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Reliable Unknown Input Observer for Continuous-Time Linear Systems

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Abstract: This work proposes a set-valued extension of the classical unknown input observers for linear continuous-time systems without using set-by-step set-membership computations or imposing the positivity property of the dynamics of the estimation error. In fact, based on an explicit set-integration method of the estimation error, the framing and convergence properties of the proposed set-valued unknown input observer are demonstrated. Moreover, thanks to the proposed design approach, robust threshold on the residual can be established *a priori*. Simulation results are reported to highlight the effectiveness of the proposed state estimator in the presence of unknown inputs.

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Keywords: State estimation, continuous-time linear systems, unknown input observer, interval observer, set-membership estimation, reachability analysis.

1. INTRODUCTION

A crucial issue in model-based fault diagnosis is the generation of reliable residual signals that indicate the presence of faults in the systems. In the literature, several approaches have been proposed to deal with this problem. For instance, one can mention factorization methods for residual generation, parity vector methods, online parameters identification approaches and observer-based approaches, the interested reader can refer to (Chen and Patton, 1999) and the references therein. The core idea behind the observer approaches consists in estimating the outputs of the system from the available data. Then, the output estimation error can be considered as a residual. In this case, the estimation of the state vector is worthless. Thus, as a matter of fact, the use of functional observers is advisable. However, for fault detection and isolation issue the use of unknown input observer is more suitable. By definition, the estimation error of unknown input observer has to converge towards zero regardless of the presence of unknown disturbances in the system. In other words, the estimation error is decoupled from the unknown inputs. This fact allows one to generate robust residual signals. In the literature, necessary and sufficient conditions are provided to show the existence of unknown input observers and a variety of synthesis methods have been proposed for both setting: deterministic (Saif, 1998; Takahashi and Peres, 1999; Valcher, 1999; Chen and Patton, 1999; Darouach et al., 1994; Hou and Müller, 1992) and stochastic (Darouach et al., 2003; Hou and Patton, 1998; Saberi et al., 2000).

Notice, in the case where the representation of the system is affected by bounded uncertainties, like additive modeling errors, process and measurement noises set-valued methods (Alamo et al., 2005; Le et al., 2013; Ben Chabane et al., 2014; Combastel, 2015; Raïssi et al., 2012; Efimov and Raïssi, 2016; Mazenc et al., 2014; Cacace et al., 2015; Meslem et al., 2018; Tang et al., 2019; Meslem and Ramdani, 2020; Meslem et al., 2020b) offer an interesting alternative to design robust unknown input observers. This promising research topic has been already tackled in the literature. For example, the concept of interval observer is extended to the case of unknown input observers in (Robinson et al., 2017; Meyer et al., 2018). In (Robinson et al., 2017), a reduced-order unknown input interval observer is proposed while in (Meyer et al., 2018) a full-order unknown input interval observer is introduced. For discrete-time systems, zonotopic computation rules (Shephard, 1974; Kühn, 1998; Combastel, 2003) are applied in (Xu et al., 2016) to compute step-by-step outer approximations of the actual state vector of the system regardless the presence of the unknown inputs. Recently, in (Meslem et al., 2020a), interval analysis (Alefeld and Mayer, 2000; Jaulin et al., 2001) is applied to design interval versions of full-order and reduced-order unknown input observers (Chen and Patton, 1999) and (Hou and Müller, 1992). These extensions are mainly based on the explicit reachability method for linear discrete-time systems introduced in (Meslem et al., 2018).

In the present contribution, a modified version of the set-integration method developed in (Meslem et al., 2020b) is used to design set-valued unknown input observer for continuous-time systems. This allows one to get: (*i*) Guaranteed bounds on the actual state vector of the system ; (*ii*) Reliable thresholds of the residual signals that are insensitive to both initial state estimation errors and unknown inputs.

The remaining parts of this paper are organized as follows. Section 2 recalls briefly the classical structure of unknown input observes for linear continuous-time systems. Section 3 presents a simple method to compute upper and lower bounds of the range of a given box by linear transformation. Then, the proposed set-valued unknown input observer is introduced in

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Section 4. Two numerical examples are studied in Section 5 to show the effectiveness of the proposed method. Conclusion and some perspectives are given in Section 6.

2. UNKNOWN INPUT OBSERVER

This section provides a straightforward and concise introduction on the design of unknown input observers (Patton et al., 1989; Hou and Müller, 1992; Maquin et al., 1994) for linear continuous-time systems. This class of dynamical systems is described by:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Rd(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ stands for the state vector, $y(t) \in \mathbb{R}^{n_y}$ represents the output vector, $u(t) \in \mathbb{R}^{n_u}$ is the input vector and $d(t) \in \mathbb{R}^{n_d}$ stands for the unknown input vector with $n_y \geq n_d$. The system matrices A, B, C, and R are of appropriate dimensions. Notice that, the matrices C and R are assumed to be a full row rank matrix and a full column rank matrix, respectively.

By definition, unknown input observers have to estimate online the state vector of (1) despite the presence of completely unknown inputs gathered in vector $d(t)$.

2.1 Conditions and design procedure

The existence of unknown input observers for system (1) is shown if the following necessary and sufficient conditions (Chen and Patton, 1999) are satisfied:

- $rank(CR) = rank(R)$
- (C, A_1) is a detectable pair

where,

$$A_1 = A - R(CR)^\star CA \tag{2}$$

and $(CR)^\star$ stands for the pseudo-inverse matrix of CR. Thus, the linear system described by

$$\begin{aligned} \dot{z}(t) &= Nz(t) + Mu(t) + Ly(t) \\ \hat{x}(t) &= z(t) + Ky(t) \end{aligned} \tag{3}$$

is an unknown input observer for system (1). Moreover, a simple procedure to compute all the matrices involved in (3) is itemized as follows:

- $K = R(CR)^\star$, $M = (I_n - KC)B$
- Choose desired eigenvalues for the matrix N
- Compute L_1 by pole assignment techniques
- $N = A_1 - L_1 C$, $L = L_1 + NK$

That is, thanks to this design procedure, one can affirm that the output vector $\hat{x}(t)$ of (3) is converging towards the actual state vector of system (1) and the dynamics of the estimation error $e(t) = x(t) - \hat{x}(t)$ is stable. More formally, one has: $\forall \|e(t_0)\| \neq 0$, $\|e(t)\|$ is bounded and

$$\lim_{t \to +\infty} \|e(t)\| = 0 \tag{4}$$

3. BOUNDS ON THE RANGE OF A LINEAR TRANSFORMATION

Let us denote by $\boldsymbol{x}$ a given box in $\mathbb{R}^n$ that can be defined as follows:

$$\boldsymbol{x} := \{x \in \mathbb{R}^n \mid x_l \leq x \leq x_u \;\; x_l,\, x_u \in \mathbb{R}^n\} \tag{5}$$

where the real vectors x_l and x_u are its upper and lower bound, respectively. The operator $\leq$ in (5) should be understood component-wise. Now, let denote by $\mathscr{P}$ the range of a linear transformation of the box $\boldsymbol{x}$. That is,

$$\mathscr{P} := \{p = Hx : x \in \boldsymbol{x}\} \tag{6}$$

where H is a real matrix of dimension $m \times n$. Lower and upper bounds on the polytope $\mathscr{P}$ can be determined as suggested in the following lemma.

Lemma 1. (Efimov and Raïssi, 2016) The bounds of the smallest box that contains the polytope defined in (6) are computed by:

$$\begin{aligned} p_u &= H^+ x_u - H^- x_l \\ p_l &= H^+ x_l - H^- x_u \end{aligned} \tag{7}$$

where H^+ and H^- are two element-wise non-negative matrices computed by the following formulas:

$$\begin{aligned} H^+ &= \max\{0_{m,n}, H\} \\ H^- &= H^+ - H \end{aligned} \tag{8}$$

Notice, in (8), $0_{m,n}$ stands for a zero matrix of dimension $m \times n$ and the max operator is applied element-wise. Thus, one can claim that for all $p \in \mathscr{P}$ one has:

$$p_l \leq p \leq p_u \tag{9}$$

It is worth pointing out that the result of this lemma plays an important role in the extension of the point-valued unknown input observer (3) to the set-valued case.

4. RELIABLE UNKNOWN INPUT OBSERVER

In this section, we will show how guaranteed bounds on the estimation error can be determined in the case where system (1) is affected by both state disturbance and measurement noises. That is, the case of systems described by:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Ew(t) + Rd(t) \\ y(t) &= Cx(t) + Fv(t) \end{aligned} \tag{10}$$

where, $w(t) \in \mathbb{R}^{n_w}$, $v(t) \in \mathbb{R}^{n_v}$ and its time derivative $\dot{v}(t)$ are assumed to be *unknown-but-bounded* vectors that represent the state disturbances, measurement noises and their time derivatives, respectively. Notice that, by *unknown-but-bounded* vectors we mean that:

- There exist bounded sets $\mathscr{W}$, $\mathscr{V}$ and $\mathscr{Z}$ such that,

$$\forall t \geq t_0, \begin{cases} w(t) \in \mathscr{W} \\ v(t) \in \mathscr{V} \\ \dot{v}(t) \in \mathscr{Z} \end{cases} \tag{11}$$

- There exists a bounded set $\mathscr{X}_0$ that contains the initial state $x(t_0)$ of (10). That is,

$$x(t_0) \in \mathscr{X}_0 \tag{12}$$

The matrices E and F are assumed constant with appropriate dimensions.

By direct computation one can show that, in this case, the estimation error is governed by the following differential equation

$$\dot{e}(t) = Ne(t) + Gr(t) \tag{13}$$

where

$$G = \begin{bmatrix} (I_n - KC)E & -L_1 F & -KF \end{bmatrix} \tag{14}$$

and

$$r(t) = \begin{pmatrix} w(t) \\ v(t) \\ \dot{v}(t) \end{pmatrix} \in \begin{pmatrix} \mathscr{W} \\ \mathscr{V} \\ \mathscr{Z} \end{pmatrix} = \mathscr{R} \tag{15}$$

Thus, the objective is to compute upper and lower bounds ($e_u(t)$ and $e_l(t)$, respectively) of the estimation error such that the following properties are satisfied:

- Framing: $\forall x_0 \in \mathscr{X}_0$ and $t \geq t_0$,
$$\hat{x}(t) + e_l(t) \leq x(t) \leq \hat{x}(t) + e_u(t)$$
- Convergence:
$$\lim_{t\to+\infty} \|e_u(t) - e_l(t)\|_\infty \leq \delta$$
where δ is a non-negative constant that depends on the size of the set $\mathscr{R}$.

4.1 Set-integration method

To achieve the above mentioned goal, we extend and improve in this work the set-integration method introduced in (Meslem et al., 2020b). This allows one to apply it in future work for solving certain fault diagnosis problem based on generalized observer scheme.

Let us denote by r_u and r_l upper and lower bounds of the set $\mathscr{R}$. That is,
$$\forall r(t) \in \mathscr{R},\ r_l \leq r(t) \leq r_u \tag{16}$$
On the other hand, let us define the set of all the possible initial estimation error by
$$\mathscr{E}_0 = \mathscr{X}_0 - \hat{x}(t_0)$$
where $\hat{x}(t_0)$ should be picked from the set $\mathscr{X}_0$. Since $\mathscr{X}_0$ is a bounded set, $\mathscr{E}_0$ is bounded too. Then, let us denote by $e_u(t_0)$ and $e_u(t_0)$ two real vectors that satisfy
$$\forall e(t_0) \in \mathscr{E}_0,\ e_l(t_0) \leq e(t_0) \leq e_u(t_0) \tag{17}$$
Moreover, we define by $\Psi_N(t,t_0)$ and $\Psi_{N,G}(t,t_0)$ the following real matrices
$$\Psi_N(t,t_0) = e^{N(t-t_0)} \tag{18}$$
$$\Psi_{N,G}(t,t_0) = e^{N(t-t_0)}G \tag{19}$$
Proposition 1. The outputs of the following bounding system
$$\begin{cases} \dot{s}_u(t) &= \Psi^+_{N,G}(t,t_0)r_u - \Psi^-_{N,G}(t,t_0)r_l \\ \dot{s}_l(t) &= \Psi^+_{N,G}(t,t_0)r_l - \Psi^-_{N,G}(t,t_0)r_u \\ e_u(t) &= s_u(t) + \Psi^+_N(t,t_0)e_u(t_0) - \Psi^-_N(t,t_0)e_l(t_0) \\ e_l(t) &= s_l(t) + \Psi^+_N(t,t_0)e_l(t_0) - \Psi^-_N(t,t_0)e_u(t_0) \end{cases} \tag{20}$$
with initial conditions $s_u(t_0) = s_l(t_0) = 0$, provide a guaranteed enclosure for all the possible estimation error solutions to (13). That is,
$$\forall e(t_0) \in \mathscr{E}_0,\ e_l(t) \leq e(t) \leq e_u(t) \tag{21}$$
Furthermore, since N is Hurwitz stable by construction, we have
$$\lim_{t\to+\infty} \|e_u(t) - e_l(t)\|_\infty \leq \delta \tag{22}$$
Proof 1. The proof of this proposition follows up the same guidelines used in (Meslem et al., 2020b). □

Remark 1. Notice, unlike the reachability method introduced in (Meslem et al., 2020b) where the bounding Lemma 1 is applied twice, the proposed set integration method applies this lemma only once. This allows reducing conservatism linked to the over-approximation of a polytope by an axis-aligned box.

Remark 2. It is worth pointing out that system (20) can be solved numerically offline. Thus, the bounds of the estimation error can be known a priori.

4.2 Robust unknown input observer

Now, thanks to the set-integration method introduced in Proposition 1, a robust unknown input observer (set-valued observer) will be designed to provide reliable estimations of the real state vector of system (10) in the presence of unknown inputs and unknown-but-bounded uncertainties.

Proposition 2. The unknown input observer (3) combined with the reliable generator of the bounds of the estimation error (20), provide a guaranteed enclosure of the actual state of (10) despite the presence of state and measurements disturbances and unknown inputs. That is, $\forall d(t) \in \mathbb{R}^{n_d}$, $\forall w(t) \in \mathscr{W}$, $v(t) \in \mathscr{V}$ and $x(t_0) \in \mathscr{X}_0$, one has:
$$\forall t \geq t_0,\ \hat{x}(t) + e_l(t) \leq x(t) \leq \hat{x}(t) + e_u(t) \tag{23}$$
and
$$\forall i \in \{1,\dots n\},\ \lim_{t\to+\infty} \left(e_{u,i}(t) - e_{l,i}(t)\right) \leq \delta_i \tag{24}$$
where δ_i are nonnegative constants and $e_{h,i}(t)$ denotes the *ith* element of the vector $e_h(t)$.

Proof 2. Note that, the framing property (23) is a direct consequence of (21) and the convergence property (24) is straightforwardly deduced form (22). □

Remark 3. In order to improve the tightness of the computed state enclosure (23), that is to get smaller values for the constants δ_i, it is recommended:

- As far as possible, consider some bounded inputs in (10) as unknown inputs
- Apply nonlinear optimization methods to compute the observer gain L_1 that minimizing the width of estimation error box.

4.3 Generating robust set-membership thresholds

Unknown input observers play an important role in the design of diagnostic algorithms. In fact, by analyzing the residual signals generated by a bundle of these observers, fault detection and localization strategies are established. By definition a residual is the different between the measured and estimated outputs of the system,
$$e_r(t) = y(t) - \hat{y}(t) \tag{25}$$
and alarms are generated at each time these residuals exceed user-defined thresholds $\varepsilon(t)$. That is, when the following condition is violated:
$$\|e_r(t)\|_2 \leq \varepsilon(t) \tag{26}$$
In practice, finding the right thresholds is not an easy task due to the uncertainties that affecting the available models of physical systems and the sensitivity of the residuals to the transient regime of the estimation error. To overcome these issues, based on the proposed set-integration method, we propose the following reliable set-membership test,
$$\varepsilon_l(t) \leq e_r(t) \leq \varepsilon_u(t) \tag{27}$$
where
$$\begin{aligned} \varepsilon_u(t) &= C^+ e_u(t) - C^- e_l(t) \\ \varepsilon_l(t) &= C^+ e_l(t) - C^- e_u(t) \end{aligned} \tag{28}$$
That is, as long as the residual signals stays inside the wrap defined by the robust upper and lower thresholds $\varepsilon_u(t)$ and $\varepsilon_l(t)$, the behavior of the system is safe and no alarm has to be activated.

5. ILLUSTRATIVE EXAMPLES

Two numerical examples borrowed from the literature are studied is this section to show the merit of the proposed method. Two experiments are performed with the first example to highlight the advantages of the introduced method, while the second example is used to compare the performance of our method with that of a selected interval observer-based method.

5.1 Example 1

In this subsection the effectiveness of the proposed reliable unknown input observer is shown on a numerical example. The considered system is borrowed from (Chen and Patton, 1999), where the system matrices are defined as follows:

$$A = \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & -1 \end{pmatrix}, \; B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix},$$

$$E = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \; R = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}, \tag{29}$$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \; F = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The bounded set of initial conditions $x(t_0)$ is a box described by,

$$x(t_0) \in \mathscr{X}_0 = \begin{pmatrix} [-1,\, 5] \\ [-1,\, 5] \\ [-1,\, 5] \end{pmatrix} \tag{30}$$

In this example, the considered state disturbance is an unknown but bounded nonlinear term,

$$\forall t \geq t_0, \; w(t) = 0.1 \sin\big(x_2(t)\big) \in \mathscr{W} = [-0.1,\, 0.1] \tag{31}$$

The measurement noises are assumed to be bounded and belong to the bounded box,

$$\forall t \geq t_0, \; v(t) \in \mathscr{V} = \begin{pmatrix} [-0.01,\, 0.01] \\ [-0.01,\, 0.01] \end{pmatrix} \tag{32}$$

with bounded time derivatives. That is,

$$\forall t \geq t_0, \; \dot{v}(t) \in \mathscr{Z} = \begin{pmatrix} [-0.1,\, 0.1] \\ [-0.1,\, 0.1] \end{pmatrix} \tag{33}$$

On the other hand, the simulated unknown input $d(t)$ is defined as follows:

$$d(t) = \begin{cases} 5 \text{ if } t \in [5,7] \\ 0 \text{ otherwise} \end{cases} \tag{34}$$

and the system input is given by

$$u(t) = 10 \sin(20t) \tag{35}$$

For the simulation purpose, the applied measurement noises vector is expressed by:

$$v(t) = \begin{pmatrix} 0.01 \cos(10t) \\ 0.01 \sin(10t) \end{pmatrix} \tag{36}$$

First experiment For this example we have $rank(CR) = rand(R)$ and the pair (C, A_1) is observable. Thus, Proposition 2 can be applied to compute a guaranteed enclosure of the state vector of the system in the presence of unknown input (34). The chosen eigenvalues for the matrix N are

$$\lambda_1 = -1, \; \lambda_2 = -2, \; \lambda_2 = -3 \tag{37}$$

Thus, all the matrices involved in (3) and (20) are given by

$$N = \begin{pmatrix} -2.0039 & 0 & 0.0481 \\ -0.2412 & 0 & 3 \\ 0.1608 & -1 & -3.9961 \end{pmatrix}, \; M = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix},$$

$$L = \begin{pmatrix} 0 & -0.0481 \\ -1 & -3 \\ 0 & 2.9961 \end{pmatrix}, \; K = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{38}$$

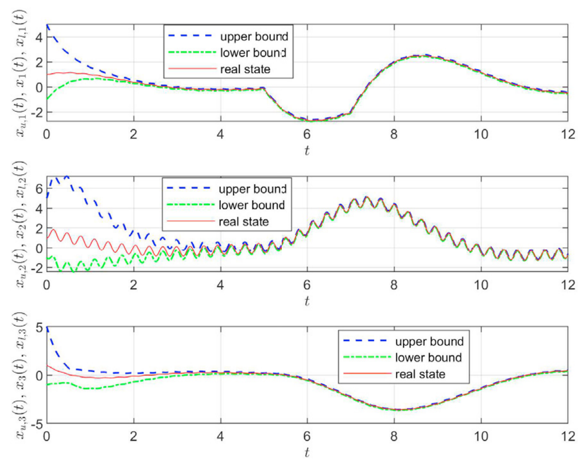

Fig. 1. Guaranteed enclosure of the actual state trajectory.

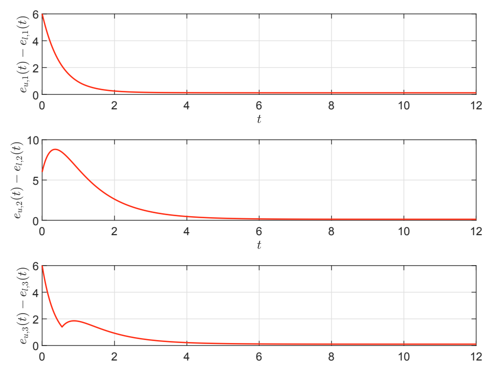

Fig. 2. The width of the estimated state enclosure.

The simulation results are presented in Figures 1 and 2. Figure 1 shows the estimated intervals for each state variable that contain the actual state trajectory of the system plotted in solid lines. Recall that, the upper and lower bounds of the state vector depicted by the dashed lines, in Figure 1, are computed by $x_u(t) = \hat{x}(t) + e_u(t)$ and by $x_l(t) = \hat{x}(t) + e_l(t)$, respectively. The convergence of the width of the estimated state enclosure is illustrated in Figure 2, were the entries of the width vector

$$e(t) = e_u(t) - e_l(t) \tag{39}$$

are plotted. As shown in this figure, after the transient regime this width vector converges towards the steady state

$$e^T = (0.122, 0.125, 0.115)$$

which is determined graphically offline. On the other hand, as observed in Figure 1, despite the presence of the unknown input between the time instants $t = 5$ and $t = 7$ the proposed state estimator keeps providing a guaranteed enclosure of the real state vector of the system. Moreover, the curves in Figure 2 confirm that the width of the estimated enclosure is not impacted by the presence of this unknown input.

Second experiment As suggested in Remark 3, to increase the accuracy of the estimated state enclosure (23) it is possible to deal with some bounded inputs in (10) as unknown inputs. In this second experiment, we consider the nonlinear term (31) as a second unknown input. Thus, the new matrices R and E are defined by $R = (E \; R)$ and $E = 0$. In this case, we still have $rank(CR) = rank(R)$ and the pair (C, A_1) is detectable. Thus,

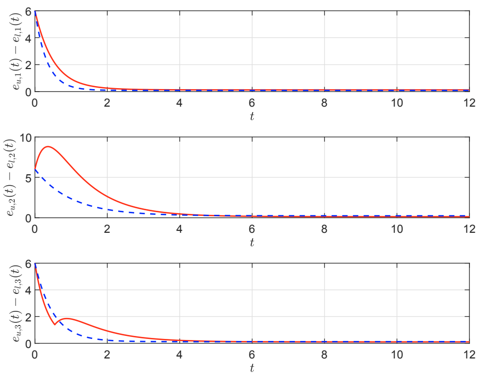

Fig. 3. The width of the estimation error in both cases: Solid lines the show results of the first experiment while the dashed lines show those of the second experiment.

Proposition 2 is still applicable. Now, for the same eigenvalues defined in (37), we get the following matrices involved in (3) and (20):

$$N = \begin{pmatrix} -3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \; M = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix},$$
$$L = \begin{pmatrix} 0 & 0 \\ -1 & 0 \\ 0 & 0 \end{pmatrix}, \; K = \begin{pmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix} \tag{40}$$

To show the effectiveness of this way of dealing with some bounded uncertainties, the width vector (39) is used as key performance indicator. For both experiments the width vector is plotted in Figure 3. Solid lines represent the performance indicator of the first experiment while the dashed lines correspond to that of the second experiment. As observed in this figure, overall the width vector obtained in the second experiment is lower than those of the first experiment, especially at the transient regime. That means, the estimated state enclosure in the second experiment is tighter than that obtained in the first experiment. Thus, we can conclude that whenever it is possible, we should consider the bounded state disturbances as unknown inputs in order to better manage their impact on the accuracy of the estimated state enclosure.

5.2 Example 2

In this second example we consider a 5th-order lateral axis model of a fixed-wing aircraft at cruise flight conditions borrowed from (Gucik-Derigny et al., 2014). The system matrices are:

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & -0.154 & -0.0042 & 1.54 & 0 \\ 0 & 0.249 & -1 & -5.2 & 0 \\ 0.0386 & -0.996 & -0.0003 & -0.117 & 0 \\ 0 & 0.5 & 0 & 0 & -0.5 \end{pmatrix}, \; B = 0$$
$$R = \begin{pmatrix} 0 & 0 \\ -0.744 & -0.032 \\ 0.337 & -1.12 \\ 0.02 & 0 \\ 0 & 0 \end{pmatrix}, \; C = \begin{pmatrix} 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{41}$$

and E, F are identity matrices of appropriate dimensions. The considered initial state vectors are:

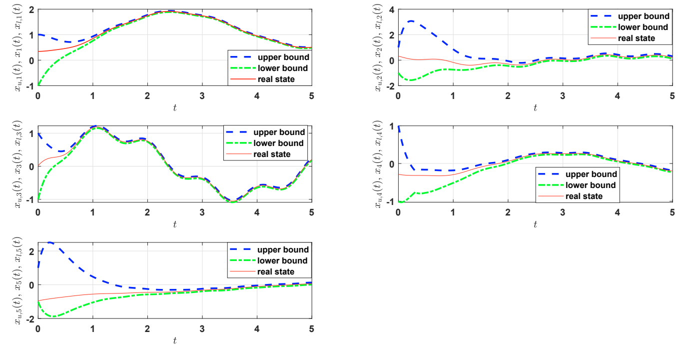

Fig. 4. Set-valued estimation of the actual state trajectory of the system.

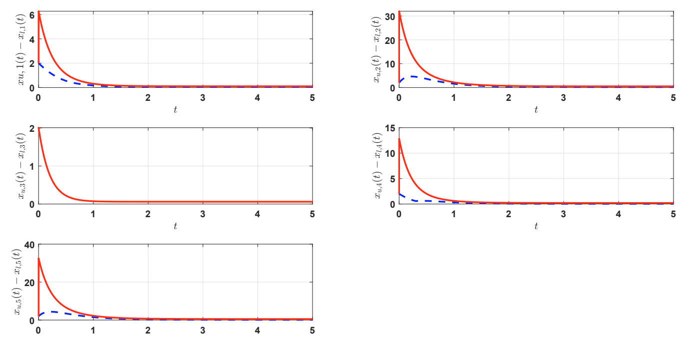

Fig. 5. The element-wise widths of the estimated state enclosure. Dashed lines correspond to the results of the proposed method while the solid ones are those of the interval observer method (Gucik-Derigny et al., 2014).

$$x(0) = (0.342, 0.32, 0.0178, -0.287, -0.9497)^T$$

and $\hat{x}(0) = (0,0,0,0,0)^T$. Moreover, the endpoints of the initial state box are known $x_u(0) = -x_l(0) = (1,1,1,1,1)^T$. The state disturbances, measurement noises and their time derivatives are unknown but belong in bounded boxes: $\forall t \geq 0$,

$$\begin{aligned} w(t) &\in [\underline{w}, \overline{w}], \text{ with } \overline{w} = -\underline{w} = 1e^{-2}(1,1,1,1,1)^T \\ v(t) &\in [\underline{v}, \overline{v}], \text{ with } \overline{v} = -\underline{v} = 1e^{-2}(1,1,1,1)^T \\ \dot{v}(t) &\in [\underline{\dot{v}}, \overline{\dot{v}}], \text{ with } \overline{\dot{v}} = -\underline{\dot{v}} = 1e^{-1}(1,1,1,1)^T \end{aligned}$$

Note that, in the simulation scenario $w(t)$ is supposed to be uniformly distributed pseudo random vector and

$$v(t) = 1e^{-2}\big(\sin(10t), \cos(10t), \sin(10t), \cos(10t)\big)^T$$

On the other hand, the applied vector of unknown inputs is $d(t) = \big(\cos(2\pi t), \sin(2\pi t)\big)^T$. As proposed in (Gucik-Derigny et al., 2014), the chosen eigenvalues for the state matrix N are:

$$\lambda_1 = -5, \; \lambda_2 = -6, \; \lambda_3 = -4, \; \lambda_4 = -3, \; \lambda_5 = -2$$

Figure 4 shows the efficiency of the proposed state estimator to provide a tight enclosure of the actual state vector of the system despite the presence of completely unknown inputs. Moreover, in terms of accuracy, Figure 5 illustrates the superiority of the proposed approach with respect to that introduced in (Gucik-Derigny et al., 2014). As observed in this figure, except for the case of the third state variable where both methods provide similar result, the widths of the others enclosures of the state variables obtained by the proposed method are always strictly lower than those generated by the interval observer approach (Gucik-Derigny et al., 2014).

6. CONCLUSION

In this work, a converging set-integration method is applied to design set-valued unknown input observers for linear continuous-time systems subject to both unkown-but-ounded state disturbances and measurement noises. It is worth pointing out that without requiring extra assumptions, guaranteed upper and lower bounds on the estimation error can be computed a priori. Moreover, based on this method, robust thresholds that are insensitive to initial estimation error are proposed. This fact allows one to design, in future works, robust fault detection and isolation schemes.

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