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Intelligent supply chain management using Quantum

Shaun D'Souza

Abstract

Algorithms provide a solutions approach to problems in computer science. A problem can be tractable (solved in better than polynomial time – NP-complete) or intractable (E.g. exponential time). Recent advances in the area of Quantum research are taking fruition as the field of quantum computing. With broad applications in Artificial intelligence and Optimization – Retail is the future application for quantum. A subset of the algorithms are now orders of magnitude faster owing to the quantum speedup that would otherwise take years to solve on a classical computer. Some of these benefits come for free owing to the nature of quantum superposition and entanglement in a Quantum realm.

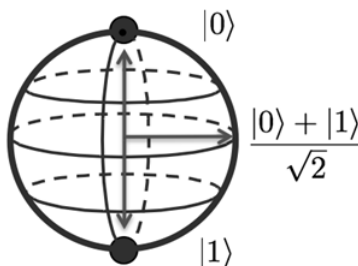


Figure 1: Qubit representation

Figure 1 shows the Bloch sphere corresponding to the state space of a qubit. It depicts a visual representation to denote quantum information.

1 Quantum will not replace classical computers

Quantum computing today lies at the intersection of computer science and quantum physics. It uses quantum laws at the algorithmic level. Classical algorithms function as Boolean logic.

With exponential growth in the internet, cloud resources have become pervasive. While these are effective at solving a large number of problems, they do not scale adequately on a set of problems such as NP hard or computationally intractable problems - Figure 2.

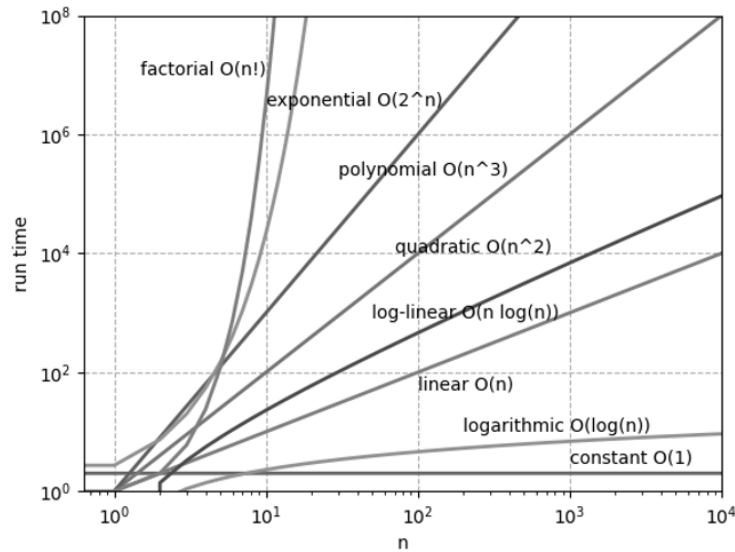


Figure 2: Algorithmic complexity

It is predicted that quantum systems will scale to >1000 qubits in the next decade opening up an array of algorithms in the areas of AI and optimization. Advanced use cases requiring higher number of qubits can be modeled on a simulator today. However, they run an order of magnitude slower as software is used simulate quantum properties. Algorithms translate into tangibles in source code. It is plausible that future quantum simulators could run on a quantum accelerator using vectors or tensors similar to the graphics and AI journey today. These could provide orders of magnitude speedup. See Appendix C.

	2019	2020	2021	2022	2023	2024+
Qubits	20	50	100	500	1000	1k - 1M
Simulator	500	1000	5000+			

Table 1: Quantum roadmap

Quantum computers exploit the properties of subatomic particles such as super position- ing and entanglement allowing us to create new algorithms for existing problems. Measuring a quantum particle through observation destroys its state for future use.

It would take a classical computer years to solve a maximum cut or traveling salesman problem for example using brute force techniques. Dynamic approaches to algorithms can alleviate complexity and tractability problems in solving an optimality similar to divide and conquer [1]. However, in the worst-case it could take exponential run time. These can be easily solved on today's quantum computers in no time. There are still fundamental constraints around the number of qubits available in the quantum cloud instances. Algorithms use a set of constraints to create a problem solution. Applications like spreadsheets, web browser, and email would continue to run in the classical paradigm. While specialized applications would run on a quantum computer.

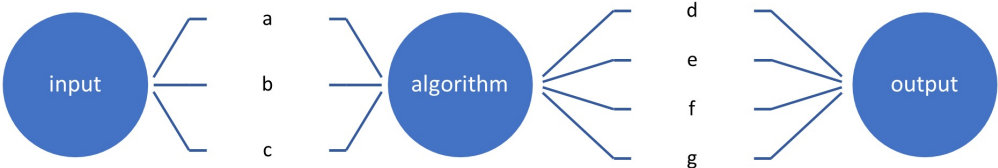


Figure 3: Dynamic programming decomposes a problem into subproblems

2 Supply chain optimization use case

Figure 4 shows a case study where an automobile manufacturer and distributor was looking to optimize their supply chains. The use case is especially relevant to Retail customers and can bring billions of dollars in savings in operational costs.

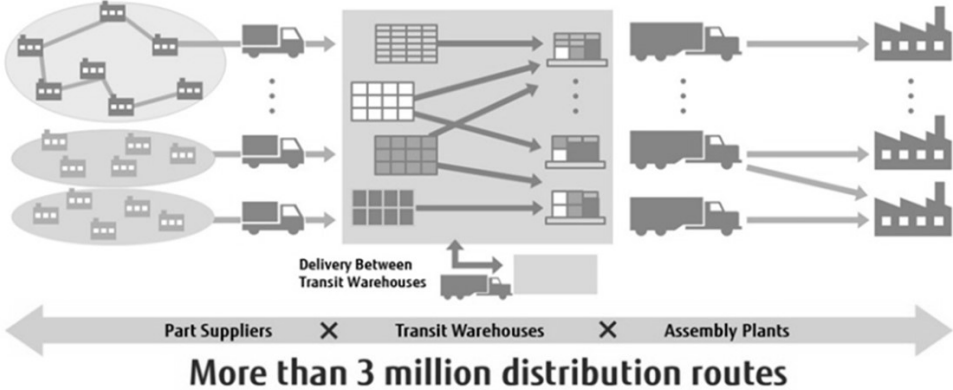


Figure 4: Optimization

Classical algorithms such as the Traveling Salesman problem have limitations owing to

their complexity. Today, these have been translated into their counterpart algorithms to run on a Quantum computer. With billions spent in the upkeep of a supply chain, even a small improvement of $\sim 1\%$ can result in millions in savings to a Retailer.

As the industry matures with ecosystem players, we will anticipate applications in optimization as well as AI and ML. Explicit and parallel programming can be cumbersome E.g. TensorFlow, Tensors. Abstractions are programmer friendly in the software paradigm E.g. Keras, OOP (Java, python), functional programming (SML). Libraries like Qiskit provide a viable API for a range of quantum applications [7]. Some applications include -

- Vehicle routing problem
- Bin packing problem
- Fleet management
- Distribution center and Inventory management

3 Fields of study

We can map some of the classical algorithms such as Linear programming or Quadratic programming to a quantum application as shown in Listing 1 – 4.

Listing 1: Quadratic programs

```
from qiskit_optimization import QuadraticProgram
quadratic_program = QuadraticProgram()
```

Listing 2: QUBO - Quadratic Unconstrained Binary Optimization

```
from qiskit_optimization.converters import QuadraticProgramToQubo
qp2qubo = QuadraticProgramToQubo()
```

Listing 3: QAOA - Quantum Approximate Optimization Algorithm [9]

```
from qiskit.algorithms import QAOA
qaoa = QAOA(optimizer = ADAM())
```

Listing 4: VQE - Variational Quantum Eigensolver

```
from qiskit.algorithms import VQE, NumPyMinimumEigensolver
vqe_optimizer = MinimumEigenOptimizer(vqe)
```

Artificial Intelligence is designed to mimic human tasks. While classical computers are compute & control bound. What that means is that a classical computer is good for general purpose programmability. The quantum computer lends itself to compute bound applications and is effective on tasks such as factorization or optimization.

Some of the areas of interest include:

- Quantum machine learning
- Quantum information science
- Quantum algorithms and software [12]

Selection tradeoffs in algorithms design include access to open-source code, pricing, licensing models, packaging, cloud deployment, security / reliability / uptime, proprietary ecosystems, legacy software, and phased lifecycles.

4 Quantum supply chain optimization

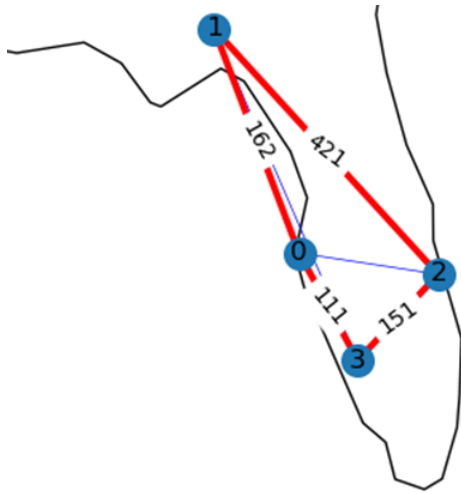
Today's clouds instances for quantum provide the user with both options of a quantum simulator and actual quantum bits. These could be used to devise algorithms for use cases in supply chain optimization allowing retailers to pass on cost savings to the consumer [10].

The traveling salesman problem (TSP) is an example of an NP-complete problem. We can express the problem using a directed graph:

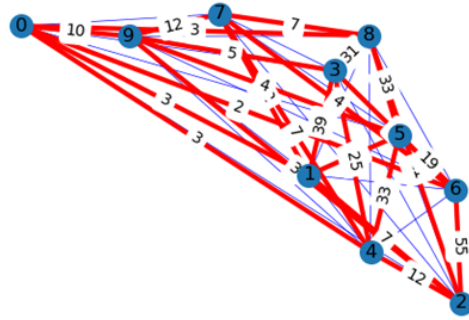
$$G = (V, E) \text{ where } V \in \text{set of vertices}$$
$$E \in \text{set of edges}$$

The goal is to discover the shortest distance over which the salesman can visit all the cities in the graph and return to his home location. With n vertices in a graph, the number of paths grow quadratically as n^2 and complexity $n!$ making it NP-hard. Even a small graph with 10 cities would take 100 qubits to solve on a quantum computer. This could be solved today on a classical computer for less than 50 cities, however, becomes intractable for larger number of nodes. As quantum computers improve in the next decade - Table 1, larger number of qubit machines would be able to take on these complex problem challenges with ease.

In this section we look at the distribution network of a leading medical retailer with stores in the regional USA. The figure shows the proposed network solution solved using a quantum algorithm for the TSP and Maximum cut (NP-hard). A classical algorithm would



(a) Traveling salesman problem ($n = 4$)



(b) Maximum cut ($n = 10$)

Figure 5: Quantum solution objective (a graph with 5–10 vertices and up to 40 edges)

we able to address up to 20 stores using today’s platforms. Figure 5 a / b shows the path using a quantum solution. A combination of quantum and unsupervised learning (clustering) could be used for more sophisticated use cases to approximate store locations or distribution centers in the vicinity.

Optimal route planning enables us to bring about operational efficiencies in the end-to-end Supply chain. Future work include quantum neural networks, quantum machine learning, and quantum programming languages.

Part II

Quantum Retail applications

In Part 2 of our edition on Quantum and Retail we continue to look at how Quantum will drive retail applications. Read Part 1 on the Intelligent supply chain. While Quantum computing will not replace classical applications, it will enable key functions around Quantum machine learning and Optimization for retail customers. The pandemic has disrupted consumer behavior and retailers have had to adapt to newer dynamics and customer trends.

Quantum instances are today available in the cloud by some of the leading vendors like IBM, Microsoft (Azure Quantum) and Google [4]. It is forecast that quantum systems will scale to > 1000 qubits in the next decade opening up an array of algorithms in the areas of

AI and optimization. Advanced use cases requiring higher number of qubits can be modeled on a quantum simulator today.

Quantum computers outperform their classical counterparts on tasks that are algorithmically intractable (solved in exponential time or NP-hard). Retailers are grappling with rising number of stores, revenues, routes, volumes, transport capacity, and utilization constraints [13]. Quantum will enhance business deliverables in an ever-expanding and complex SCM ecosystem. We explore a realm where quantum meets retail.

5 Industry perspectives on Quantum

Following Artificial Intelligence, Quantum is the next iteration in intelligent computing. It sits along other technologies such as the distributed ledger (cryptocurrency and blockchain) and extended reality (metaverse). These are collectively referred to as DARQ.

- **Human-machine.** AI and software are today ubiquitous. In the conventional paradigm parallel programming, concurrency, and synchronization are used to achieve speedup goals. Algorithms translate into tangibles in source code. Quantum will leap us into the Fifth industrial revolution with AI, blockchain, and data analytics being the Fourth industrial revolution [3].
- **NP-completeness.** A subset of the algorithms are now orders of magnitude faster owing to the nature of quantum superposition and entanglement. Some of these benefits come for free owing to the nature of quantum particles.

6 Platforms / algorithms as an abstraction

Some vendors for Quantum computing programming languages are IBM (Qiskit), Rigetti (PyQuil), Google (Cirq), and Microsoft (Q#). Quantum computing today lies at the intersection of computer science and quantum physics. It uses quantum laws at the algorithmic level. Classical algorithms function as Boolean logic. Developers will continue to use the same tools, languages, and code in a quantum paradigm - Figure 6.

Windows was the hot application for the classical computer. Retail is the next hot application for the quantum computer! Quantum applications will deliver Retail value creation through numerous venues. Some of these are:

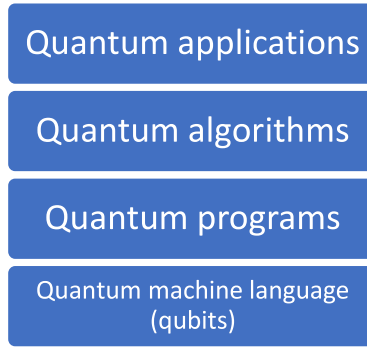


Figure 6: Quantum programming languages taxonomy

7 Vehicle routing problem

Vehicle routing problem (VRP) is a variation of the Traveling salesman problem. These encompass:

- **Fleet management.** The routing of delivery vehicles, their loading and fill.

We can conceptualize the supply chain as a complex network of graphs. Nodes in the graph corresponding to stores or DCs and the edges as the links connecting these entities. The goal is to minimize the travel distance and overlapping routes and/or additional constraints around capacity.

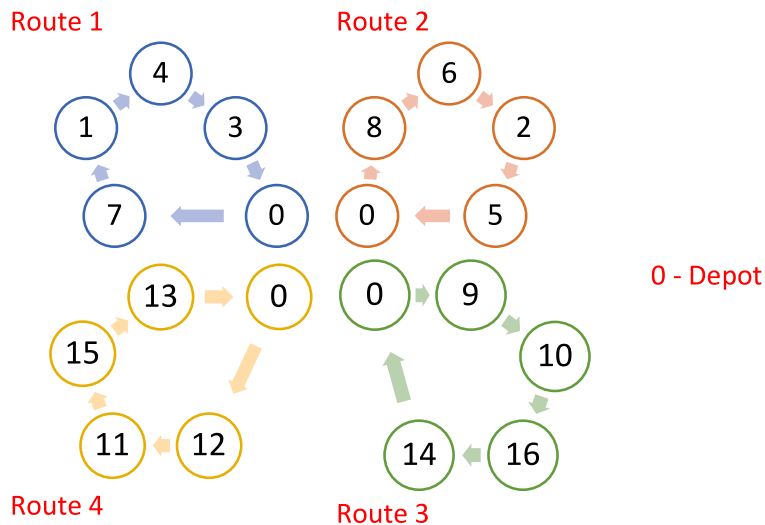


Figure 7: Vehicle routing problem

Variations of VRP include Capacitated vehicle routing problem (CVRP) and Pickup

and deliver problem (PDP). Optimal route planning can help in curbing emissions, energy consumptions and related costs through sustainable and green practices.

8 Bin packing problem

The bin packing or knapsack problem are variations of the optimal packing problem. And has applications in:

- **Scheduler planning.** Scheduling and planning are used in Retail applications such as distribution and packing. Some of the classical algorithms use Shortest path or Dynamic programming approaches to determine constraint optimality's.
- **Distribution center and Inventory management.** Applications include store arrangement or allocation of shelf space to determine optimal products to use in a promotional campaign.

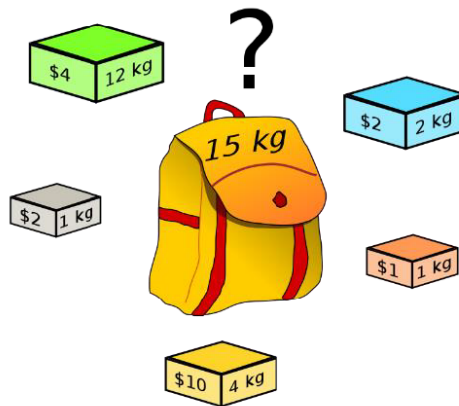


Figure 8: Knapsack problem

9 Supply chain forecasting

Retailers are looking to provide forecast guidance in applications such as demand forecasting, product end-of-life, and promotional forecasting. With ability to learn from time series and regression data Quantum models will enable future supply chain forecasting tasks.

- **Stochastic modeling.** AI/ML models learn outcomes from real data/parameters and business rules. These can be applied to supply chains, i.e., predicting store sales for a given calendar day based on historical sales data. Typically, such promotions are governed by retailer goals and constraints / business rules. The data points provided are typically evaluated in conjunction with business rules such as seasonality, promotional events, day of the week, and state or school holidays.

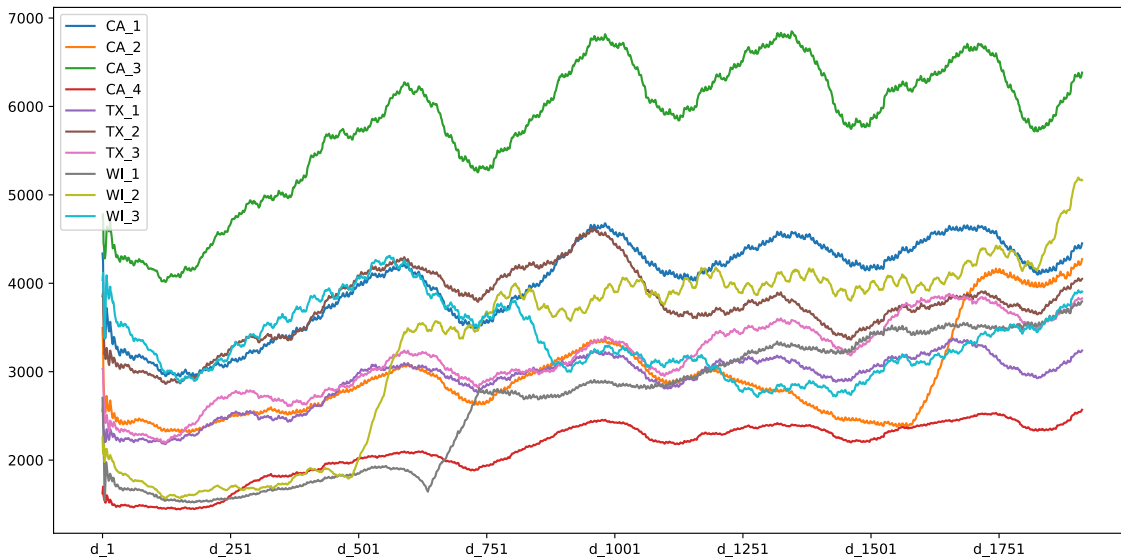


Figure 9: Retail forecasting - daily sales

10 Quantum feature maps

Quantum ML has large applications in kernel feature estimation [11]. These are traditionally done using classical approaches to reduce the feature size of a training dataset. Some techniques for dimensionality reduction are Principal Component analysis (PCA) and Linear Discriminant analysis (LDA). These are used to map higher dimensional datasets into a representable space to be used by a Machine learning algorithm - Figure 10. A novel feature space representation can be the difference in training a robust ML model vs. one with ambiguity. Use cases for text include the creation of Topic models to classify unstructured text [6].

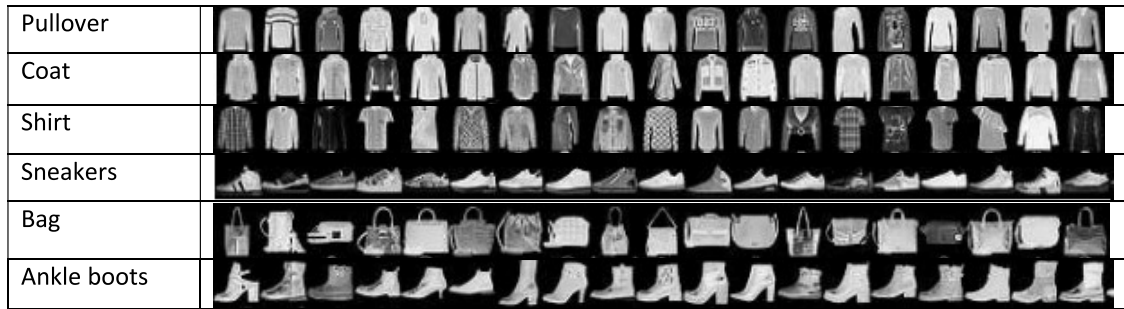


Figure 10: Fashion MNIST [2]

11 GPT-3 Quantum dataset

Datasets like the GPT-3 [5] could be trained using Quantum models. It is predicted that the answer to life and everything in the universe could be described in 5 trillion parameters from state of the art models such as the OpenAI GPT-3¹. Modern day applications use fewer parameters and can be trained with precision in the present generation of AI models. Data sets like the GPT-3 are used in a wide variety of tasks for natural language. Their deep learning models can produce human-like texts. With applications in text classification, translation and chatbots. However, these take long amounts of time to train with costs in the millions of dollars. GPT-3 with 175 billion parameters has been trained on 500 billion words at a cost of 5 million dollars. Future models can be trained using Quantum AI.

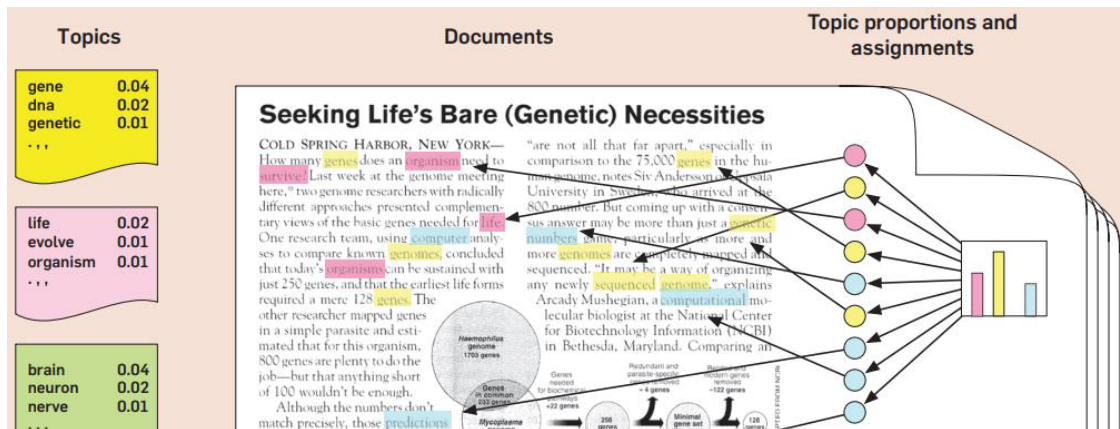


Figure 11: Topic modeling

¹GPT-3

12 Conclusions and Future work

Customers today are moving from brick-and-mortar stores to online in a globalized economy. Retailers are ramping inventory to cope with rising demand with increasing number of products (SKUs) and fulfillment channels. Current approaches in supply chain optimization look at a combination of greedy heuristics. These provide an adequate balance of business value and customer satisfaction. With rising number of stores, routes, transport capacity and utilization constraints, our goal is to further enhance business deliverables in an ever-expanding and complex SCM ecosystem. Note: Protocols like one-time passwords (OTP) and OAuth are the norm today in e-commerce transactions. Some research challenges include software, more algorithms (cryptography, randomness) and interdisciplinary collaboration (social), GDPR. Organizations would grow these functions organically as quantum applications become mainstream in business / entrepreneurial and government initiatives. IBM does have a foothold in this area with Qiskit. However, I am sure Microsoft (Azure Quantum), and Google would catch up here soon. It is the right time to invest into the quantum market [8].

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A Bra–ket notation

The state of a qubit can be described as –

$$|q\rangle = \alpha |0\rangle + \beta |1\rangle, \text{ where } \alpha, \beta \in C \text{ and } |\alpha|^2 + |\beta|^2 = 1$$

So that when a quantum particle is measured it would observe a $|0\rangle$ with probability $|\alpha|^2$ and $|1\rangle$ with probability $|\beta|^2$. Their corresponding vector notations are denoted by

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

These provide an orthogonal basis to represent the state of a quantum system. We can view the state of a quantum system as a complex linear superposition of its basis states $x_n \in 0, 1$

$$|\psi\rangle = \sum_{x \in \{0,1\}^n} \alpha_x |x_1, \dots, x_n\rangle \text{ where } \sum_x |\alpha_x|^2 = 1$$

This demonstrates a property of quantum particles. E.g. An n-qubit system can exist in any superposition of the 2^n basis states.

$$\alpha_0 |000000\rangle + \alpha_1 |000001\rangle + \dots + \alpha_{2^n-1} |111111\rangle$$

As per Heisenberg uncertainty principle, quantum physics limits how we can measure certain properties of a sub-atomic particle and its state such as position or momentum accurately.

B 4 points traveling salesman problem expressed as a quadratic optimization

Minimize

$$\begin{aligned} \text{obj: } [& 96 x_{0_0}x_{1_1} + 96 x_{0_0}x_{1_3} + 182 x_{0_0}x_{2_1} + 182 x_{0_0}x_{2_3} \\ & + 66 x_{0_0}x_{3_1} + 66 x_{0_0}x_{3_3} + 96 x_{0_1}x_{1_0} + 96 x_{0_1}x_{1_2} \\ & + 182 x_{0_1}x_{2_0} + 182 x_{0_1}x_{2_2} + 66 x_{0_1}x_{3_0} + 66 x_{0_1}x_{3_2} \\ & + 96 x_{0_2}x_{1_1} + 96 x_{0_2}x_{1_3} + 182 x_{0_2}x_{2_1} + 182 x_{0_2}x_{2_3} \\ & + 66 x_{0_2}x_{3_1} + 66 x_{0_2}x_{3_3} + 96 x_{0_3}x_{1_0} + 96 x_{0_3}x_{1_2} \\ & + 182 x_{0_3}x_{2_0} + 182 x_{0_3}x_{2_2} + 66 x_{0_3}x_{3_0} + 66 x_{0_3}x_{3_2} \\ & + 126 x_{1_0}x_{2_1} + 126 x_{1_0}x_{2_3} + 142 x_{1_0}x_{3_1} + 142 x_{1_0}x_{3_3} \\ & + 126 x_{1_1}x_{2_0} + 126 x_{1_1}x_{2_2} + 142 x_{1_1}x_{3_0} + 142 x_{1_1}x_{3_2} \\ & + 126 x_{1_2}x_{2_1} + 126 x_{1_2}x_{2_3} + 142 x_{1_2}x_{3_1} + 142 x_{1_2}x_{3_3} \\ & + 126 x_{1_3}x_{2_0} + 126 x_{1_3}x_{2_2} + 142 x_{1_3}x_{3_0} + 142 x_{1_3}x_{3_2} \\ & + 184 x_{2_0}x_{3_1} + 184 x_{2_0}x_{3_3} + 184 x_{2_1}x_{3_0} + 184 x_{2_1}x_{3_2} \\ & + 184 x_{2_2}x_{3_1} + 184 x_{2_2}x_{3_3} + 184 x_{2_3}x_{3_0} + 184 x_{2_3}x_{3_2} \\ &]/2 \end{aligned}$$

Subject To

$$\begin{aligned} \text{c0: } & x_{0_0} + x_{0_1} + x_{0_2} + x_{0_3} = 1 \\ \text{c1: } & x_{1_0} + x_{1_1} + x_{1_2} + x_{1_3} = 1 \\ \text{c2: } & x_{2_0} + x_{2_1} + x_{2_2} + x_{2_3} = 1 \\ \text{c3: } & x_{3_0} + x_{3_1} + x_{3_2} + x_{3_3} = 1 \\ \text{c4: } & x_{0_0} + x_{1_0} + x_{2_0} + x_{3_0} = 1 \\ \text{c5: } & x_{0_1} + x_{1_1} + x_{2_1} + x_{3_1} = 1 \\ \text{c6: } & x_{0_2} + x_{1_2} + x_{2_2} + x_{3_2} = 1 \\ \text{c7: } & x_{0_3} + x_{1_3} + x_{2_3} + x_{3_3} = 1 \end{aligned}$$

Bounds

$$\begin{aligned} 0 & \leq x_{0_0} \leq 1 \\ 0 & \leq x_{0_1} \leq 1 \\ 0 & \leq x_{0_2} \leq 1 \\ 0 & \leq x_{0_3} \leq 1 \\ 0 & \leq x_{1_0} \leq 1 \\ 0 & \leq x_{1_1} \leq 1 \\ 0 & \leq x_{1_2} \leq 1 \\ 0 & \leq x_{1_3} \leq 1 \end{aligned}$$

```
0 <= x_2_0 <= 1
0 <= x_2_1 <= 1
0 <= x_2_2 <= 1
0 <= x_2_3 <= 1
0 <= x_3_0 <= 1
0 <= x_3_1 <= 1
0 <= x_3_2 <= 1
0 <= x_3_3 <= 1
```

Binaries

```
x_0_0 x_0_1 x_0_2 x_0_3 x_1_0 x_1_1 x_1_2 x_1_3 x_2_0 x_2_1 x_2_2 x_2_3 x_3_0
x_3_1 x_3_2 x_3_3
```

End

C Qiskit Aer simulator backend supports graphic accelerator

```
from qiskit import Aer, transpile
Aer.backends()
[AerSimulator('aer_simulator'),
AerSimulator('aer_simulator_statevector_gpu'),
AerSimulator('aer_simulator_density_matrix_gpu'),
AerSimulator('aer_simulator_stabilizer'),
AerSimulator('aer_simulator_matrix_product_state'),
AerSimulator('aer_simulator_extended_stabilizer'),
AerSimulator('aer_simulator_unitary_gpu'),
AerSimulator('aer_simulator_superop'),
QasmSimulator('qasm_simulator'),
StatevectorSimulator('statevector_simulator'),
UnitarySimulator('unitary_simulator'),
PulseSimulator('pulse_simulator')]
```


D Classical solution objective

Appendix D shows a solution for a network with up to 20 cities but becomes NP-hard to solve classically for larger number of nodes. It is forecast that quantum computers with greater than 10's of thousands of qubits will be available in the coming decade. These would outperform today's computers and simulators providing linear speedup to some of our most complex network problems.

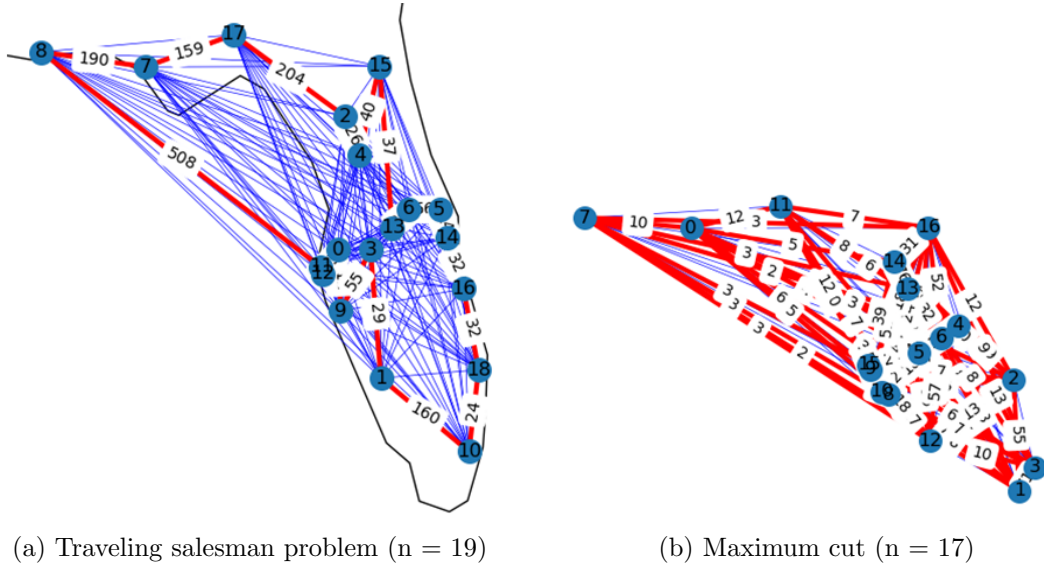


Figure 12: Classical solution objective (a graph with 20 vertices and up to 200 edges)

E Quantum Approximate Optimization Algorithm (QAOA) to solve the maximum cut problem

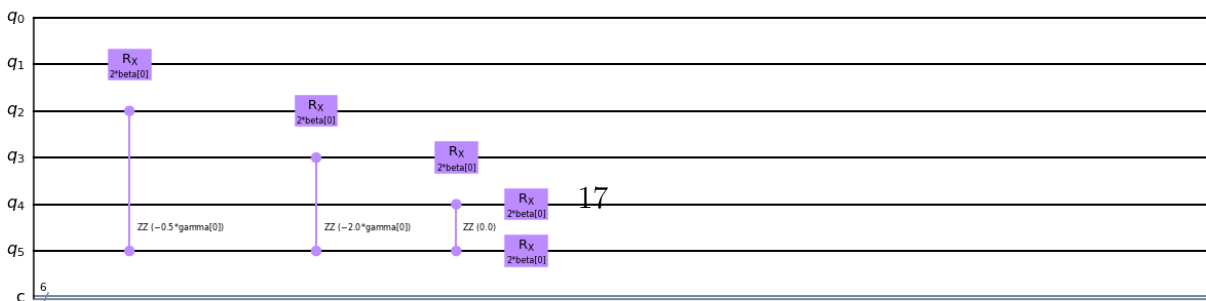
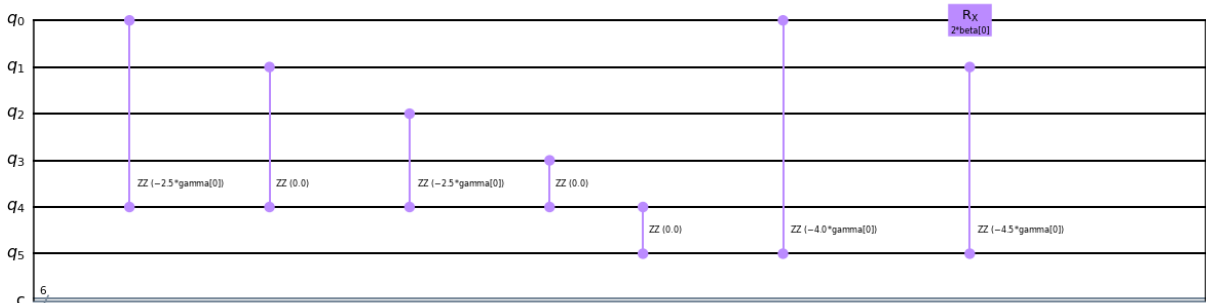
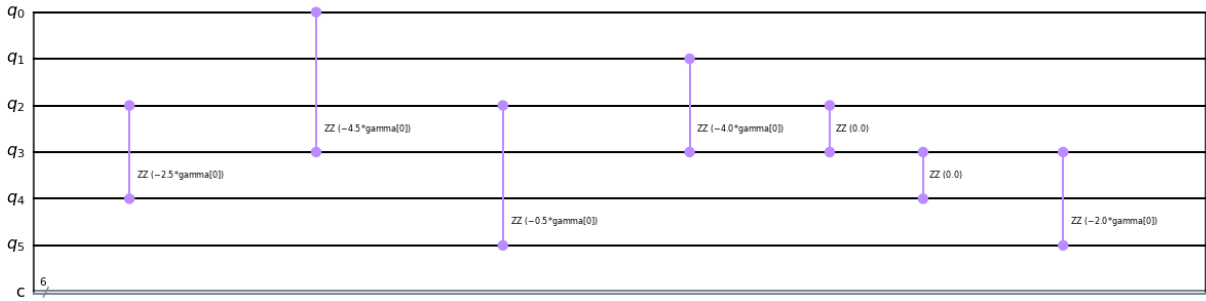
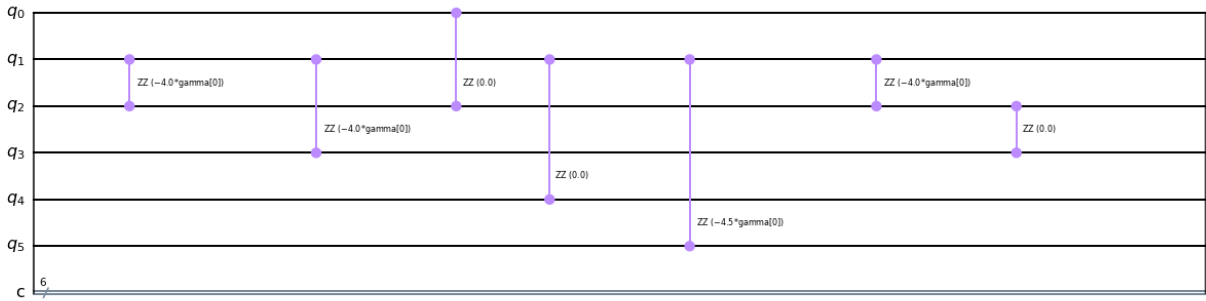
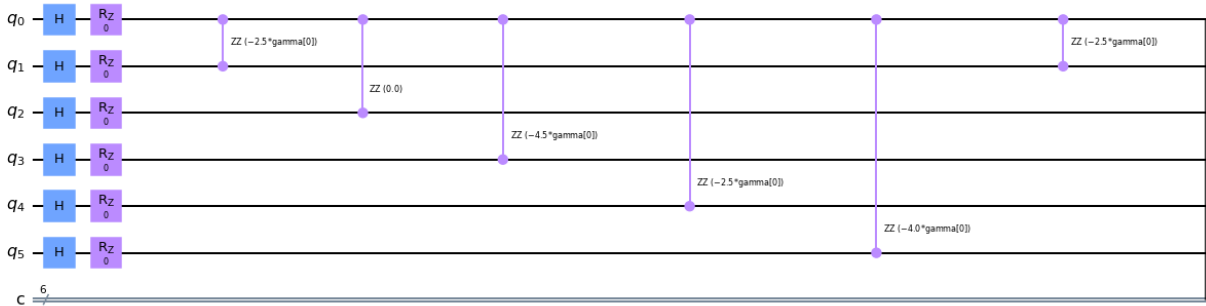


Figure 13: QAOA quantum algorithm