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Quantum Gravitational Energy Simplifies Gravitational Physics

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Abstract

We show the simplest form with which one can express the gravity force, and that still gives all the same predictions of observable phenomena as does standard Newton gravity. This new form to express gravity, through quantum gravitational energy, requires less constants to predict gravity phenomena than does standard gravity theory. This alone should make the physics community interested in investigating this approach. It shows that gravitational energy, and other types of energy, are a collision-length in their most complete and deepest form. This is consistent with a recently published collision-space-time theory.

While general relativity theory needs two constants to predict gravity phenomena, that is G and c , our new theory, based on gravity energy, only needs one constant, c_g , that is easily found from gravitational observations with no prior knowledge of any constants. Further, we will show that, at the deepest quantum level, quantum gravity needs two constants, c_g and the Planck length, while the standard formulation here needs c , h and l_p . Thus our theory gives a reduction in constants and simpler formulas than does standard gravity theory. Further, it gives insight into the quantum aspects of gravity. This is not ment as a replacement of existing gravitational theories, but a new approach that look at gravity from a deeper perspective that deserves further investigation by more researchers.

Keywords: Gravity force, Newton gravity, gravitational energy, gravity constant, quantum gravity

1 Short about the modern Newtonian formula

Today, Newton's gravitational force formula, as found in modern text books and research papers, is given by:

$$F = G \frac{Mm}{R^2} \quad (1)$$

where M and m are the mass of the large and the small mass in kilograms, G is the gravitational constant, and R is the center-to-center distance from M to m . It is of great importance to understand that the gravity force itself has never been directly observed, but only indirectly through observable gravitational phenomena, and the gravity force is not among these. In all observable gravitational phenomena predicted from this formula, the small mass m always cancels out in the derivation of something that actually can be observed, something we soon will look at in detail¹. Also be aware that this version of the gravity formula with a gravity constant was introduced in 1873 by Cornu and Baille [1]. They were the first to introduce the gravitational constant and used notation f for it. Boys [2] in 1894 was likely the first to use the symbol G for the gravity constant. Isaac Newton's original gravity force formula was:

$$F = \frac{Mm}{R^2} \quad (2)$$

as he only stated by word in Principia [3]. However, Newton's mass definition was very different than today's kilogram mass definition (so M and m , in this formula, mean something different); see [4] for an in-depth analysis of the original Newton gravity force formula. The so-called Newton's gravitational constant was pointed out by Thüuring [5] in 1961 to have been introduced somewhat ad-hoc; see also Gillies [6]. Thüuring pointed out that the gravitational constant cannot be associated with a unique property of nature. The gravitational constant has dimensions $|G| = \frac{L^3}{MT^2}$ or, in SI units, $m^3 \cdot kg^{-1} \cdot s^{-2}$. In nature this is, at a fundamental level, length cubed divided by mass times time squared. One must have a good imagination to come up with something that fits the bill. We think it is no coincidence that the gravitational constant has such output units. We have good reasons to think it is a composite constant, something that is discussed in detail in Haug [53].

¹In real, two-body problems where both masses act significantly on each other, then the gravity parameter is changed to $G(M_1 + M_2) = GM_1 + GM_2$

Einstein's [7] general relativity theory took Newton's gravitational constant for granted. Already, in the same 1916 paper on general relativity, Einstein pointed out that the next step in gravity was to develop a quantum gravity theory or, in his own words:

“Because of the intra-atomic movement of electrons, the atom must radiate not only electromagnetic but also gravitational energy, if only in minute amounts. Since, in reality, this cannot be the case in nature, then it appears that the quantum theory must modify not only Maxwell's electro-dynamics but also the new theory of gravitation.” – A. Einstein

One should clearly still be allowed to question even the very foundation of gravity theories. We think that in particular one should be allowed to question the gravitational constant, what it represents, and if it can be replaced, as it was inserted ad-hoc in 1873. Einstein also mentioned *gravitational energy*. Here we will demonstrate that one can easily come up with a formula that replaces Newton's formula using G and kilogram masses, with a formula based on gravitational energy and the speed of this gravitational energy, which we will see is the speed of gravity that again is identical to the speed of light. Further, our suggestions in this paper are directly linked to a recent quantum gravity theory known as collision-space-time [8–10], something we will get back to soon.

2 A new and simpler gravitational formula rooted in gravitational energy

We will here introduce a new gravitational force formula that we will demonstrate can replace Newton's gravity formula. It is given by:

$$F = c_g \frac{E_g \varepsilon_g}{R^2} \quad (3)$$

where E_g and ε_g are the gravitational energy of the large and small mass (M_g and m_g). The gravitational energy is in form of collision-length as defined in collision space-time [8–10]. This length is unknown for any mass, but we will soon show how to measure it. Secondly, c_g is the speed of gravity, which we can also easily find from gravity observations. We will also demonstrate that the formula above will remarkably give exactly the same predictions for observable gravity phenomena as Newton's gravitational force formula $F = G \frac{Mm}{r^2}$, both in values and in output units. The only exception is for non-observable phenomena, such as the gravity force itself, something we will soon come back to.

This new gravity force formula has output units $m \cdot s^{-1}$ in SI units, or in dimensions $|F| = L/T$. In other words, the gravity force is a speed in this formulation. This is in contrast to the standard modern version of Newton's gravitational force which has output $m \cdot kg \cdot s^{-2}$. So, one could easily make the mistake of thinking that our gravity force formula must be wrong as it doesn't even match the output units for the standard gravity force. A basic first check in physics, that even I myself typically use when coming up with a formula, is whether one has at least got the output units right. If not, that is typically a sign one has done something wrong or based the derivation on wrong assumptions. Still, the gravity force has never been observed, so the output units have partly been arbitrary chosen, they where for example not the same in Newton's time, he did not even have a gravity constant in his formula. What is important is that the formula predicts accurately everything that can be observed with respect to both values and, naturally, the correct output units. Also, our new gravitational energy is a length. this seems totally inconsistent with standard joule energy. However, as we [10] have already demonstrated in a paper on a new quantum gravity theory, this view is fully consistent with such things as the standard relativistic energy momentum relation.

It is, as we have demonstrated, true that both standard energy (joule) and Einstein's relativistic energy momentum relations are derivatives of simpler and deeper relations.

We will show that this gravitational model can be calibrated and used to predict a long series of observable gravitational phenomena. We have previously [8, 11] shown that a formula that predicts the same as Newton's gravitational force formula is given by:

$$F = c_g^3 \frac{M_g m_g}{R^2} \quad (4)$$

where M_g is the collision-time mass. That is, indeed mass as time which we come to by also incorporating gravity in the mass, something that is missing in the standard kilogram mass. Well, it is the duration of the aggregates of the collisions of the indivisible particles making up the mass; see [8] for detailed discussion. Further, this means we have $E_g = M_g c_g$, which at first glance seems to be totally inconsistent with $E = mc^2$, but it is not; it is fully consistent also with this. The reason for the difference is simply different energy and mass definitions.

For example, we could define a new energy $E_2 = E/c$. Then we would have $E_2 = mc$. There is nothing mathematically wrong with this as it is simply a change of units in the energy done in a consistent way. However, why should energy be re-defined as joule divided by the speed of light? A re-definition of energy and/or mass

must be able to explain something new or make things more intuitive. Simply by taking the joule energy and dividing by c to define a new energy does not seem to simplify intuition or teach us something new. Our new energy definition of E_g , on the other hand, is just a length, and we can go from the joule energy to this energy (collision-length) by multiplying the joule energy, E , with $\frac{l_p^2}{hc}$, and in $E = Mc^2$ we need to do the same on both sides, so we get $M_g = M \frac{l_p^2}{h}$. It is hard to see intuition here yet, except that we end up with a length for energy and time for mass. Length is something most of us find easier to understand than joule. To go from E and M to E_g and M_g in the way just described, one needs to know \hbar and l_p and also c . So, is this not some fancy change of units? As we soon will see, E_g (and also M_g) can easily be extracted from gravity phenomena with no knowledge of G , h or c . We will end up needing knowledge of less constants than are used in Newton's and Einstein's gravity to make the same predictions. Just to briefly demonstrate that our new energy and mass definitions are consistent with the relativistic energy momentum relation, we must have:

$$\begin{aligned}
E_g &= m_g \gamma c_g \\
E_g^2 &= m_g^2 c_g^2 \gamma^2 \\
E_g^2 &= m_g^2 c^2 \gamma^2 - m_g^2 c^2 + m_g^2 c^2 \\
E_g^2 &= \frac{m_g^2 c^2}{1 - v^2/c^2} - m_g^2 c^2 + m_g^2 c^2 \\
E_g^2 &= \frac{m_g^2 c^2}{1 - v^2/c^2} - \frac{m_g^2 c^2 (1 - v^2/c^2)}{1 - v^2/c^2} + m_g^2 c^2 \\
E_g^2 &= \frac{m_g^2 c^2}{1 - v^2/c^2} - \frac{m_g^2 c^2 - m_g^2 v^2}{1 - v^2/c^2} + m_g^2 c^2 \\
E_g^2 &= \frac{m_g^2 v^2}{1 - v^2/c^2} + m_g^2 c^2 \\
E_g^2 &= p_g^2 + m_g^2 c^2 \\
E_g &= \sqrt{p_g^2 + m_g^2 c^2}
\end{aligned} \tag{5}$$

where $p_g = m_g v \gamma$ is the relativistic gravitational momentum and γ is, as usual, the Lorentz factor, $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$. That is, it's the same as the standard relativistic momentum except m is replaced with m_g . Now it is only necessary to multiply each side of equation 5 with $\frac{\hbar}{l_p^2} c$ (or $\frac{c^4}{G}$) as well as setting $c_g = c$ (as we know it is from measurements and theory) and we end up with the standard $E = \sqrt{p^2 c^2 + m^2 c^4}$. We will claim the standard mass and standard energy are incomplete mass and energy definitions; they can almost be seen as derivatives of a deeper theory, where the deeper relation is the first line in the derivation above. The incomplete mass and energy are enough to describe energy and mass relations not related to gravity, but they fall short when we work with gravity.

The standard mass and energy have no information about the Planck scale as the Planck length embedded in E_g is taken out, something that will soon be clearer. We will claim the Planck scale is the essence of gravity. When the Planck scale is not incorporated in standard mass and energy, this is, in our view, one of the main reasons why one has not been able to unify gravity and quantum mechanics, at least until perhaps very recently; see [10] for a much more in-depth discussion about this point.

Actually the speed of gravity c_g constant which by calibration only from gravity phenomena can be shown to be equal to the speed of light can be seen as a pure scaling factor that adjust for human conventions in how we define length in relation to time. The speed of gravity can be found without knowledge of the speed of light and without any detection of gravitational waves, this have recently been demonstrated, see [11, 12]. If we take the standard gravity force formula and multiply it with R we get $G \frac{Mm}{R}$ which has output units joule, which is energy. In our formula if we multiply it with R we get $c_g \frac{E_g E_g}{R}$ which gives output units when using meters and seconds as meters squared divided by second. However if we set $c_g = 1$ then we get output dimensions $\frac{E_g E_g}{R}$ which is length as one of the collision-length energies dimensions cancel with the length dimensions of R , which is collision-length energy, when space and time are connected through the speed of light.

3 Finding the speed of gravity and the gravitational energy without any knowledge off G , h or c

In our new gravitational model, we need to know E_g and c_g . They are both unknown, so even if we can assume $c_g = c$, we want to see if we can find it "experimentally", with no knowledge of c , and in a simple way by utilizing the implications of our theory. The radius R from the center of the gravitational object to the center of the small mass the gravitational field acts on, can typically be easily measured directly or indirectly. Remarkably, there

is an easy way to find both the speed of gravity c_g and the gravitational energy without any prior knowledge of any constants or of the mass of the gravitational object. Also, in our formulation we must have:

$$m_g a = F \quad (6)$$

That, when we replace F with our new gravity force formula, leads to:

$$m_g a = c_g \frac{E_g E_g}{R^2} \quad (7)$$

and in our theory we have $m_g = \frac{E_g}{c}$, and as discussed in the section above this is fully consistent with $E = mc^2$, so we get

$$\begin{aligned} \frac{E_g}{c_g} a &= c_g \frac{E_g}{R^2} \\ a &= c_g^2 \frac{E_g}{R^2} \end{aligned} \quad (8)$$

That is, the gravitational acceleration field is given by $g = a = c_g^2 \frac{E_g}{R^2}$. This only dependent on one constant: the speed of gravity, c_g . There are two unknowns here, both c_g and the gravitational energy E_g of the mass in question. We could as standard theory assume $c_g = c$, something that also have been experimentally tested to at least be very close, but we will based on our own theory show it is a much easier way to extract c_g from gravity observations and therefore demonstrate we are totally independent on any assumptions about the value for c_g . Let us solve the gravitational acceleration field with respect to E_g ; this gives:

$$E_g = \frac{gR^2}{c_g^2} \quad (9)$$

However, we still do not know c_g so we cannot, from this, find E_g yet. The gravitational red shift for a beam sent in a weak gravitational acceleration field from R_h to R_L ($R_h > R_L$) is given by:

$$z = \frac{f_h - f_L}{f_L} \approx \frac{\sqrt{1 - \frac{2E_g}{R_L}}}{\sqrt{1 - \frac{2E_g}{R_h}}} - 1 \quad (10)$$

Next, replace E_g with $E_g = g \frac{R^2}{c_g^2}$ in the formula above and we get:

$$\begin{aligned} z = \frac{f_h - f_L}{f_L} &\approx \frac{\sqrt{1 - \frac{2g_L \frac{R_L^2}{c_g^2}}{R_L}}}{\sqrt{1 - \frac{2g_L \frac{R_L^2}{c_g^2}}{R_h}}} - 1 \\ z &\approx \frac{\sqrt{1 - \frac{2g_L R_L}{c_g^2}}}{\sqrt{1 - \frac{2g_L \frac{R_L^2}{c_g^2}}{R_h}}} - 1 \end{aligned} \quad (11)$$

$$(12)$$

Solved with respect to c_g this gives:

$$c_g \approx \frac{\sqrt{2g_L R_L \left(\frac{R_h - R_L}{z} + 2R_L + R_L z \right)}}{\sqrt{R_h (2 + z)}} \quad (13)$$

This is actually equal to the strong field exact solution one can get in general relativity theory as we have recently demonstrated [13], but it is in our view still just an approximation, so far in our model, we have not taken into account Lorentz's [14] relativistic mass, something we will get back to. That is, to find the speed of gravity all we need to do is to measure the gravitational acceleration at the surface of the Earth, for example at sea level (R_L), and then also measure the gravitational red-shift from a laser beam going from R_h to R_L , where $R_h > R_L$. This result in itself is remarkable, because it means one can measure the speed of gravity easily by combining two types of gravitational observations, and thus there is no need for advanced LIGO measurements of gravitational waves to do this; see also [11]. Inputting measured values into this formula reveals that the

speed of gravity, c_g , is indeed identical to the speed of light, c , as also assumed in general relativity theory and it seems confirmed by complex experiments; see [15, 16].

To measure the speed of gravity c_g , all we need is a measurement of the gravitational acceleration and the gravitational red shift. This can easily be done without any knowledge of any other constants. One can easily misunderstand here and think we are simply getting out c as we have inputted c , but this is not the case. In standard physics, we have $g = \frac{GM}{R^2}$ and $z \approx \frac{GM}{c^2 R}$, so one could think we are here getting c out since it is an input in the red shift formula. However, this is only if one predicts the gravitational red shift. There is no need to know c to measure the frequency shift in a light beam in a gravitational field; we only need to know c to predict the gravitational red shift in standard theory. However, here we are not predicting these, but measuring them and then finding c_g , which indeed has the same value as c . Based on our deeper understanding of our theory, we also know that $c_g = c$, naturally based on some assumptions; see [10].

Next, we can now find E_g from formula 9 as we now know c_g , g , and R . As soon as we know E_g and c_g , we can predict all kinds of other observable gravitational phenomena from the gravitational object's gravitational energy, E_g , which is the gravitational energy of the large mass. For example, we can predict all types of gravitational effects from the Earth. A long series of predictions we can do are illustrated in Table 1. Here we show both the modern standard Newton and GR formulations as well as the predictions from our new framework. For all observable phenomena, they give the same output in values and in terms of output units. Actually, the standard method is less accurate as it needs G that typically is calculated first from a smaller test mass, for example by using a Cavendish apparatus. This will add an additional measurement error when we work with larger masses such as, for example, the mass of the Earth. This is outside the scope of this paper but will be looked at in detail in another paper we are working on.

	Standard:	Alternative:
Mass	M (kg)	M_g (collision-time mass)
Energy	$E = Mc^2$ (joule)	$E_g = M_g c_g$ (gravitational energy)
Gravitational constant	G	c_g m/s
Non observable (contains GMm)		
Gravity force	$F = G \frac{Mm}{R^2}$ ($kg \cdot m \cdot s^{-2}$)	$F = c_g \frac{E_g E_g}{R^2}$ m/s
Observable predictions, identical for the two methods: (contains only GM)		
Gravity acceleration	$g = \frac{GM}{R^2}$	$g = c_g^2 \frac{E_g}{R^2}$
Orbital velocity	$v_o = \sqrt{\frac{GM}{R}}$	$v_o = c_g \sqrt{\frac{E_g}{R}}$
Orbital time	$T = \frac{2\pi R}{\sqrt{\frac{GM}{R}}}$	$T = \frac{2\pi \sqrt{R^3}}{c_g \sqrt{E_g}}$
Velocity ball Newton cradle	$v_{out} = \sqrt{2 \frac{GM}{R^2} H}$	$v_{out} = \frac{c_g}{R} \sqrt{E_g H}$
Periodicity Pendulum (clock)	$T = 2\pi \sqrt{\frac{L}{g}} = 2\pi R \sqrt{\frac{L}{GM}}$	$T = \frac{2\pi R}{c_g} \sqrt{\frac{L}{E_g}}$
Frequency Newton spring	$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi R} \sqrt{\frac{GM}{x}}$	$f = \frac{c_g}{2\pi R} \sqrt{\frac{E_g}{x}}$
Gravitational red shift	$z = \frac{\sqrt{1 - \frac{2GM}{R_1 c^2}}}{\sqrt{1 - \frac{2GM}{R_2 c^2}}} - 1$	$z = \frac{\sqrt{1 - \frac{2E_g}{R_L}}}{\sqrt{1 - \frac{2E_g}{R_h}}} - 1$
Time dilation	$T_R \approx T_f \sqrt{1 - \frac{2GM}{R} \frac{1}{c^2}}$	$T_R \approx T_f \sqrt{1 - \frac{2E_g}{R}}$
Time dilation (beyond GR) exact	$T_R = T_f \sqrt{1 - \frac{2GM}{Rc^2} + \frac{G^2 M^2}{c^2 R^2}}$	$T_R = T_f \sqrt{1 - \frac{2E_g}{R} + \frac{E_g^2}{R^2}}$
Gravitational deflection (GR)	$\delta = \frac{4GM}{c^2 R}$	$\delta = \frac{4E_g}{R}$
Advance of perihelion	$\sigma = \frac{6\pi GM}{a(1-e^2)c^2}$	$\sigma = \frac{6\pi E_g}{a(1-e^2)}$
Micro lensing	$\theta = \sqrt{\frac{4GM}{c^2} \frac{d_s - d_L}{d_s d_L}}$	$\theta = 2\sqrt{E_g \frac{d_s - d_L}{d_s d_L}}$
Cosmology: (contains only GM)		
Cosmological redshift	$z_H \approx \frac{dH_0}{c} = \frac{1}{2GM_c} \frac{1}{c^2 d}$	$z_H \approx \frac{d}{E_g}$
Hubble constant	$H_0 = \frac{c^3}{2GM_c}$	$H_0 = \frac{c_g}{E_g}$
Hubble radius	$R_H = \frac{c}{H_0} = \frac{2GM_c}{c^2}$	$R_H = E_g$
Indirectly/“hypothetical“ observable predictions: (contains only GM)		
Gravitational parameter	$\mu = GM$	$\mu = c_g^2 E_g$
Two body problem	$\mu = G(M_1 + M_2)$	$\mu = c_g^2 (E_{g,1} + E_{g,2})$
Constants needed	G, c and H_0	Only c_g ($c_g = c$)

Table 1: The table shows that all observable gravity is linked to GM in the standard Newton and general relativity formulation. Pay attention to how standard gravity theory needs knowledge of two constants to predict gravity phenomena, namely G and the speed of light. The alternative theory only needs one constant, namely c_g . The gravitational energy E_g as well as c_g can be found directly from gravity observations without knowledge of any known physical constants. Standard theory needs G, c , and M . Both theories naturally, in addition, need to know the distance to the center of the gravitational object.

4 What about quantum gravity energy?

At a deeper level, the gravitational energy (collision-length) is given by (see [10]):

$$E_g = l_p \frac{l_p}{\bar{\lambda}} \quad (14)$$

where l_p is the Planck length first described by Max Planck [17, 18] in 1899, and $\bar{\lambda}$ is the reduced Compton [19] wavelength. Max Planck introduced the Planck length in 1899 by the formula $l_p = \sqrt{\frac{G\hbar}{c^3}}$. So, it looks like we need to know G and \hbar to find the Planck length. In recent years it has been shown this is not the case. The Planck length can be found totally independently of G and \hbar ; see [11, 20, 21]. We will also, in this paper, devote a short section on how to find the Planck length independently of knowledge of G and h .

So the first l_p in the gravitational energy formula represents the collision-length of a single collision (Planck event). This collision-length of one Planck event is always the Planck length; see [9, 22] for a discussion of why it is invariant. The next part, $\frac{l_p}{\bar{\lambda}}$, is the number of Planck mass events in the gravitational mass over an observational time window of the Planck time. It is not that we need to observe anything in the Planck time, but it is what this represents, and we can measure it indirectly. For a Planck mass then, the reduced Compton wavelength is the Planck length, and then this factor is 1. For a mass smaller than the Planck mass, then $\frac{l_p}{\bar{\lambda}}$ is less than one. It is then a probability for a Planck mass event in the Planck time. Elementary particles consist, in this view, of Planck mass events (collisions) happening at the reduced Compton frequency. If the gravitational mass is larger than the Planck mass (and it is typically much larger) then $\frac{l_p}{\bar{\lambda}}$ typically consist of a large integer number plus a small fraction. The integer part then represents the number of Planck mass events in the Planck time. In other words, this is where the quantization comes in. The quantization is linked to the Planck length, that in our view is the diameter of an indivisible particle; see [8, 9].

Our simple gravitational force formula $F = c_g \frac{E_g E_g}{R^2}$ has embedded quantum gravity. That is, gravitational energy and gravitational mass come in quanta. The quanta is linked to the Planck length and the reduced Compton wavelength through the factor $\frac{l_p}{\bar{\lambda}}$ that is embedded in the gravitational energy as it is, at the deepest level, described by equation 14. We will even claim standard Newton gravity has hidden quantum gravity in it, not on purpose or by design, but by coincidence, as gravity when calibrating G to observations incorporates l_p ; see a lengthy discussion and review of the composite view of the gravitational constant view by Haug [53].

Further the gravitational mass is equal to

$$M_g = \frac{E_g}{c} = \frac{l_p}{c} \frac{l_p}{\bar{\lambda}} = t_p \frac{l_p}{\bar{\lambda}} \quad (15)$$

In other words mass is time, it is collision-time, It is how many collisions we have in the gravitational mass per Planck time. Where again $\frac{l_p}{\bar{\lambda}}$ gives the numbers off such events, so even mass is quantized. Again one can easily think it must be inconsistent that mass is simply energy divided by c as it in standard theory is c^2 , but this is fully consistent with that as demonstrated in the derivation in the end of section two.

5 Finding the Planck length and the Compton wavelength independently of any knowledge of G , \hbar , and even c

We can do all gravitational predictions simply from the gravitational energy, E_g , and the speed of gravity, c_g , and these we have already shown how to find. It is only when we want to understand the deeper aspects of E_g and M_g that we need the Planck length. The Planck length can easily be found without any knowledge of G , c and \hbar ; something that is controversial in standard gravity theory, but that we have recently demonstrated, in a series of published papers, is possible. Still, we will also demonstrate here that it is also possible when we write our formulas from gravitational energy.

The gravitational acceleration as before (section 3) is given by:

$$g = c_g^2 \frac{E_g}{R^2} \quad (16)$$

Further, E_g at the quantum level is given by $E_g = l_p \frac{l_p}{\bar{\lambda}}$, where $\bar{\lambda}$ is the reduced Compton wavelength of the gravitational energy in question. We can now simply solve the formula above for l_p , and this gives:

$$l_p = \sqrt{g \frac{R^2}{c_g^2} \bar{\lambda}} \quad (17)$$

That is, to find the Planck length independently of G and \hbar , we need to find g and the reduced Compton wavelength independently of these. The gravitational acceleration we can find simply by measuring how long it takes for a ball dropped at high H above the ground to hit the ground. It is given by $g = \frac{2H}{T^2}$.

Still, how do we find the reduced Compton wavelength of, for example, the Earth? The reduced Compton wavelength [19] for a fundamental particle like the electron is given by:

$$\bar{\lambda} = \frac{\hbar}{mc} \quad (18)$$

That is, if you know the kilogram mass of the particle m , the reduced Planck constant, and the speed of light. However, it is not necessary to know the kilogram mass of the particle or the Planck constant to find the Compton wavelength (or reduced Compton wavelength). In photon scattering of an electron, we have:

$$\lambda_e = \frac{\lambda_{\gamma,2} - \lambda_{\gamma,1}}{1 - \cos \theta} \quad (19)$$

where λ_e is the Compton wavelength of the electron, and $\lambda_{\gamma,1}$ and $\lambda_{\gamma,2}$ are respectively the wavelength of the incoming and outgoing photon; see [23]. So, there is no need to know the Planck constant to measure the Compton wavelength. The reduced Compton wavelength is simply this divided by 2π .

Next we can utilize the knowledge that electrons and protons have the same absolute value of the charge. The cyclotron frequency is given by:

$$f = \frac{qB}{2\pi m} \quad (20)$$

This means we must have:

$$\frac{f_e}{f_p} = \frac{\frac{qB}{2\pi m_e}}{\frac{qB}{2\pi m_p}} = \frac{m_p}{m_e} = \frac{\bar{\lambda}_e}{\lambda_P} \approx 1836.15 \quad (21)$$

That is, to find the reduced Compton wavelength of the proton we can simply take the reduced Compton wavelength of the electron and divided it by 1836.15. This is why cyclotron experiments have also been used to find the proton electron mass ratio; see [24–26]. Research on the proton Compton wavelength goes back to at least 1958 in the paper of Levitt [27]. Next, we can simply count the number of protons in the mass in question, divide the Compton wavelength of the single proton by this number, and we have the Compton wavelength of the mass in question. For simplicity and even practical purposes, we can treat neutrons as the same mass as protons, or alternatively by doing the small corrections for the slightly different mass. This way of finding the Compton wavelength of a large mass ignores nuclear binding energy (see for example D’Auria [28]), but this will, at a maximum, give an error in the Compton wavelength of about 1%. This is naturally a considerable additional error, but we can even more-or-less remove it by treating the binding energy as mass equivalent, $m = E/c^2$, and adjust for it.

To count the number of atoms in a mass is, in practice, no easy task, but for smaller macroscopic masses one can also, in practice, count the number of atoms. This was one of the competing methods to re-define the kilogram; see [29–32]. Silicon 28 has a very uniform crystal structure, so if one can count the number of atoms in a very small volume of this material and next create a very accurate sphere of such material, then one can find with high precision the number of atoms in this sphere. Such a sphere can then, for example, be used in a Cavendish apparatus to measure gravitation effects, such as the gravitational acceleration from this uniform sphere.

Next, to find the Compton wavelength of much larger objects like, for example, the Earth, we can utilize the following relation:

$$\frac{g_1 R_1^2}{g_2 R_2^2} = \frac{\bar{\lambda}_2}{\lambda_1} \quad (22)$$

After we have done this, we know the Planck length as well as the Compton wavelength of the gravitational object in question and we can next predict all gravity phenomena by using only two constants, namely the Planck length and the speed of gravity (light) plus variables. We can now actually also directly see that the modern Newton formulation and our new gravity theory are, at the deepest level, identical. To see this, we need to replace G in the standard gravitational framework with $G = \frac{l_p^2 c^3}{\hbar}$, which is simply the Max Planck length formula $l_p = \sqrt{\frac{\hbar G}{c^3}}$ solved with respect to G ; see [33, 53]. One should be aware that the idea of expressing G from Planck units goes back to at least 1984 when Cahill [34, 35] suggested $G = \frac{\hbar c}{m_p^2}$. However, as pointed out by Cohen [36] in 1987, this seemed to lead to a circular problem, as it seemed one had to know G to find m_p , so to express G from Planck units seemed to be useless. This has been a view held until very recently. It was first in 2017 that we showed one could find the Planck length independently of any knowledge off G . In addition to writing G as a composite constant, one needs to solve the reduced Compton wavelength formula with respect to m . This means we can express any kilogram mass as $m = \frac{\hbar}{\lambda} \frac{1}{c}$. Now, by inserting this composite gravity constant as well as this way to express the mass in the standard Newtonian framework, we see that it leads to exactly the same formulas for all observable phenomena as our new gravitational energy framework. In other

words, they give the same output both in terms of values and units as can be seen from Table 2. Non-observable phenomena such as the gravitational force itself have different output units in the two approaches.

The two approaches are only identical when we ignore Lorentz's relativistic mass, as do general relativity theory and standard Newtonian gravity theory. We are of the opinion that for strong gravity fields one should in addition consider Lorentz's relativistic mass. Considering this means, for example, that a micro black hole in our theory matches all the properties of the Planck scale, while general relativity theory that ignores Lorentz relativistic mass will lead to micro black holes that only can match a few properties of the Planck scale, see [37].

table

6 Finding the gravitational energy and the Planck length using a Cavendish apparatus

Here we will look at how to find the collision-length, that again is the gravitational energy, by using a Cavendish [38] apparatus. Moment of force, better known as torque, is given by:

$$\kappa\theta \quad (23)$$

where θ is the deflection angle of the balance and κ is the torsion coefficient of the suspending wire. Next, we have the following well-known relationship:

$$\kappa\theta = LF \quad (24)$$

where L is the length between the two small balls in the Cavendish apparatus. Further, F can be set equal to the gravitational force given by:

$$F = c_g \frac{E_g E_g}{R^2} \quad (25)$$

This means we have:

$$\kappa\theta = Lc_g \frac{E_g E_g}{R^2} \quad (26)$$

Further, the natural resonant oscillation period of a torsion balance is given by:

$$T = 2\pi\sqrt{\frac{I}{\kappa}} \quad (27)$$

The moment of inertia I of the balance is given by:

$$I = m_g \left(\frac{L}{2}\right)^2 + m_g \left(\frac{L}{2}\right)^2 = \frac{m_g L^2}{2} \quad (28)$$

from this we must have

$$T = 2\pi\sqrt{\frac{m_g L^2}{2\kappa}} \quad (29)$$

We now solve this with respect to κ , and this gives:

$$\begin{aligned} \frac{T^2}{2^2\pi^2} &= \frac{m_g L^2}{2\kappa} \\ \kappa &= \frac{m_g L^2 2\pi^2}{T^2} = \frac{E_g L^2 2\pi^2}{c_g T^2} \end{aligned} \quad (30)$$

Now in the equation 26 replace κ with this expression, and then we solve this equation with respect to the gravitational energy, and this gives:

$$\begin{aligned} \kappa\theta &= LF \\ \frac{E_g L^2 2\pi^2}{c_g T^2} \theta &= Lc_g \frac{E_g E_g}{R^2} \\ E_g &= \frac{L 2\pi^2 R^2 \theta}{T^2 c_g^2} \end{aligned} \quad (31)$$

And the collision-time mass of the large ball in the Cavendish apparatus is given by

$$M_g = \frac{E_g}{c_g} = \frac{L2\pi^2 R^2 \theta}{T^2 c_g^3} \quad (32)$$

Again, T is the oscillation time; further, L is the distance between the small balls in the Cavendish apparatus, and R is the distance from center to center between the small ball and the large ball in the Cavendish apparatus. Further, θ is the angle of the arm when the arm is deflected. In other words, there is no need to know G or h to measure this in a Cavendish apparatus. However, there the speed of gravity is, as we already know from previous sections, identical to the speed of light.

That is, we know the gravitational energy in the large ball in the Cavendish apparatus. It is indeed an incredibly short length, the collision-length. It is the aggregated collision-length of all the collisions in the mass making up the gravitational energy in the mass during only the Planck time. Pay attention to how we need less information to find this than to find G or M from a Cavendish apparatus. To find G in a Cavendish apparatus, one uses:

$$G = \frac{L2\pi^2 R^2 \theta}{T^2 M} \quad (33)$$

That is, one needs to know M in addition to L , R , and T is all that is needed to find E_g . One can only find the large mass in the Cavendish apparatus by simply weighting the mass, but doing so adds measurement errors compared to only finding E_g . If the G found from the Cavendish apparatus is only used in combination with the large mass in the Cavendish apparatus, then this error from measure M will cancel out with the error this gave to G , but if G measured from the Cavendish apparatus is next used in combination with the mass from the Earth to predict observable gravitational phenomena from the gravitational field of the Earth, then this will give bigger errors than simply using E_g from the Earth directly.

We can also find M in a Cavendish by:

$$M = \frac{L2\pi^2 R^2 \theta}{T^2 G} \quad (34)$$

But then one needs G , and one needed M to find G so this makes little sense. To find the gravitational energy (or the collision-time mass) of the large mass in the Cavendish apparatus requires no G and no kilogram measurements

To separate out the Planck length, we additionally need to know the reduced Compton wavelength of the gravitational mass, and we get:

$$\begin{aligned} \kappa\theta &= LF \\ \frac{E_g L^2 2\pi^2}{c_g T^2} \theta &= L c_g \frac{E_g E_g}{R^2} \\ l_p &= \sqrt{\frac{L2\pi^2 R^2 \theta \bar{\lambda}}{T^2 c_g^2}} \end{aligned} \quad (35)$$

That is, the Planck scale (here the Planck length) can be measured (detected) without any knowledge off G or h , but also here we see that we need the reduced Compton wavelength to do so, and this can be found independently of knowledge of G , h , and c . This is rather amazing as it has been thought for more than 100 years that to detect the Planck scale is almost impossible, despite large efforts to do so. That we can indirectly measure the Planck length without knowledge of G , c or h means, in our view, simply that to detect gravity is to detect the Planck scale. This is highly controversial and perhaps a shocking view. What one has been searching for, the Planck scale, is already something one has detected all the time, namely almost any observable effects of the gravity force. However, we are measuring an aggregate of these Planck events, and it is when one first understands the relationship between Compton frequency and Planck mass events that one really gets to the depth of it. This paper is, however, not going into great detail about the depth of the quantum scale as this has been a topic in other papers we have recently published and cited here. This paper is mainly about how we can replace the Newtonian formula with a considerably simpler gravitational energy formula $F = c_g \frac{E_g E_g}{R^2}$. The gravitational constant here is linked to something physically understandable; this is not the case for G . Even our energy is simpler to understand, as it is simply a length (a collision-length), while joule is very diffuse or, as beautifully illustrated by Feynman:

“ *It is important to realize that in physics today, we have no knowledge what energy is.* ” Richard Feynman

Our new collision-length energy demystifies energy, and it is a more complete energy definition that incorporates gravity into standard energy. Standard joule energy is an incomplete derivative of this energy, not by purpose, but based on physics, which, like most sciences, has mostly developed top-down. It is therefore natural that energy, over time, has been understood better and better. We suspect collision-length (energy) and

collision-time (mass) is the ultimate depth of reality. This is just one paper in a series of papers trying to explain different aspects of this new and promising theory. We naturally do not ask other researchers to take any of this for granted, but to take time to study this new theory before prematurely rejecting it.

7 Equivalence principle

We will here briefly discuss the equivalence principle and also how it is related to our theory. A full analysis of our theory in respect to this will certainly require a series of papers by multiple researchers over time, but at least this will give a good indication of where we currently think our new theory stands in relation to the equivalence principle. Normally, a distinguishment is made between the strong and the weak equivalence principle. In general relativity theory, it is assumed that both the weak and strong equivalence principle should hold. Part of Einstein's equivalence principle basically means that acceleration in flat space-time is locally indistinguishable from acceleration caused by a gravitational field.

One aspect and test of the weak equivalence principle goes all the way back to Galileo Galilei, and is simply related to that all bodies should fall at the same rate in a gravitational field in a vacuum. That this holds is due to the fact that it has been extremely accurately tested in recent times; see, for example, [39–41]. This is also fully in line with our theory.

Our theory is, like standard theory, also in line with recent tests of sub-millimeter scale deviations of the Newtonian $1/R^2$, which have confirmed this law is also valid at such short scales; see [42].

The weak equivalence principle is, in addition, linked to so-called inertial mass and gravitational mass through the following well known relation :

$$m_i a = G \frac{M m}{R^2} \quad (36)$$

That is, the weak equivalence principle is also about the inertial mass being equal to the gravitational mass, $m_i = m$ (see, for example, [43–45]). The equivalence principle is not only discussed inside Newton and general relativity theory, but across several gravitational theories, and its importance and interpretations are still often discussed; see [46, 47].

The mass m_i , on the left side of equation 36, is thought of as inertial mass and is why we have marked the mass with subscript i . The mass M and m on the right side of the equation are considered gravitational masses. The gravitational force formula used here in modern physics is the 1873 modified Newton formula. This basically means a mass acted upon by a uniform gravitational field g behaves identically to a mass of the same size acted upon by a force (pseudo-force) different than the gravitational force (used for acceleration) are indistinguishable, something that has also been well tested. At least it appears to be so, and we are not questioning the experimental test results, but we will here actually question the interpretation of this in relation to the standard Newton gravity force formula (Eq. 36).

In our view there is, however, actually only one form of mass; it is the collision-time mass, which is given by $M_g = \frac{l_p}{c} \frac{l_p}{\lambda} = \frac{G}{c^3} M$. This means we can just as well write:

$$\begin{aligned} m_g a &= c_g^3 \frac{M_g m_g}{R^2} = c_g \frac{E_g E_g}{R^2} \\ \frac{G}{c^3} m a &= c_g^3 \frac{G}{c^3} \frac{M \frac{G}{c^3} m}{R^2} \end{aligned} \quad (37)$$

After dividing by $\frac{G}{c^3} m$ on each side, we are left with:

$$a = \frac{GM}{R^2} \quad (38)$$

So, we see our theory is identical to standard theory when it comes to $a = g$. Still, it is an illusion that the inertial mass m_i , if expressed as just kilograms, as in the standard theory, is identical to the (in our view) true gravitational mass $m_g = \frac{G}{c^3} m$; that is, we naturally have $m \neq m_g$. The true gravitational mass is not M and m , but $M_g = \frac{l_p^2}{h} M = \frac{G}{c^3} M$ and $m_g = \frac{l_p^2}{h} m = \frac{G}{c^3} m$. The reason, in our view, that modern physics mistakenly thinks the kilogram mass, which is the unit of inertial mass, is the same as the gravitational mass, is that using the modern 1873 version of Newton's theory entails unknowingly using two different mass definitions in the same gravity force formula. That is, in the formula $F = G \frac{M m}{R^2}$, the gravitational mass should in reality not be seen as M and m , but rather as M fixed by its multiplication with G ; see also [48]. The other incomplete mass, m , is only used for derivations with another incomplete inertial mass m_i , that always cancels out in derivations to get formulas for something that can also be observed, such as g (see Table 2). The kilogram mass is incomplete and does not have enough information to also account for gravity, and is why any kilogram mass that has a significant gravitational field needs to be fixed (multiplied) with a gravitational constant. However, for standard energy mass relations such as $E = mc^2$, then the kilogram mass has enough information embedded in it, as this does not incorporate any information about gravitational mass or gravitational energy.

The inertial mass and the gravitational mass are the same when both are correctly expressed as collision-time mass, but they are not the kilogram mass. In standard theory “*even the unit of inertial mass is the kilogram is now per definition via h the unit of inertial mass. However, only the atom count determines the inertial mass of the kilogram realization without any reference to the equivalence principle*”; see Mana and Schlamminger [49]. This is in line with the research undertaken to decide on a new kilogram standard, culminating in the new 2019 NIST CODATA standard to link the kilogram to the Planck constant and the watt balance; see [50, 51]. The other alternative and competing method for defining the kilogram was counting atoms; see [29–32]. Still, none of these methods make the kilogram contain any information needed to make it a gravitational mass, so it is not the same as a gravitational mass.

Inertial mass is, however, identical to gravitational mass when it is defined as collision-time mass. We can easily find the collision-time mass of any macroscopic-sized object in a Cavendish apparatus, as demonstrated in section 6. This can be done without any knowledge of any constants except c and no need for counting atoms or including the watt balance and the Planck constant. This contrasts with the kilogram mass of the large ball in the Cavendish apparatus that we cannot find directly from the Cavendish apparatus without knowing G first. Further, we basically need to know the kilogram mass of the large ball in the apparatus to find G , so this leads to a circular problem. The way one avoids this circular problem in standard physics is to find the kilogram mass of the large ball used in the Cavendish apparatus from an independent method, such as the watt balance, or to count atoms. These methods to define the kilogram are directly or indirectly constructing an arbitrary human-made clump of matter as the standard kilogram mass, but we do not need any of this to find the collision-time mass or collision-length energy of the large balls in the Cavendish apparatus, as it can be found directly in the Cavendish apparatus by only knowing one constant, namely the speed of light c . This constant also has no uncertainty in it, unlike G .

For larger astronomical objects, we can also easily find the collision-time mass for any object such as planets, the sun, and other stars without finding G first. Not only that, but also the uncertainty in these mass measures will be smaller than in the kilogram mass measure for the same objects. This is because to find the kilogram mass of astronomical objects, we generally need to first find G ; see [52]. For example, the collision-time mass of the Earth we can simply find by measuring gravitational acceleration from a drop ball; the collision-time mass of the Earth is then given by:

$$M_g = \frac{2HR^2}{T_b^2 c^3} = g \frac{R^2}{c^3} \quad (39)$$

where H is the height of the drop from above the ground, R is the radius of the Earth, T_b is the time it took from when the ball was dropped until it hit the ground, and c is the speed of light (or gravity, as they are the same). We naturally do not claim this method is very accurate, The point is that all we need is to measure g accurately without any knowledge of G ; for a detailed discussion of why the collision-time mass (and thereby the gravitational energy) can be found with higher precision than the kilogram mass, see [52].

Even if the kilogram mass (and therefore the kilogram inertial mass) is not the same as the gravitational mass, it is true, in our theory, that two masses with the same amount of kilogram also have the same gravitational mass, since any kilogram mass is simply a collision-time mass multiplied by a constant $m = m_g \frac{h}{\Gamma^2} = m_g \frac{c^3}{G}$. This is, however, much more than just a change of units. This unit change when going from kilogram to collision-time is made to get information about the Planck scale into the mass definition, and the Planck scale is what is needed to do any gravity modeling, but this is done unknowingly in standard theory through the calibrated G . We need to know the Planck constant and the Planck length, or G and c , to find the kilogram mass from the collision-time mass.

The kilogram mass has not incorporated the Planck length (or Planck time), which is related to the essence of gravity, but by multiplying G with M one unknowingly gets the Planck length into the mass. This is also discussed in more detail in section 5 of the recently published paper [53].

The kilogram mass is incomplete and contains no information about the Planck scale (except for the Planck mass itself) and therefore is an incomplete mass definition, which is why the only mass that is significant for gravity in two-body problems, wherein $m \ll M$ problems the mass M is multiplied with G , so the real gravitational mass is linked to GM not just M in the Newton formula. In two-body problems where we do not have $m \ll M$, then the gravitational parameter is $\mu = G(M + m) = GM + Gm$; in other words, then both the kilogram masses are corrected by being multiplied by G , and one is thus unknowingly turning the kilogram mass into collision-time mass, which is the real gravitational mass in our view, but in standard theory this has not been discovered and therefore not been understood. Be aware that when G is multiplied by M , as is needed for prediction of any directly observable gravitational phenomena, then the kilogram unit falls out. This is because G has output units $m^3 \cdot kg^{-11} \cdot s^{-2}$ and the kilogram mass naturally outputs unit kg , so the kilogram cancels. This is because gravity does not depend on humanly-constructed mass units, but rather depends on the foundation of mass and energy that is linked to the Planck length and Planck time.

The equivalence principle in general relativity theory is considered incompatible with the local presence of the gravitational energy; see, for example, Misner, Thorne and Wheeler [54] (section 20.4). According to them, the local gravitational energy-momentum has no weight and do not curve space-time. Further, they write: “*It does*

not produces any geodesic deviation of two nearby world lines that pass through the region of space in question. It is not observable". Further, they write: "At issue is not the existence of gravitational energy, but the localizability of gravitational energy. It is not localizable. The equivalence principle forbids it."

The gravitational energy, collision-length energy, that we have presented here and in other papers is not directly comparable with what is considered gravitational-momentum energy in general relativity theory. Our gravitational energy is directly related to the Planck scale as discussed in detail in [10]. Our model does not assume curved space-time, so it is not clear exactly how to compare our model on this point. For an observational area larger than two Planck lengths, one will in principle be able to clearly observe gravity caused by gravitational energy and, in this way, localize the gravitational energy to be inside a sphere with a radius equal to the Planck length (and a diameter of two Planck lengths). However, we could go even further and try to find out if the localization was inside a volume smaller than inside an area with a radius of the Planck length. This is impossible in our theory, as time would stand still and nothing could be observed inside an area of space smaller than the Planck volume; to understand this, one needs to study our paper on the deeper foundation of this theory; see [10]. So, in our theory we can localize the gravitational energy and indirectly find out the gravitational energy is rest-mass energy at the Planck scale inside mass, but we cannot localize the gravitational energy in the sense that our theory has no information below the Planck scale: below the Planck volume, and the Planck time.

There is likely no way, either now or in the future, that we can detect a single Planck event directly. To do so, we ourselves would have to be a photon, as in our model at the deepest level these Planck events are photon-photon collisions inside matter; again, see the paper just referred to. These Planck events, in our theory, consist of a mass of about 10^{-51} kg in a one-second observational time-window, and the Planck event is related to collisions of photons internally in matter; such collisions have only a duration of the Planck time, and a radius equal to the Planck length. These are all much smaller than anything we can measure directly with our best instruments now or, probably, in the future. Still, surprisingly we can measure the aggregates of such events in macroscopic objects, and even measure how many such events happen in any observational time window. This is namely the $\frac{l_p}{\lambda}$ factor we discussed above (section 4). That is, in our theory, gravity is linked to the Planck scale. We are boldly claiming to have been the first to have measured the Planck length independently of G , \hbar and even c ; see again [11, 20, 55]. Indeed, we have somehow indirectly localized the gravitational energy, by measuring its aggregate that is directly linked to the observed gravity, but we have further been able to break it down into its components by measuring the Compton wavelength: $\bar{\lambda}$, the speed of gravity: c_g , and the Planck length: l_p , totally independently of any knowledge of G , \hbar , or c . Table 1 shows we can model all gravity phenomena from the speed of gravity and the Planck length, in addition to, naturally, the distance of the center of the gravitational field R .

That is, we have a quantum gravity theory that, even by practical experiments, can and has detected the Planck scale. Tests done in relation to gravitational energy, and that are interpreted through general relativity theory, are not even close to testing the Planck scale. In our view, the Planck scale is, however, misunderstood in standard theory. To detect any effect from gravity is, in our model, to detect gravitational energy and thereby the Planck scale and thereby gravitational energy indirectly.

Be aware that the formulas presented in this paper so far are the weak field approximations of collision space-time quantum gravity theory, just as standard Newton theory is the weak field approximation in general relativity theory. Particularly for strong gravitational fields, this theory needs to take into account Lorentz relativistic mass; see [37, 56]. Our gravity theory when considering Lorentz relativistic mass, which can be reformulated as relativistic gravitational energy, basically means we have:

$$F = c_g \frac{E_g E_g \gamma}{R^2} \quad (40)$$

where γ is the Lorentz factor: $\gamma = 1/\sqrt{1-v^2/c^2}$, so this simply means Lorentz relativistic mass is identical to relativistic gravitational energy. To take such relativistic effects into account is important for light sent out from strong gravitational fields such as from quasars. Haug and Spavieri [57] have recently shown that the unexpected lack of observed velocity time dilation in high Z quasars (see [58]) can be explained in this model taking into account Lorentz relativistic mass or, alternatively, relativistic gravitational energy. Also, when taking into account Lorentz relativistic mass, we are able to predict supernova Ia (one-A) without the hypothesis of dark energy; see [59]. Our theory is also incompatible with general relativity for strong gravitational fields where the escape velocity is close to c , so mostly this will affect how to interpret black hole type objects; see [37, 56].

There is still considerable discussion around whether all the different parts of Einstein's equivalence principle actually hold; see, for example, [60] for a discussion. So, there are clearly many more aspects and tests we need to consider to compare our quantum gravitational energy model before we can conclude which parts of the equivalence principle our theory is compatible with or not.

8 Mercury precessions and bending of light

Two types of gravitational observations that early on led to Newton assuming the theory was incomplete were Mercury precession and the observed size of the deflection of light in gravitational fields. We will shortly look

at these here and suggest that Newton theory, in particular after also considering some relativistic effects, is potentially in line with observations.

Corda [61] has recently shown that also taking into account the gravitational effects of Mercury, and not only that of the sun, as well as relativistic effects seems to give predictions in line with the observed precession of Mercury. Haug [62] has also discussed that Mercury precession can possibly be predicted correctly when taking into account Lorentz relativistic mass.

Another cornerstone test of gravitational theories is if they can predict light bending correctly. Already, in 1794, Cavendish had pointed out that Newtonian gravity would lead to bending of light. Soldner [63] in 1881, as well as Einstein [64] in 1911, calculated that Newtonian theory would predict a light bending of $\delta = \frac{2GM}{Rc^2}$, which is exactly half of what general relativity theory predicts: $\delta = \frac{4GM}{Rc^2}$ and what has been observed. Einstein basically derived this by integrating over the gravitational potential in the following way:

$$\delta = \frac{1}{c^2} \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \frac{GM}{R} \cos \theta ds = \frac{2GM}{Rc^2} \quad (41)$$

As pointed out by Rybczyk [65] in a working paper in 2009, the gravitational potential $\Phi = \frac{GM}{R}$ is equal to the orbital velocity squared $v_o^2 = \frac{GM}{R}$. So another potential way to get to exactly the same light bending prediction in Newton theory is, as Rybczyk points out, to integrate over $\frac{v_o^2}{c^2}$:

$$\delta = \frac{1}{c^2} \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \frac{GM}{R} \cos \theta ds = \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \frac{v_o^2}{c^2} \cos \theta d\theta = \frac{1}{c^2} \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} v_o^2 \cos \theta d\theta = \frac{2GM}{Rc^2} = \frac{2v_o^2}{c^2} \quad (42)$$

This is the same result of predicted light bending as derived by Soldner and Einstein for Newton theory. If this should also be an adequate and logical way to do so, then one can ask if it is not the escape velocity that should be used in such derivation, rather than the orbital velocity? Our point is not to indicate that Einstein assumed light bending was related to orbital velocity, which he never claimed, but our point is to ask if light bending could be related to escape velocity rather than to his derivation from gravitational potential. The Newton escape velocity we get by solving the following equation with respect to v :

$$\frac{1}{2}mv^2 - G\frac{Mm}{R} = 0 \quad (43)$$

This gives the well-known result:

$$v_e = v = \sqrt{\frac{2GM}{R}} \quad (44)$$

If we now integrate over $\frac{v_e^2}{c^2}$ (as first suggested by Rybczyk) we get:

$$\delta = \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \frac{v_e^2}{c^2} \cos \theta d\theta = \frac{1}{c^2} \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} v_e^2 \cos \theta d\theta = \frac{4GM}{Rc^2} = \frac{2v_e^2}{c^2} \quad (45)$$

If this is the relevant way to look at light bending, then it would mean Newton theory does, after all, predict the same as general relativity theory. It is well known that general relativity theory and Newton theory lead to the same escape velocity; see [66].

Are we fully sure it is the gravitational potential (or orbital velocity) that is relevant and not the escape velocity when dealing with photons and light bending?

Next, if we now derive an escape velocity where we consider relativistic mass, then we simply need to solve the following equation with respect to v :

$$\frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2 - G\frac{Mm\gamma}{R} = 0 \quad (46)$$

This gives:

$$v_e = \sqrt{\frac{2GM}{R} - \frac{G^2M^2}{R^2c^2}} \quad (47)$$

See [67] for more details on this escape velocity. If we now integrate based on this, we get:

$$\delta = \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \frac{v_e^2}{c^2} \cos \theta d\theta = \frac{1}{c^2} \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} v_e^2 \cos \theta d\theta = \frac{4GM}{Rc^2} - \frac{2G^2M^2}{R^2c^4} = \frac{2v_e^2}{c^2} \quad (48)$$

This result is also given by Rybczyk [65]. We see the first term is identical to the predictions from general relativity light bending $\frac{4GM}{Rc^2}$ and then there is another second order term. In a weak gravitational field, this would

give the same prediction as general relativity and would therefore be indistinguishable in weak gravitational field measurements. Gravitational bending has only been tested in weak gravitational fields.

The same can be derived from our new way of presenting gravity through quantum gravitational energy. We have:

$$E_g \gamma c - E_g c - c_g \frac{E_g E_g \gamma}{R^2} = 0 \quad (49)$$

This gives:

$$v_e = c \sqrt{\frac{2E_g}{R} - \frac{E_g^2}{R^2}} \quad (50)$$

which is identical to $v_e = \sqrt{\frac{2c^3 M_g}{R} - \frac{cM_g^2}{R^2}}$ as $E_g = M_g c$, so this is identical to the escape velocity derived in Haug [10]. Next, we integrate over this:

$$\delta = \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \frac{v_e^2}{c^2} \cos \theta d\theta = \frac{2v_e^2}{c^2} = \frac{2c^2 \left(\frac{2E_g}{R} - \frac{E_g^2}{R^2} \right)}{c^2} = \frac{4E_g}{R} - \frac{2E_g^2}{R^2} \quad (51)$$

The gravitational energy in terms of collision-length for the sun is equal to:

$$E_g = l_p \frac{l_p}{\lambda_s} \approx 1480 \text{ m} \quad (52)$$

where l_p is the Planck length and λ_s is the reduced Compton wavelength of the sun. The factor $\frac{l_p}{\lambda_s} \approx 9.15 \times 10^{37}$ is the number of Planck events in the sun per Planck time; it is the quantization (number) of gravity in the sun.

However, we do not even need to find the Planck length or the reduced Compton wavelength to find $l_p \frac{l_p}{\lambda_s} = E_g$. We can find this factor directly by simply observing the orbital time of the Earth around the sun and in addition knowing the distance from the Earth to the sun; then we have:

$$\begin{aligned} T_o &= \frac{2\pi R}{v_o} \\ T_o &= \frac{2\pi R}{c \sqrt{\frac{E_g}{R}}} \\ T_o^2 &= \frac{4\pi^2 R^2}{c^2 \frac{E_g}{R}} \\ E_g &= \frac{4\pi^2 R^3}{T_o^2 c^2} \end{aligned} \quad (53)$$

We can check in numerically; the orbital time of the earth is about 365 days, and the distance earth to the sun is approximately 149,600,000,000 meters. So, we have:

$$E_g = \frac{4\pi^2 \times 149600000000^3}{(365 \times 24 \times 60 \times 60)^2 \times c^2} \approx 1479 \text{ m}$$

Further, the radius of the sun is 696,340,000 meters. So the predicted light bending of the sun is:

$$\delta = \frac{4E_g}{R} - \frac{2E_g^2}{R^2} \approx \frac{4 \times 1479}{696340000} - \frac{2 \times 1479^2}{696340000^2} \approx 8.49 \times 10^{-6} \text{ radians}$$

This must be multiplied by $\frac{648000}{\pi}$ to get arcseconds. This gives:

$$\delta \approx 8.49 \times 10^{-6} \times \frac{648000}{\pi} \approx 1.75 \text{ arcseconds}$$

which is nicely in line with the observed deflection from the sun. The first accurate observation was performed in 1919 by Eddington and later confirmed with a series of observations.

We could have done this from just the first term, that in our theory is only valid in weak gravitational fields, where a weak gravitational field is where $v_e \ll c$, and it is a strong gravitational field when v_e is significant to the strong field. In the weak field we get:

$$\delta = \frac{4E_g}{R} \approx \frac{4 \times 1479}{696340000} \approx 8.49 \times 10^{-6} \text{ radians}$$

This must be multiplied by $\frac{648000}{\pi}$ to get arcseconds. This gives:

$$\delta \approx 8.49 \times 10^{-6} \times \frac{648000}{\pi} \approx 1.75 \text{ arcseconds}$$

So this is clearly in line with observations. Still, we have to ask if an observation of bending of light is simply related to indirectly measuring the escape velocity divided by the speed of light (both squared) as we have speculated on here? Could this make sense or is it just an ad hoc coincidence that such an approach gives the correct prediction of light bending?

Whether this is a sound or unsound way of deriving gravitational deflection predictions is unclear at the moment and should be further investigated and discussed. It is clearly purely mathematically correct, but it must also be linked to correct physical interpretation of gravity. So, an outstanding question is if the escape velocity would be relevant for the light bending rather than the gravitational potential? This should be studied more closely before we make a conclusion. At least Lorentz relativistic mass seems to open potential new ways of interpreting observed gravity phenomena.

We have to keep in mind that many gravity observations are interpreted through a mathematical lens. Here, we have presented a mathematical lens where light bending is seen as escape velocity relative to the speed of light and also how this can be done through quantum gravitational energy. Only time can tell if it can give a full consistent framework and if it can add something to, or replace parts of, established theories.

9 Conclusion

We have demonstrated how the modern and very well-known Newtonian formula $F = G \frac{Mm}{R^2}$ can be replaced with the simpler and more intuitive formula $F = c_g \frac{E_g E_g}{R^2}$. These formulas give the same predictions, but the latter one only needs the speed of light (gravity) as a constant, so it does not need G . This is likely the simplest way one can express Newtonian gravity. We have further shown that there is a deeper quantum gravity hidden behind this as well as the Newtonian formula. Both formulas are consistent with weak field gravitation based on the new quantum gravity theory known as collision-space-time. In strong gravitational fields we claim in addition needs to take into account Lorentz relativistic mass or relativistic quantum gravitational energy. We also discuss how Newton theory actually possibly predict the correct bending of light.

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