20.01	22.53	17.66

Table 5: Average errors on the Sauter mean diameter  $D_{32}^n$ , volume weighted mean diameter  $D_{43}^n$ , the  $D_{10}^v$ ,  $D_{50}^v$  and  $D_{90}^v$  diameters for the model of Pan and Render [10] as well as the present power, Weibull and lognormal laws for the experimental data of Vargas *et al.* [9]. The minimum average error in each column is displayed in bold.



Experiments of Guégan <i>et al.</i> [12]	Average errors [%]				
	$D_{32}^n$	$D_{43}^n$	$D_{10}^v$	$D_{50}^v$	$D_{90}^v$
Pan and Render [10], Weibull law	56.82	46.85	<b>34.05</b>	26.92	21.52
Current, power law	37.87	29.33	42.55	<b>18.21</b>	17.59
Current, Weibull law	20.07	<b>21.64</b>	91.16	21.85	14.96
Current, lognormal law	<b>17.87</b>	23.30	105.89	18.52	<b>14.80</b>

Table 6: Average errors on the Sauter mean diameter  $D_{32}^n$ , volume weighted mean diameter  $D_{43}^n$ , the  $D_{10}^v$ ,  $D_{50}^v$  and  $D_{90}^v$  diameters for the present fits as well as the model of Pan and Render [10] for the experimental data of Guégan *et al.* [12]. The minimum average error in each column is displayed in bold.

#### 410 4. Conclusion

411 The fragmentation of ice particles resulting from their impact on solid  
412 walls was examined in an attempt to model size characteristics of the reemitted  
413 fragments. First, a new semi-empirical model predicting the diameter of the  
414 largest reemitted fragment was proposed. The strong compression result-  
415 ing from impact is assumed to remain limited to the vicinity of the contact  
416 area, while inducing increasing tensile strain elsewhere, resulting in crack  
417 formation and eventually fragmentation. Accordingly, an energy formalism  
418 considering fragmentation to be driven by the conversion of tensile and ki-  
419 netic energies into fracture surface energy was applied. The tensile strain  
420 causing fragmentation was related to the indentation radius in the contact  
421 area assuming plastic deformation. The resulting model may incorporate  
422 strain rate and temperature dependent compressive yield strength as well  
423 as a temperature dependent surface fracture energy. Model predictions ap-  
424 pear in good agreement with all considered experimental data, thus covering  
425 broad normal impact velocity ( $[20\text{-}200\text{ m s}^{-1}]$ ) and initial particle diameter  
426 ( $[30\text{ }\mu\text{m}\text{-}20\text{ mm}]$ ) ranges.

427 Modeling of the fragment volume distribution was then considered. In  
428 particular, the respective accuracy of power, lognormal and Weibull laws  
429 was evaluated to determine the best fitting candidate to the present frag-  
430 ment volume data. For the lognormal and Weibull laws, the fitting strategy  
431 is based on an estimate of the Sauter mean diameter and the mean volume  
432 diameters using the previously derived strain rate estimate. For the power  
433 law, a maximum likelihood estimator was used to determine its exponent  
434 when possible. Alternatively, a regression method adapted to the presence

435 of outliers on a log-log scale was applied. The current Weibull fit seems best  
436 suited to approximate the impact data up to moderate impact velocities, i.e.  
437 up to  $80 \text{ m s}^{-1}$ . However, this fit yields huge errors in the low diameter range  
438 of the fragment volume distributions at higher normal impact velocities. In  
439 particular, average errors on the  $D_{10}^v$  diameter are unacceptably large for the  
440 current Weibull and lognormal fits. This is mainly due to the fact that the  
441 current Sauter mean and mean volume diameter estimates, which the current  
442 Weibull and lognormal fits are based on, exhibit large errors when extrapo-  
443 lated to higher impact velocities. Due to these uncertainties, a simple power  
444 law fit based on a constant exponent seems to yield higher yet more sta-  
445 ble average error levels over the entire available fragment volume data. The  
446 fragment volume distribution fit proposed by Pan and Render [10] performs  
447 almost as well as the current Weibull fit for the moderate impact velocity  
448 fragment data. As for the current fits, the accuracy of Pan and Render's fit  
449 deteriorates when considering fragment distribution data at higher impact  
450 velocities, in particular for the Sauter mean and volume mean diameters.  
451 Moreover, the fragment volume distribution model of Pan and Render may  
452 not cover the entire normal impact velocity range representative of ice crys-  
453 tal icing ( $50\text{-}250 \text{ m s}^{-1}$  approximately) as it loses validity for normal impact  
454 velocities exceeding  $186 \text{ m s}^{-1}$ .

455       Considering the outlooks, it seems worthwhile to further confirm the main  
456 underlying assumption of a link between the indentation radius and the ten-  
457 sile strain magnitude causing fragmentation. Detailed numerical simulations  
458 of ice particle impact seem best suited for such purpose. Regarding the frag-  
459 ment volume distributions, it seems important to gather additional fragment

460 data for high velocity impacts to assess whether the resulting volume distri-  
461 butions follow significantly different trends. More generally, a better char-  
462 acterization and understanding of the statistical variability of experimental  
463 impact data with respect to impact parameters seems crucial to improve  
464 modeling accuracy.

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### 470 **Appendix A. Accounting for varying material properties**

The energy per unit area required to create new fracture surface  $G_c$  is assumed to depend on the material temperature [15]:

$$G_c = G_{c,0} \exp\left(\frac{Q_s}{\mathcal{R}T} - \frac{Q_s}{\mathcal{R}T_0}\right) \quad (\text{A.1})$$

471 with  $G_{c,0} = 0.12 \text{ J m}^{-2}$  representing the reference surface energy per unit  
472 area [22],  $T_0 = 253 \text{ K}$  the reference temperature,  $Q_s = 4.82 \times 10^4 \text{ J mol}^{-1}$   
473 the reference activation energy [39] and  $\mathcal{R} = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$  the universal  
474 molar gas constant.

Regarding the compressive yield strength, experimental evidence suggests that it depends on both strain rate and temperature [13, 21]. Regarding strain rate dependency, a polynomial fit to the average curve given by Tippmann [13] and gathering experimental measurements from several sources

is used. The temperature dependency of the quasi-static compressive yield strength of ice is described with the experimental data compiled by Petrovic [21]. It appears that a simple linear fit yields a satisfactory accuracy for the latter quantity. The resulting expression reads:

$$Y_c(T, \dot{\varepsilon}) = Y_{c,0} f_{y_c}(\dot{\varepsilon}) g_{y_c}(T) \quad (\text{A.2})$$

475 with:

$$f_{y_c}(\dot{\varepsilon}) = \exp \left( 3.595 \times 10^{-4} \ln^3(\dot{\varepsilon}/\dot{\varepsilon}_0) - 7.470 \times 10^{-3} \ln^2(\dot{\varepsilon}/\dot{\varepsilon}_0) + 1.118 \times 10^{-1} \ln(\dot{\varepsilon}/\dot{\varepsilon}_0) + 1.062 \right) \quad (\text{A.3})$$

$$g_{y_c}(T) = -28.70 (T/T_{c,0}) + 30.21 \quad (\text{A.4})$$

476 with  $Y_{c,0} = 5.2 \times 10^6$  Pa representing a reference value for the quasi-static  
 477 compressive yield strength at  $T_{c,0} = 273$  K [13, 21]. The maximum error re-  
 478 mains below 2% for the strain rate dependency with respect to the numerized  
 479 data of Tippmann [13] and below 0.5% for the temperature dependency with  
 480 respect to the numerized data of Petrovic [21]. Note that the use of eq. A.2  
 481 requires an iterative procedure to determine the characteristic strain rate in  
 482 eq. 12.

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