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Homogeneous non-split superstrings of odd dimension 4

Michail Bashkin

Abstract. Let \mathbf{L}_k be the holomorphic line bundle of degree $k \in \mathbb{Z}$ on the projective line. The tuples $(k_1 k_2 k_3 k_4)$ for which there exists no homogeneous non-split supermanifolds $\mathcal{CP}_{k_1 k_2 k_3 k_4}^{1|4}$ associated with the vector bundle $\mathbf{L}_{-k_1} \oplus \mathbf{L}_{-k_2} \oplus \mathbf{L}_{-k_3} \oplus \mathbf{L}_{-k_4}$ are classified.

For many types of the remaining tuples, there are listed cocycles that determine homogeneous non-split supermanifolds.

Proofs follow the lines indicated in the paper Bunegina V.A., Onishchik A.L., Homogeneous supermanifolds associated with the complex projective line. *J. Math. Sci. V. 82 (1996) 3503–3527.*

1 Introduction

In this paper, I summarize the results of classification (up to a diffeomorphism) of homogeneous complex (more precisely, almost complex, see [BGLS*], since the vanishing of the Nijenhuis tensor is never required) supermanifolds $\mathcal{M} := (M, \mathcal{O})$, where $M = \mathbb{CP}^1$ and $\dim \mathcal{M} = 1|n$. (*Comments with starred references are added by the editor of this Special Volume. D.L.*)

For the case where \mathcal{M} is split, the classification is known, see [BuO1]: the non-diffeomorphic supermanifolds are in one-to-one correspondence with n -tuples of non-negative integers.

If \mathcal{M} is non-split, the classification is considerably more complicated and reduces to computation of cohomology of split homogeneous supermanifolds with coefficients in the tangent sheaf.

For $n = 2$ and 3 , V. A. Bunegina and A. L. Onishchik completely investigated the case, see [BuO1], [BuO2].

For $n = 4$, see below (summary of the results of [B1] – [B4], [BaO1], [BaO2]). For the method of the proof, see [BuO1], [BuO2].

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2 Results

As is known, any holomorphic bundle \mathbf{E} over $\mathbb{C}\mathbb{P}^1$ can be uniquely decomposed into a direct sum of line bundles: Grothendieck's theorem, see [HM*]. (For interesting applications of Linear Superalgebra (with elements of category theory) to the description of vector bundles over projective spaces, see the review [BG*]. For the latest results on non-splitness of supermanifolds whose retract is the Grassmann manifold, see [Vi*], [Vi1*], [Vi2*].) Let \mathbf{L}_k be the holomorphic line bundle of degree $k \in \mathbb{Z}$.

Consider a holomorphic bundle

$$\mathbf{E} = \mathbf{L}_{-k_1} \oplus \mathbf{L}_{-k_2} \oplus \mathbf{L}_{-k_3} \oplus \mathbf{L}_{-k_4}, \text{ where } k_1 \geq k_2 \geq k_3 \geq k_4 \geq 0.$$

If \mathcal{M} is homogeneous, then the k_i must be non-negative, see [BuO1].

Let $\mathcal{CP}_{k_1 k_2 k_3 k_4}^{1|4}$ designate the split supermanifold determined by \mathbf{E} .

Let us cover $\mathbb{C}\mathbb{P}^1$ by two affine charts U_0 and U_1 with local coordinates x and $y = \frac{1}{x}$, respectively. Then, the transition functions on $\mathcal{CP}_{k_1 k_2 k_3 k_4}^{1|4}$ in $U_0 \cap U_1$ are of the form

$$\begin{aligned} y &= x^{-1}, \\ \eta_i &= x^{-k_i} \xi_i \text{ for } i = 1, \dots, 4, \end{aligned}$$

where ξ_i and η_i are basis sections of \mathbf{E} over U_0 and U_1 , respectively.

Let M be a compact complex manifold. We will sometimes need general statements about the $m|n$ -dimensional supermanifold $\mathcal{M} = (M, \mathcal{O})$. Let $\mathcal{I} \subset \mathcal{O}$ be the subsheaf of ideals generated by the subsheaf $\mathcal{O}_{\bar{1}}$. Consider the filtration of \mathcal{O} by powers of \mathcal{I} :

$$\mathcal{O} = \mathcal{I}^0 \supset \mathcal{I} \supset \mathcal{I}^2 \supset \dots \supset \mathcal{I}^n \supset \mathcal{I}^{n+1} = 0.$$

The graded sheaf $\text{gr } \mathcal{O} = \bigoplus_{0 \leq i \leq n} \text{gr}_i \mathcal{O}$ with $\text{gr}_i \mathcal{O} := \mathcal{I}^i / \mathcal{I}^{i+1}$ defines the split supermanifold $(M, \text{gr } \mathcal{O})$ called the *retract* of (M, \mathcal{O}) . Let $\mathcal{T}_{\text{gr}} := \bigoplus_{-1 \leq p \leq 4} (\mathcal{T}_{\text{gr}})_p$ denote the graded tangent sheaf of the split supermanifold $(M, \mathcal{O}_{\text{gr}})$. Consider the subsheaf

$$\mathcal{Aut}_{(2)} \mathcal{O}_{\text{gr}} = \exp((\mathcal{T}_{\text{gr}})_2 \oplus (\mathcal{T}_{\text{gr}})_4)$$

of the sheaf $\mathcal{Aut } \mathcal{O}_{\text{gr}}$. Thanks to a theorem due to Green ([G]), the set of supermanifolds with a given retract $(M, \mathcal{O}_{\text{gr}})$ is isomorphic to the set of orbits of the group $\text{Aut } \mathbf{E}$ in $H^1(M, \mathcal{Aut}_{(2)} \mathcal{O}_{\text{gr}})$.

In what follows we often assume that $H^0(M, (\mathcal{T}_{\text{gr}})_2) = 0$. This is needed for existence of a bijection (see [O3, Section 3.2 pp.23–37])

$$H^1(M, \mathcal{Aut}_{(2)} \mathcal{O}_{\text{gr}}) \longleftrightarrow H^1(M, (\mathcal{T}_{\text{gr}})_2) \oplus H^1(M, (\mathcal{T}_{\text{gr}})_4).$$

2.1 Statement ([BaO2]). For $n \leq 5$, let $H^0(M, (\mathcal{T}_{\text{gr}})_2) = 0$ and let there be given subspaces $Q_{2p} \subset Z^1(\mathfrak{U}, (\mathcal{T}_{\text{gr}})_{2p})$, where $p = 1, 2$, such that every cohomology class in $H^1(M, (\mathcal{T}_{\text{gr}})_{2p})$ contains precisely one cocycle in Q_{2p} .

Then, every cohomology class in $H^1(M, \mathcal{Aut}_{(2)} \mathcal{O}_{\text{gr}})$ can be represented by a unique cocycle of the form $z = \exp(u^2 + u^4)$, where $u^2 \in Q_2$ and $u^4 \in Q_4$.

Hereafter, the expression “ (M, \mathcal{O}) is determined by the cocycle $u^2 + u^4$ ” means that (M, \mathcal{O}) actually corresponds to the cocycle $z = \exp(u^2 + u^4)$.

In [BuO1, Prop. 12], there is given a description of the algebra $\text{End } \mathbf{E}$. Let us reformulate this Proposition in terms more convenient for us here. Any endomorphism $a \in \text{End } \mathbf{E}$ can be considered as an endomorphism of the sheaf \mathcal{E} of \mathcal{F} -modules. In U_0 , we have

$$a(\xi_i) = \sum_{1 \leq j \leq n} a_{ji} \xi_j \quad \text{for } i = 1, \dots, n,$$

where $A = (a_{ji})$ is a matrix with elements $a_{ji} \in \mathcal{F}(U_0)$. The matrix A completely determines the endomorphism a , and $a \in \text{Aut } \mathbf{E}$ if and only if A is invertible.

The following statement is needed in the classification of homogeneous non-split supermanifolds up to a diffeomorphism.

2.2 Statement ([BaO2]). The matrix $A = (a_{ji})$ over $\mathcal{F}(U_0)$ corresponds to an element in $\text{End } \mathbf{E}$ if and only if

$$a_{ij} = \begin{cases} 0 & \text{if } k_i < k_j, \\ \text{is a polynomial of degree } \leq k_i - k_j & \text{if } k_i \geq k_j. \end{cases}$$

First, consider the tuples $(k, k, 2, 0)$ for $k \geq 2$. Let $\mathfrak{v}(\mathbb{CP}^1, \mathcal{O}_{\text{gr}})$ be the Lie superalgebra of vector fields on $\mathbb{CP}_{kk20}^{1|4}$. Theorem 14 in [BaO1] implies the following

2.3. Lemma. For a basis of $H^1(\mathbb{CP}^1, (\mathcal{T}_{\text{gr}})_q)$, where $q = 1, 2, 4$, we can take (the classes of) the following cocycles defining $\mathcal{CP}_{kk20}^{1|4}$.

1) $q = 1$ for $k = 2$

$$\begin{aligned} &x^{-1}\xi_1\xi_2\partial_{\xi_2}, \quad x^{-1}\xi_1\xi_4\partial_{\xi_4}, \quad x^{-1}\xi_2\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_2\xi_4\partial_{\xi_4}, \quad x^{-1}\xi_3\xi_1\partial_{\xi_1}, \\ &x^{-1}\xi_3\xi_4\partial_{\xi_4}, \quad x^{-1}\xi_1\xi_2\partial_{\xi_3}, \quad x^{-1}\xi_1\xi_3\partial_{\xi_2}, \quad x^{-1}\xi_2\xi_3\partial_{\xi_1}, \\ &x^{-r}\xi_1\xi_2\partial_{\xi_4}, \quad x^{-r}\xi_1\xi_3\partial_{\xi_4}, \quad x^{-r}\xi_2\xi_3\partial_{\xi_4} \quad \text{for } r = 1, 2, 3 \end{aligned}$$

for $k = 3$

$$\begin{aligned} &x^{-1}\xi_3\xi_4\partial_{\xi_4}, \quad x^{-1}\xi_3\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_1\xi_3\partial_{\xi_2}, \quad x^{-r}\xi_1\xi_3\partial_{\xi_4} \quad (r = 1, 2, 3, 4), \quad x^{-1}\xi_2\xi_3\partial_{\xi_1}, \\ &x^{-r}\xi_2\xi_3\partial_{\xi_4} \quad (r = 1, 2, 3, 4), \quad x^{-r}\xi_1\xi_2\partial_{\xi_3} \quad (r = 1, 2, 3), \quad x^{-r}\xi_1\xi_2\partial_{\xi_4} \quad (r = 1, 2, 3, 4, 5), \\ &x^{-r}\xi_1\xi_j\partial_{\xi_j} \quad (j = 2, 3, 4, r = 1, 2), \quad x^{-r}\xi_2\xi_j\partial_{\xi_j} \quad (j = 1, 3, 4, r = 1, 2); \end{aligned}$$

for $k \geq 4$

$$\begin{aligned} &x^{-1}\xi_3\xi_4\partial_{\xi_4}, \quad x^{-1}\xi_3\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_1\xi_3\partial_{\xi_2}, \quad x^{-r}\xi_1\xi_3\partial_{\xi_4} \quad (r = 1, \dots, k + 1), \\ &x^{-r}\xi_1\xi_2\partial_{\xi_3} \quad (r = 1, \dots, 2k - 3), \quad x^{-1}\xi_2\xi_3\partial_{\xi_1}, \quad x^{-r}\xi_2\xi_3\partial_{\xi_4} \quad (r = 1, \dots, k + 1), \\ &x^{-r}\xi_1\xi_2\partial_{\xi_4} \quad (r = 1, \dots, 2k - 1), \quad x^{-r}\xi_i\partial_x \quad (i = 1, 2, j = 1, \dots, k - 3), \\ &x^{-r}\xi_1\xi_j\partial_{\xi_j} \quad (j = 2, 3, 4, r = 1, \dots, k - 1), \quad x^{-r}\xi_j\xi_4\partial_{\xi_3} \quad (j = 1, 2, r = 1, \dots, k - 3), \\ &x^{-r}\xi_2\xi_j\partial_{\xi_j} \quad (j = 1, 3, 4, r = 1, \dots, k - 1); \end{aligned}$$

2) $q = 2$ for $k = 2$

$$\begin{aligned}
& x^{-1}\xi_1\xi_2\partial_x, \quad x^{-r}\xi_1\xi_2\xi_j\partial_{\xi_j} \quad (j = 3, 4, \quad r = 1, 2, 3), \quad x^{-1}\xi_1\xi_4\xi_2\partial_{\xi_2}, \quad x^{-1}\xi_2\xi_4\xi_1\partial_{\xi_1}, \\
& x^{-1}\xi_1\xi_3\partial_x, \quad x^{-r}\xi_1\xi_3\xi_j\partial_{\xi_j} \quad (j = 2, 4, \quad r = 1, 2, 3), \quad x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}, \\
& x^{-1}\xi_2\xi_3\partial_x, \quad x^{-r}\xi_2\xi_3\xi_j\partial_{\xi_j} \quad (j = 1, 4, \quad r = 1, 2, 3), \quad x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}, \quad x^{-1}\xi_1\xi_2\xi_4\partial_{\xi_3}, \\
& x^{-r}\xi_1\xi_2\xi_3\partial_{\xi_4} \quad (r = 1, 2, 3, 4, 5);
\end{aligned}$$

for $k = 3$

$$\begin{aligned}
& x^{-r}\xi_1\xi_2\partial_x \quad (r = 1, 2, 3), \quad x^{-r}\xi_1\xi_2\xi_j\partial_{\xi_j} \quad (j = 3, 4, \quad r = 1, 2, 3, 4, 5), \quad x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}, \\
& x^{-r}\xi_1\xi_3\partial_x \quad (r = 1, 2), \quad x^{-r}\xi_1\xi_3\xi_j\partial_{\xi_j} \quad (j = 2, 4, \quad r = 1, 2, 3, 4), \quad x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}, \\
& x^{-r}\xi_2\xi_3\partial_x \quad (r = 1, 2), \quad x^{-r}\xi_2\xi_3\xi_j\partial_{\xi_j} \quad (j = 1, 4, \quad r = 1, 2, 3, 4), \quad x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}, \\
& x^{-r}\xi_1\xi_2\xi_3\partial_{\xi_4} \quad (r = 1, \dots, 7), \quad x^{-r}\xi_1\xi_4\xi_j\partial_{\xi_j} \quad (j = 2, 3, \quad r = 1, 2), \\
& x^{-r}\xi_1\xi_2\xi_4\partial_{\xi_3} \quad (r = 1, 2, 3), \quad x^{-r}\xi_2\xi_4\xi_j\partial_{\xi_j} \quad (j = 1, 3, \quad r = 1, 2);
\end{aligned}$$

for $k \geq 4$

$$\begin{aligned}
& x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}, \quad x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}, \\
& x^{-r}\xi_1\xi_2\partial_x \quad (r = 1, \dots, 2k - 3), \quad x^{-r}\xi_1\xi_2\xi_j\partial_{\xi_j} \quad (j = 3, 4, \quad r = 1, \dots, 2k - 1), \\
& x^{-r}\xi_1\xi_3\partial_x \quad (r = 1, \dots, k - 1), \quad x^{-r}\xi_1\xi_3\xi_j\partial_{\xi_j} \quad (j = 2, 4, \quad r = 1, \dots, k + 1), \\
& x^{-r}\xi_2\xi_3\partial_x \quad (r = 1, \dots, k - 1), \quad x^{-r}\xi_2\xi_3\xi_j\partial_{\xi_j} \quad (j = 1, 4, \quad r = 1, \dots, k + 1), \\
& x^{-r}\xi_1\xi_2\xi_3\partial_{\xi_4} \quad (r = 1, \dots, 2k + 1), \quad x^{-r}\xi_1\xi_2\xi_4\partial_{\xi_3} \quad (r = 1, \dots, 2k - 3), \\
& x^{-r}\xi_1\xi_4\partial_x \quad (r = 1, \dots, k - 3), \quad x^{-r}\xi_1\xi_4\xi_j\partial_{\xi_j} \quad (j = 2, 3, \quad r = 1, \dots, k - 1), \\
& x^{-r}\xi_2\xi_4\partial_x \quad (r = 1, \dots, k - 3), \quad x^{-r}\xi_2\xi_4\xi_j\partial_{\xi_j} \quad (j = 1, 3, \quad r = 1, \dots, k - 1);
\end{aligned}$$

4) $q = 4$

$$x^{-r}\xi_1\xi_2\xi_3\xi_4\partial_x \quad (r = 1, \dots, 2k - 1).$$

Consider the exact sequence (see [BuO1])

$$0 \rightarrow \text{End } \mathbf{E} \rightarrow \mathfrak{v}(\mathbb{C}\mathbb{P}^1, \mathcal{O}_{\text{gr}})_0 \xrightarrow{\beta} \mathfrak{sl}_2(\mathbb{C}) \rightarrow 0. \quad (1)$$

The subalgebra $\mathfrak{a} \subset \mathfrak{v}(\mathbb{C}\mathbb{P}^1, \mathcal{O}_{\text{gr}})_0$ splits the sequence (1) if β is an isomorphism with $\mathfrak{sl}_2(\mathbb{C})$ or, equivalently, $\mathfrak{v}(\mathbb{C}\mathbb{P}^1, \mathcal{O}_{\text{gr}})_0 = \text{End } \mathbf{E} \oplus \mathfrak{a}$ is a direct sum of Lie algebras. In [BuO1], it was shown that the supermanifold with retract $(\mathbb{C}\mathbb{P}^1, \mathcal{O}_{\text{gr}})$ is even-homogeneous if and only if a subalgebra \mathfrak{a} splitting (1) can be lifted to it.

If this is the case, we will say that the $(\mathbb{C}\mathbb{P}^1, \mathcal{O})$ is *even-homogeneous relative* \mathfrak{a} . Then, there exist only the following (up to an automorphism in $\text{Aut } \mathbf{E}$) splitting subalgebras $\mathfrak{a}_i \simeq \mathfrak{sl}_2(\mathbb{C})$ (see [BuO1]) for the supermanifold $\mathcal{CP}_{kk20}^{1|4}$ (spanned by \mathbf{e} , \mathbf{f} , and $\mathbf{h} = [\mathbf{e}, \mathbf{f}]$) for $k = 2$

$$\mathfrak{a}_1 : \mathbf{e} = \partial_x, \quad \mathbf{f} = -x^2\partial_x - 2x(\xi_1\partial_{\xi_1} + \xi_2\partial_{\xi_2} + \xi_3\partial_{\xi_3});$$

$$\mathfrak{a}_2 : \mathbf{e} = \xi_2\partial_{\xi_1} + \partial_x, \quad \mathbf{f} = \xi_1\partial_{\xi_2} - x^2\partial_x - 2x(\xi_1\partial_{\xi_1} + \xi_2\partial_{\xi_2} + \xi_3\partial_{\xi_3});$$

$$\mathfrak{a}_3 : \mathbf{e} = \xi_3\partial_{\xi_2} + \xi_2\partial_{\xi_1} + \partial_x,$$

$$\mathbf{f} = 2\xi_2\partial_{\xi_3} + 2\xi_1\partial_{\xi_2} - x^2\partial_x - 2x(\xi_1\partial_{\xi_1} + \xi_2\partial_{\xi_2} + \xi_3\partial_{\xi_3});$$

for $k \geq 3$ and $\nabla = k\xi_1\partial_{\xi_1} + k\xi_2\partial_{\xi_2} + 2\xi_3\partial_{\xi_3}$

\mathbf{a}_1 : $\mathbf{e} = \partial_x$, $\mathbf{f} = -x^2\partial_x - x\nabla$;

\mathbf{a}_2 : $\mathbf{e} = \xi_2\partial_{\xi_1} + \partial_x$, $\mathbf{f} = \xi_1\partial_{\xi_2} - x^2\partial_x - x\nabla$.

Designate by $H^1(\mathbb{CP}^1, (\mathcal{T}_{\text{gr}}))^{\mathbf{a}}$ the set of \mathbf{a} -invariants.

2.4 Lemma ([B1]). *Let $n = 4$, $k_1 \geq k_2 \geq k_3 \geq k_4 \geq 0$ and $H^0(\mathbb{CP}^1, (\mathcal{T}_{\text{gr}})_2) = \{0\}$. Then,*

1) $H^1(\mathbb{CP}^1, (\mathcal{T}_{\text{gr}})_2)^{\mathbf{s}} \neq \{0\}$ if and only if (k_1, k_2, k_3, k_4) is one of the following

$$\begin{aligned} & (1, 1, 1, 0), \quad (2, 2, 1, 0), \quad (3, 3, 1, 0), \quad (4, 4, 1, 0), \quad (4, 4, 3, 0), \\ & (6, 4, 3, 0), \quad (2, 2, 2, 1), \quad (3, 3, 2, 1), \quad (5, 3, 2, 1), \quad (k+1, k, 1, 1)_{k \geq 1}, \\ & (k+1, k, 2, 0)_{k \geq 2}, \quad (k, k, 2, 0)_{k \geq 2}, \quad (k+3, k, 2, 2)_{k \geq 2}, \quad (k+1, k, 2, 2)_{k \geq 2}, \\ & (k, k, 2, 2)_{k \geq 2}, \quad (k+3, k, 3, 1)_{k \geq 3}, \quad (k+1, k, 3, 1)_{k \geq 3}, \quad (k, k, 3, 1)_{k \geq 3}, \\ & (k+3, k, 4, 0)_{k \geq 4}, \quad (k+2, k, 4, 0)_{k \geq 4}, \quad (k+1, k, 4, 0)_{k \geq 4}, \quad (k, k, 4, 0)_{k \geq 4}, \\ & (k_2 + k_3 + k_4 - 2, k_2, k_3, k_4)_{k_3 \geq 1}. \end{aligned}$$

2) $H^1(\mathbb{CP}^1, (\mathcal{T}_{\text{gr}})_4)^{\mathbf{s}} \neq \{0\}$ if and only if $(k_1, k_2, k_3, k_4) = (1, 1, 1, 1)$.

2.5. Lemma. *For a basis of $H^1(\mathbb{CP}^1, (\mathcal{T}_{\text{gr}})_2)^{\mathbf{a}_i}$ we can take (the classes of) the following cocycles for the supermanifold $\mathcal{CP}_{kk20}^{1|4}$.*

for $k = 2$

1) $i = 1$:

$$\begin{aligned} & x^{-1}\xi_1\xi_2\partial_x + x^{-2}(\xi_1\xi_2\xi_3\partial_{\xi_3} + \xi_1\xi_2\xi_4\partial_{\xi_4}), \quad x^{-1}\xi_1\xi_4\xi_2\partial_{\xi_2}, \quad x^{-1}\xi_2\xi_4\xi_1\partial_{\xi_1}, \\ & x^{-1}\xi_1\xi_3\partial_x + x^{-2}(\xi_1\xi_3\xi_2\partial_{\xi_2} + \xi_1\xi_3\xi_4\partial_{\xi_4}), \quad x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_1\xi_2\xi_4\partial_{\xi_3}, \\ & x^{-1}\xi_2\xi_3\partial_x + x^{-2}(\xi_2\xi_3\xi_1\partial_{\xi_1} + \xi_2\xi_3\xi_4\partial_{\xi_4}), \quad x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}, \quad x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}; \end{aligned}$$

2) $i = 2$: $x^{-1}\xi_1\xi_2\xi_4\partial_{\xi_3}$;

3) $i = 3$: $x^{-1}\xi_1\xi_2\xi_4\partial_{\xi_3}$, $2x^{-3}\xi_1\xi_2\xi_3\partial_{\xi_3} + x^{-2}\xi_1\xi_3\xi_2\partial_{\xi_2} + x^{-1}\xi_2\xi_3\xi_1\partial_{\xi_1}$,
 $2x^{-3}\xi_1\xi_2\xi_4\partial_{\xi_4} + x^{-2}\xi_1\xi_3\xi_4\partial_{\xi_4} + x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_4}$;

for $k = 3$ and $k \geq 5$

1) $i = 1$: $x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}$, $x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}$, $x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}$;

2) $i = 2$: $x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}$, $x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}$;

for $k = 4$

1) $i = 1$:

$$\begin{aligned} & x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}, \quad x^{-1}\xi_1\xi_4\partial_x + 2x^{-2}\xi_1\xi_4\xi_2\partial_{\xi_2} + x^{-2}\xi_1\xi_4\xi_3\partial_{\xi_3}, \\ & x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}, \quad x^{-1}\xi_2\xi_4\partial_x + 2x^{-2}\xi_2\xi_4\xi_1\partial_{\xi_1} + x^{-2}\xi_2\xi_4\xi_3\partial_{\xi_3}; \end{aligned}$$

2) $i = 2$: $x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}$, $x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}$.

Proof. follows from Theorem 15 in [BaO2]. □

The proof of the following Lemma is similar.

2.6. Lemma. $H^1(\mathbb{CP}^1, (\mathcal{T}_{\text{gr}})_4)^{\mathfrak{a}} = \{0\}$ for any splitting subalgebra \mathfrak{a} and supermanifold $\mathcal{CP}_{kk20}^{1|4}$.

Let $\lambda_2 : \mathcal{A}ut_{(2)}\mathcal{O}_{\text{gr}} \rightarrow (\mathcal{T}_{\text{gr}})_2$ be a sheaf homomorphism which to any germ of the automorphism a assigns the 2-component of $\log a$ in $(\mathcal{T}_{\text{gr}})_2 \oplus (\mathcal{T}_{\text{gr}})_4$.

Denote by $H^1(\mathbb{CP}^1, \mathcal{A}ut_{(2)}\mathcal{O}_{\text{gr}})^{\mathfrak{a}}$ the set of classes that determine supermanifolds even-homogeneous relative \mathfrak{a} .

2.7. Proposition. If \mathfrak{a} splits the sequence (1), then λ_2^* maps $H^1(\mathbb{CP}^1, \mathcal{A}ut_{(2)}\mathcal{O}_{\text{gr}})^{\mathfrak{a}}$ bijectively to $H^1(\mathbb{CP}^1, (\mathcal{T}_{\text{gr}})_2)^{\mathfrak{a}}$.

Proof. follows from Statement 2.1 and Lemma 2.6. □

Using the lifting condition of the vector field on non-split supermanifold described in [O2], and Lemma 2.6, we deduce that any supermanifold $\bar{0}$ -homogeneous relative \mathfrak{a} is determined by cocycles u^2 such that the class $[u^2]$ is \mathfrak{a} -invariant and $[u^2, u^2] = 0$. Since $[u^2, u^2] = 0$ is true for all cocycles in Lemma 2.5, the following Theorem holds.

2.8. Theorem. For any splitting subalgebra \mathfrak{a} , the even-homogeneous relative \mathfrak{a} supermanifolds are determined by the cocycles listed in Lemma 2.5.

Thus, all even-homogeneous relative \mathfrak{a} non-split supermanifolds with retract $\mathcal{CP}_{kk20}^{1|4}$ are described for any $k \geq 2$.

Let us find out if any of these even-homogeneous relative \mathfrak{a} non-split supermanifolds are homogeneous. For this we use the following Proposition analogous to Proposition 15 in [BuO1]:

2.9. Proposition. Under conditions of Proposition 2.7, let $(\mathbb{CP}^1, \mathcal{O})$ be even-homogeneous relative \mathfrak{a}_i for one of $i = 1, 2, 3$.

If $i = 1$, then $(\mathbb{CP}^1, \mathcal{O})$ is homogeneous if and only if the vector fields ∂_{ξ_j} , where $j = 1, \dots, 4$ can be lifted to $(\mathbb{CP}^1, \mathcal{O})$;

If $i = 2$, then $(\mathbb{CP}^1, \mathcal{O})$ is homogeneous if and only if the vector fields ∂_{ξ_j} , where $j = 1, 3, 4$, can be lifted to $(\mathbb{CP}^1, \mathcal{O})$.

If $i = 3$, then $(\mathbb{CP}^1, \mathcal{O})$ is homogeneous if and only if the vector fields ∂_{ξ_1} and ∂_{ξ_4} can be lifted to $(\mathbb{CP}^1, \mathcal{O})$.

Proposition 2.9 applied to the cocycles of Lemma 2.5 implies the classification (Theorems 2.10–2.20). For other 4-tuples, homogeneous non-split supermanifolds do not exist.

2.10. Theorem. There does not exist homogeneous non-split supermanifolds with retract $\mathcal{CP}_{kk20}^{1|4}$ for any $k \geq 2$.

2.11. Theorem. *For the following tuples $(k_1 k_2 k_3 k_4)$, there does not exist homogeneous non-split supermanifolds $\mathcal{CP}_{k_1 k_2 k_3 k_4}^{1|4}$.*

$$(1, 1, 1, 0), (3, 3, 1, 0), (4, 4, 1, 0), (4, 4, 3, 0), (6, 4, 3, 0), (5, 4, 3, 0), (k+2, k, 4, 0)_{k \geq 4}, \\ (3, 3, 2, 1), (5, 3, 2, 1), (k+3, k, 4, 0)_{k \geq 4}, (k+3, k, 3, 1)_{k \geq 3}, (k+1, k, 4, 0)_{k \geq 4}, \\ (k+1, k, 3, 1)_{k \geq 3}, (k+1, k, 2, 0)_{k > 2}, (k, k, 4, 0)_{k \geq 4}, (k, k, 3, 1)_{k \geq 3}.$$

2.12. Theorem. *For every of the following tuples $(k_1 k_2 k_3 k_4)$, there exists one homogeneous non-split supermanifold with retract $\mathcal{CP}_{k_1 k_2 k_3 k_4}^{1|4}$ which can be represented, up to an isomorphism, by (the classes of) the following cocycles.*

$$(2, 2, 1, 0) \quad x^{-1} \xi_1 \xi_4 \xi_3 \partial_{\xi_3}; \\ (2, 2, 2, 1) \quad x^{-1} \xi_1 \xi_2 \partial_x; \\ (3, 2, 2, 0) \quad 2x^{-1} \xi_1 \xi_3 \partial_x + 3x^{-2} \xi_2 \xi_3 \xi_1 \partial_{\xi_1}; \\ (4, 3, 2, 1) \quad x^{-1} \xi_2 \xi_3 \xi_4 \partial_{\xi_1}; \\ (k+1, k, 1, 1)_{k > 1} \quad x^{-1} \xi_3 \xi_4 \xi_1 \partial_{\xi_1}; \\ (k+3, k, 2, 2)_{k \geq 2} \quad 2x^{-1} \xi_3 \xi_4 \partial_x + (k+3)x^{-2} \xi_3 \xi_4 \xi_1 \partial_{\xi_1} + kx^{-2} \xi_3 \xi_4 \xi_2 \partial_{\xi_2}; \\ (k+1, k, 2, 2)_{k > 2} \quad 2x^{-1} \xi_3 \xi_4 \partial_x + (k+1)x^{-2} \xi_3 \xi_4 \xi_1 \partial_{\xi_1} + kx^{-2} \xi_3 \xi_4 \xi_2 \partial_{\xi_2}; \\ (k, k, 2, 2)_{k \geq 4} \quad x^{-1} \xi_3 \xi_4 \partial_x.$$

2.13. Theorem. *If $k_4 \neq 0$, then for every $(k_2 + k_3 + k_4 - 2, k_2, k_3, k_4)_{k_3 \geq 1}$ which is different from $(k, k, 1, 1)_{k \geq 1}$, there exists one homogeneous non-split supermanifold which can be represented, up to an isomorphism, by the (class of) cocycle*

$$x^{-1} \xi_2 \xi_3 \xi_4 \partial_{\xi_1}.$$

2.14. Theorem. *There exist two homogeneous non-split supermanifolds with their retracts being $\mathcal{CP}_{2222}^{1|4}$, and which can be represented, up to an isomorphism, by (the classes of) the following cocycles.*

$$x^{-1} \xi_1 \xi_2 \partial_x + x^{-2} \xi_1 \xi_2 \xi_3 \partial_{\xi_3} + x^{-2} \xi_1 \xi_2 \xi_4 \partial_{\xi_4}, \\ x^{-1} \xi_1 \xi_2 \partial_x + x^{-2} \xi_1 \xi_2 \xi_3 \partial_{\xi_3} + x^{-2} \xi_1 \xi_2 \xi_4 \partial_{\xi_4} + x^{-1} \xi_3 \xi_4 \partial_x + \\ + x^{-2} \xi_3 \xi_4 \xi_1 \partial_{\xi_1} + x^{-2} \xi_3 \xi_4 \xi_2 \partial_{\xi_2}.$$

2.15. Theorem. *There exist two homogeneous non-split supermanifolds with their retracts being $\mathcal{CP}_{3222}^{1|4}$, and which can be represented, up to an isomorphism, by (the classes of) the cocycles*

$$x^{-1} \xi_2 \xi_3 \partial_x + x^{-2} \xi_2 \xi_3 \xi_1 \partial_{\xi_1} + x^{-2} \xi_2 \xi_3 \xi_4 \partial_{\xi_4}, \\ 2x^{-3} \xi_2 \xi_3 \xi_1 \partial_{\xi_1} + x^{-2} \xi_2 \xi_4 \xi_1 \partial_{\xi_1} + x^{-1} \xi_3 \xi_4 \xi_1 \partial_{\xi_1}.$$

2.16. Theorem. *There exist three homogeneous non-split supermanifolds with their retracts being $\mathcal{CP}_{3322}^{1|4}$, and which can be represented, up to an isomorphism, by (the classes of) the following cocycles.*

$$x^{-1} \xi_3 \xi_4 \partial_x, \quad x^{-2} \xi_1 \xi_3 \partial_x + x^{-1} \xi_2 \xi_3 \partial_x, \\ x^{-1} \xi_3 \xi_4 \partial_x + x^{-2} \xi_1 \xi_3 \partial_x + x^{-1} \xi_2 \xi_3 \partial_x.$$

2.17. Theorem. *There exist five homogeneous non-split supermanifolds with their retracts being $\mathcal{CP}_{kk11}^{1|4}$, where $k > 2$, which can be represented, up to an isomorphism, by (the classes of) the following cocycles.*

$$\begin{aligned} & x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}, \quad x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1} + x^{-1}\xi_3\xi_4\xi_2\partial_{\xi_2}, \\ & x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2} + x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}, \quad x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1} + x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2} + x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}. \end{aligned}$$

2.18. Theorem. *There exist eight homogeneous non-split supermanifolds with their retracts being $\mathcal{CP}_{2111}^{1|4}$, which can be represented, up to an isomorphism, by (the classes of) the following cocycles.*

$$\begin{aligned} & x^{-1}\xi_2\xi_3\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_4}, \quad x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}, \\ & x^{-1}\xi_2\xi_3\xi_1\partial_{\xi_1} + x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_4}, \quad x^{-1}\xi_2\xi_3\xi_1\partial_{\xi_1} + x^{-1}\xi_3\xi_4\xi_2\partial_{\xi_2}, \\ & x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_4} + x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1} + x^{-1}\xi_3\xi_4\xi_2\partial_{\xi_2}, \\ & x^{-1}\xi_2\xi_3\xi_1\partial_{\xi_1} + x^{-1}\xi_2\xi_4\xi_3\partial_{\xi_3} + x^{-1}\xi_3\xi_4\xi_2\partial_{\xi_2}. \end{aligned}$$

2.19. Theorem. *There exist nine homogeneous non-split supermanifolds with their retracts being $\mathcal{CP}_{2211}^{1|4}$, which can be represented, up to an isomorphism, by (the classes of) the following cocycles.*

$$\begin{aligned} & x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}, \\ & x^{-2}\xi_2\xi_3\xi_1\partial_{\xi_1} - x^{-1}\xi_2\xi_4\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1} + x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}, \\ & x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1} + x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1} + x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}, \\ & x^{-2}\xi_2\xi_3\xi_1\partial_{\xi_1} - x^{-1}\xi_2\xi_4\xi_1\partial_{\xi_1} + x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}, \\ & x^{-2}\xi_2\xi_3\xi_1\partial_{\xi_1} - x^{-1}\xi_2\xi_4\xi_1\partial_{\xi_1} + x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}, \\ & x^{-2}\xi_2\xi_3\xi_1\partial_{\xi_1} - x^{-1}\xi_2\xi_4\xi_1\partial_{\xi_1} + x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1} + x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}, \\ & x^{-2}\xi_2\xi_3\xi_1\partial_{\xi_1} - x^{-1}\xi_2\xi_4\xi_1\partial_{\xi_1} + x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1} + \\ & \quad + x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1} + x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}. \end{aligned}$$

2.20 Theorem ([B5]). *There exist four homogeneous non-split superstrings and one 1-parameter family of homogeneous non-split superstrings with retract $\mathcal{CP}_{1111}^{1|4}$, which can be represented, up to an isomorphism, by (the classes of) the following cocycles.*

$$\begin{aligned} & x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}, \quad x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1} - x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}, \\ & x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1} - x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2} + x^{-1}\xi_1\xi_2\xi_4\partial_{\xi_3}, \\ & x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1} - x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2} + x^{-1}\xi_1\xi_2\xi_4\partial_{\xi_3} - x^{-1}\xi_1\xi_2\xi_3\partial_{\xi_4}, \\ & t(x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1} - x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2} + x^{-1}\xi_1\xi_2\xi_4\partial_{\xi_3} - x^{-1}\xi_1\xi_2\xi_3\partial_{\xi_4}) + \\ & \quad + x^{-1}\xi_1\xi_2\xi_3\xi_4\partial_x, \text{ where } t \in \mathbb{C}^\times. \end{aligned}$$

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¹Certain attributions in this paper are wrong. For example, the Lie superalgebra $\Gamma(\sigma_1, \sigma_2, \sigma_3)$ — the deformation of $\mathfrak{osp}(4|2)$ — was discovered by Kaplansky [Kap*], [Kapp*], see [KIE*]; later Kac redenoted $\Gamma(\sigma_1, \sigma_2, \sigma_3)$ by $D(2, 1; \alpha)$; for reasons given in [CCLL*], this is an ill-chosen notation. For a reasonable and meaningful notation which gradually started to replace the initial *ad hoc* names of the exceptional Lie superalgebras given by Kaplansky and later ones, due to Kac, see [Sr*], [CCLL*].

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