



**HAL**  
open science

# Homogeneous non-split superstrings of odd dimension 4

Mikhail Bashkin

► **To cite this version:**

Mikhail Bashkin. Homogeneous non-split superstrings of odd dimension 4. Communications in Mathematics, In press, Volume 30 (2022), Issue 3 (Special issue: in memory of Arkady Onishchik), 10.46298/cm.9843 . hal-03736767v2

**HAL Id: hal-03736767**

**<https://hal.science/hal-03736767v2>**

Submitted on 4 Jan 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Homogeneous non-split superstrings of odd dimension 4

*Michail Bashkin*

**Abstract.** Let  $\mathbf{L}_k$  be the holomorphic line bundle of degree  $k \in \mathbb{Z}$  on the projective line. The tuples  $(k_1 k_2 k_3 k_4)$  for which there exists no homogeneous non-split supermanifolds  $\mathcal{CP}_{k_1 k_2 k_3 k_4}^{1|4}$  associated with the vector bundle  $\mathbf{L}_{-k_1} \oplus \mathbf{L}_{-k_2} \oplus \mathbf{L}_{-k_3} \oplus \mathbf{L}_{-k_4}$  are classified.

For many types of the remaining tuples, there are listed cocycles that determine homogeneous non-split supermanifolds.

Proofs follow the lines indicated in the paper Bunegina V.A., Onishchik A.L., Homogeneous supermanifolds associated with the complex projective line. *J. Math. Sci. V. 82 (1996) 3503–3527.*

## 1 Introduction

In this paper, I summarize the results of classification (up to a diffeomorphism) of homogeneous complex (more precisely, almost complex, see [BGLS\*], since the vanishing of the Nijenhuis tensor is never required) supermanifolds  $\mathcal{M} := (M, \mathcal{O})$ , where  $M = \mathbb{CP}^1$  and  $\dim \mathcal{M} = 1|n$ . (*Comments with starred references are added by the editor of this Special Volume. D.L.*)

For the case where  $\mathcal{M}$  is split, the classification is known, see [BuO1]: the non-diffeomorphic supermanifolds are in one-to-one correspondence with  $n$ -tuples of non-negative integers.

If  $\mathcal{M}$  is non-split, the classification is considerably more complicated and reduces to computation of cohomology of split homogeneous supermanifolds with coefficients in the tangent sheaf.

For  $n = 2$  and  $3$ , V. A. Bunegina and A. L. Onishchik completely investigated the case, see [BuO1], [BuO2].

For  $n = 4$ , see below (summary of the results of [B1] – [B4], [BaO1], [BaO2]). For the method of the proof, see [BuO1], [BuO2].

---

*MSC 2020:* Primary 32C11, 32M10 Secondary 53Z05, 81T30

*Keywords:* Non-split homogeneous supermanifold, retract, holomorphic vector bundle

*Affiliation:* P.A. Solovyov Rybinsk State Aviation Technical University, Rybinsk, Russia

*E-mail:* mbashkin@rsatu.ru

## 2 Results

As is known, any holomorphic bundle  $\mathbf{E}$  over  $\mathbb{C}\mathbb{P}^1$  can be uniquely decomposed into a direct sum of line bundles: Grothendieck’s theorem, see [HM\*]. (For interesting applications of Linear Superalgebra (with elements of category theory) to the description of vector bundles over projective spaces, see the review [BG\*]. For the latest results on non-splitness of supermanifolds whose retract is the Grassmann manifold, see [Vi\*], [Vi1\*], [Vi2\*].) Let  $\mathbf{L}_k$  be the holomorphic line bundle of degree  $k \in \mathbb{Z}$ .

Consider a holomorphic bundle

$$\mathbf{E} = \mathbf{L}_{-k_1} \oplus \mathbf{L}_{-k_2} \oplus \mathbf{L}_{-k_3} \oplus \mathbf{L}_{-k_4}, \text{ where } k_1 \geq k_2 \geq k_3 \geq k_4 \geq 0.$$

If  $\mathcal{M}$  is homogeneous, then the  $k_i$  must be non-negative, see [BuO1].

Let  $\mathcal{CP}_{k_1 k_2 k_3 k_4}^{1|4}$  designate the split supermanifold determined by  $\mathbf{E}$ .

Let us cover  $\mathbb{C}\mathbb{P}^1$  by two affine charts  $U_0$  and  $U_1$  with local coordinates  $x$  and  $y = \frac{1}{x}$ , respectively. Then, the transition functions on  $\mathcal{CP}_{k_1 k_2 k_3 k_4}^{1|4}$  in  $U_0 \cap U_1$  are of the form

$$\begin{aligned} y &= x^{-1}, \\ \eta_i &= x^{-k_i} \xi_i \text{ for } i = 1, \dots, 4, \end{aligned}$$

where  $\xi_i$  and  $\eta_i$  are basis sections of  $\mathbf{E}$  over  $U_0$  and  $U_1$ , respectively.

Let  $M$  be a compact complex manifold. We will sometimes need general statements about the  $m|n$ -dimensional supermanifold  $\mathcal{M} = (M, \mathcal{O})$ . Let  $\mathcal{I} \subset \mathcal{O}$  be the subsheaf of ideals generated by the subsheaf  $\mathcal{O}_{\bar{1}}$ . Consider the filtration of  $\mathcal{O}$  by powers of  $\mathcal{I}$ :

$$\mathcal{O} = \mathcal{I}^0 \supset \mathcal{I} \supset \mathcal{I}^2 \supset \dots \supset \mathcal{I}^n \supset \mathcal{I}^{n+1} = 0.$$

The graded sheaf  $\text{gr } \mathcal{O} = \bigoplus_{0 \leq i \leq n} \text{gr}_i \mathcal{O}$  with  $\text{gr}_i \mathcal{O} := \mathcal{I}^i / \mathcal{I}^{i+1}$  defines the split supermanifold  $(M, \text{gr } \mathcal{O})$  called the *retract* of  $(M, \mathcal{O})$ . Let  $\mathcal{T}_{\text{gr}} := \bigoplus_{-1 \leq p \leq 4} (\mathcal{T}_{\text{gr}})_p$  denote the graded tangent sheaf of the split supermanifold  $(M, \mathcal{O}_{\text{gr}})$ . Consider the subsheaf

$$\mathcal{Aut}_{(2)} \mathcal{O}_{\text{gr}} = \exp((\mathcal{T}_{\text{gr}})_2 \oplus (\mathcal{T}_{\text{gr}})_4)$$

of the sheaf  $\mathcal{Aut } \mathcal{O}_{\text{gr}}$ . Thanks to a theorem due to Green ([G]), the set of supermanifolds with a given retract  $(M, \mathcal{O}_{\text{gr}})$  is isomorphic to the set of orbits of the group  $\text{Aut } \mathbf{E}$  in  $H^1(M, \mathcal{Aut}_{(2)} \mathcal{O}_{\text{gr}})$ .

In what follows we often assume that  $H^0(M, (\mathcal{T}_{\text{gr}})_2) = 0$ . This is needed for existence of a bijection (see [O3, Section 3.2 pp.23–37])

$$H^1(M, \mathcal{Aut}_{(2)} \mathcal{O}_{\text{gr}}) \longleftrightarrow H^1(M, (\mathcal{T}_{\text{gr}})_2) \oplus H^1(M, (\mathcal{T}_{\text{gr}})_4).$$

**2.1 Statement ([BaO2]).** For  $n \leq 5$ , let  $H^0(M, (\mathcal{T}_{\text{gr}})_2) = 0$  and let there be given subspaces  $Q_{2p} \subset Z^1(\mathfrak{U}, (\mathcal{T}_{\text{gr}})_{2p})$ , where  $p = 1, 2$ , such that every cohomology class in  $H^1(M, (\mathcal{T}_{\text{gr}})_{2p})$  contains precisely one cocycle in  $Q_{2p}$ .

Then, every cohomology class in  $H^1(M, \mathcal{Aut}_{(2)} \mathcal{O}_{\text{gr}})$  can be represented by a unique cocycle of the form  $z = \exp(u^2 + u^4)$ , where  $u^2 \in Q_2$  and  $u^4 \in Q_4$ .

Hereafter, the expression “ $(M, \mathcal{O})$  is determined by the cocycle  $u^2 + u^4$ ” means that  $(M, \mathcal{O})$  actually corresponds to the cocycle  $z = \exp(u^2 + u^4)$ .

In [BuO1, Prop. 12], there is given a description of the algebra  $\text{End } \mathbf{E}$ . Let us reformulate this Proposition in terms more convenient for us here. Any endomorphism  $a \in \text{End } \mathbf{E}$  can be considered as an endomorphism of the sheaf  $\mathcal{E}$  of  $\mathcal{F}$ -modules. In  $U_0$ , we have

$$a(\xi_i) = \sum_{1 \leq j \leq n} a_{ji} \xi_j \quad \text{for } i = 1, \dots, n,$$

where  $A = (a_{ji})$  is a matrix with elements  $a_{ji} \in \mathcal{F}(U_0)$ . The matrix  $A$  completely determines the endomorphism  $a$ , and  $a \in \text{Aut } \mathbf{E}$  if and only if  $A$  is invertible.

The following statement is needed in the classification of homogeneous non-split supermanifolds up to a diffeomorphism.

**2.2 Statement ([BaO2]).** The matrix  $A = (a_{ji})$  over  $\mathcal{F}(U_0)$  corresponds to an element in  $\text{End } \mathbf{E}$  if and only if

$$a_{ij} = \begin{cases} 0 & \text{if } k_i < k_j, \\ \text{is a polynomial of degree } \leq k_i - k_j & \text{if } k_i \geq k_j. \end{cases}$$

First, consider the tuples  $(k, k, 2, 0)$  for  $k \geq 2$ . Let  $\mathfrak{v}(\mathbb{C}\mathbb{P}^1, \mathcal{O}_{\text{gr}})$  be the Lie superalgebra of vector fields on  $\mathbb{C}\mathbb{P}_{kk20}^{1|4}$ . Theorem 14 in [BaO1] implies the following

**2.3. Lemma.** For a basis of  $H^1(\mathbb{C}\mathbb{P}^1, (\mathcal{T}_{\text{gr}})_q)$ , where  $q = 1, 2, 4$ , we can take (the classes of) the following cocycles defining  $\mathcal{CP}_{kk20}^{1|4}$ .

1)  $q = 1$  for  $k = 2$

$$\begin{aligned} & x^{-1}\xi_1\xi_2\partial_{\xi_2}, \quad x^{-1}\xi_1\xi_4\partial_{\xi_4}, \quad x^{-1}\xi_2\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_2\xi_4\partial_{\xi_4}, \quad x^{-1}\xi_3\xi_1\partial_{\xi_1}, \\ & x^{-1}\xi_3\xi_4\partial_{\xi_4}, \quad x^{-1}\xi_1\xi_2\partial_{\xi_3}, \quad x^{-1}\xi_1\xi_3\partial_{\xi_2}, \quad x^{-1}\xi_2\xi_3\partial_{\xi_1}, \\ & x^{-r}\xi_1\xi_2\partial_{\xi_4}, \quad x^{-r}\xi_1\xi_3\partial_{\xi_4}, \quad x^{-r}\xi_2\xi_3\partial_{\xi_4} \quad \text{for } r = 1, 2, 3 \end{aligned}$$

for  $k = 3$

$$\begin{aligned} & x^{-1}\xi_3\xi_4\partial_{\xi_4}, \quad x^{-1}\xi_3\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_1\xi_3\partial_{\xi_2}, \quad x^{-r}\xi_1\xi_3\partial_{\xi_4} \quad (r = 1, 2, 3, 4), \quad x^{-1}\xi_2\xi_3\partial_{\xi_1}, \\ & x^{-r}\xi_2\xi_3\partial_{\xi_4} \quad (r = 1, 2, 3, 4), \quad x^{-r}\xi_1\xi_2\partial_{\xi_3} \quad (r = 1, 2, 3), \quad x^{-r}\xi_1\xi_2\partial_{\xi_4} \quad (r = 1, 2, 3, 4, 5), \\ & x^{-r}\xi_1\xi_j\partial_{\xi_j} \quad (j = 2, 3, 4, r = 1, 2), \quad x^{-r}\xi_2\xi_j\partial_{\xi_j} \quad (j = 1, 3, 4, r = 1, 2); \end{aligned}$$

for  $k \geq 4$

$$\begin{aligned} & x^{-1}\xi_3\xi_4\partial_{\xi_4}, \quad x^{-1}\xi_3\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_1\xi_3\partial_{\xi_2}, \quad x^{-r}\xi_1\xi_3\partial_{\xi_4} \quad (r = 1, \dots, k + 1), \\ & x^{-r}\xi_1\xi_2\partial_{\xi_3} \quad (r = 1, \dots, 2k - 3), \quad x^{-1}\xi_2\xi_3\partial_{\xi_1}, \quad x^{-r}\xi_2\xi_3\partial_{\xi_4} \quad (r = 1, \dots, k + 1), \\ & x^{-r}\xi_1\xi_2\partial_{\xi_4} \quad (r = 1, \dots, 2k - 1), \quad x^{-r}\xi_i\partial_x \quad (i = 1, 2, j = 1, \dots, k - 3), \\ & x^{-r}\xi_1\xi_j\partial_{\xi_j} \quad (j = 2, 3, 4, r = 1, \dots, k - 1), \quad x^{-r}\xi_j\xi_4\partial_{\xi_3} \quad (j = 1, 2, r = 1, \dots, k - 3), \\ & x^{-r}\xi_2\xi_j\partial_{\xi_j} \quad (j = 1, 3, 4, r = 1, \dots, k - 1); \end{aligned}$$

2)  $q = 2$ for  $k = 2$ 

$$\begin{aligned} & x^{-1}\xi_1\xi_2\partial_x, \quad x^{-r}\xi_1\xi_2\xi_j\partial_{\xi_j} \quad (j = 3, 4, \quad r = 1, 2, 3), \quad x^{-1}\xi_1\xi_4\xi_2\partial_{\xi_2}, \quad x^{-1}\xi_2\xi_4\xi_1\partial_{\xi_1}, \\ & x^{-1}\xi_1\xi_3\partial_x, \quad x^{-r}\xi_1\xi_3\xi_j\partial_{\xi_j} \quad (j = 2, 4, \quad r = 1, 2, 3), \quad x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}, \\ & x^{-1}\xi_2\xi_3\partial_x, \quad x^{-r}\xi_2\xi_3\xi_j\partial_{\xi_j} \quad (j = 1, 4, \quad r = 1, 2, 3), \quad x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}, \quad x^{-1}\xi_1\xi_2\xi_4\partial_{\xi_3}, \\ & x^{-r}\xi_1\xi_2\xi_3\partial_{\xi_4} \quad (r = 1, 2, 3, 4, 5); \end{aligned}$$

for  $k = 3$ 

$$\begin{aligned} & x^{-r}\xi_1\xi_2\partial_x \quad (r = 1, 2, 3), \quad x^{-r}\xi_1\xi_2\xi_j\partial_{\xi_j} \quad (j = 3, 4, \quad r = 1, 2, 3, 4, 5), \quad x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}, \\ & x^{-r}\xi_1\xi_3\partial_x \quad (r = 1, 2), \quad x^{-r}\xi_1\xi_3\xi_j\partial_{\xi_j} \quad (j = 2, 4, \quad r = 1, 2, 3, 4), \quad x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}, \\ & x^{-r}\xi_2\xi_3\partial_x \quad (r = 1, 2), \quad x^{-r}\xi_2\xi_3\xi_j\partial_{\xi_j} \quad (j = 1, 4, \quad r = 1, 2, 3, 4), \quad x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}, \\ & x^{-r}\xi_1\xi_2\xi_3\partial_{\xi_4} \quad (r = 1, \dots, 7), \quad x^{-r}\xi_1\xi_4\xi_j\partial_{\xi_j} \quad (j = 2, 3, \quad r = 1, 2), \\ & x^{-r}\xi_1\xi_2\xi_4\partial_{\xi_3} \quad (r = 1, 2, 3), \quad x^{-r}\xi_2\xi_4\xi_j\partial_{\xi_j} \quad (j = 1, 3, \quad r = 1, 2); \end{aligned}$$

for  $k \geq 4$ 

$$\begin{aligned} & x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}, \quad x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}, \\ & x^{-r}\xi_1\xi_2\partial_x \quad (r = 1, \dots, 2k - 3), \quad x^{-r}\xi_1\xi_2\xi_j\partial_{\xi_j} \quad (j = 3, 4, \quad r = 1, \dots, 2k - 1), \\ & x^{-r}\xi_1\xi_3\partial_x \quad (r = 1, \dots, k - 1), \quad x^{-r}\xi_1\xi_3\xi_j\partial_{\xi_j} \quad (j = 2, 4, \quad r = 1, \dots, k + 1), \\ & x^{-r}\xi_2\xi_3\partial_x \quad (r = 1, \dots, k - 1), \quad x^{-r}\xi_2\xi_3\xi_j\partial_{\xi_j} \quad (j = 1, 4, \quad r = 1, \dots, k + 1), \\ & x^{-r}\xi_1\xi_2\xi_3\partial_{\xi_4} \quad (r = 1, \dots, 2k + 1), \quad x^{-r}\xi_1\xi_2\xi_4\partial_{\xi_3} \quad (r = 1, \dots, 2k - 3), \\ & x^{-r}\xi_1\xi_4\partial_x \quad (r = 1, \dots, k - 3), \quad x^{-r}\xi_1\xi_4\xi_j\partial_{\xi_j} \quad (j = 2, 3, \quad r = 1, \dots, k - 1), \\ & x^{-r}\xi_2\xi_4\partial_x \quad (r = 1, \dots, k - 3), \quad x^{-r}\xi_2\xi_4\xi_j\partial_{\xi_j} \quad (j = 1, 3, \quad r = 1, \dots, k - 1); \end{aligned}$$

4)  $q = 4$ 

$$x^{-r}\xi_1\xi_2\xi_3\xi_4\partial_x \quad (r = 1, \dots, 2k - 1).$$

Consider the exact sequence (see [BuO1])

$$0 \rightarrow \text{End } \mathbf{E} \rightarrow \mathfrak{v}(\mathbb{C}\mathbb{P}^1, \mathcal{O}_{\text{gr}})_0 \xrightarrow{\beta} \mathfrak{sl}_2(\mathbb{C}) \rightarrow 0. \quad (1)$$

The subalgebra  $\mathfrak{a} \subset \mathfrak{v}(\mathbb{C}\mathbb{P}^1, \mathcal{O}_{\text{gr}})_0$  splits the sequence (1) if  $\beta$  is an isomorphism with  $\mathfrak{sl}_2(\mathbb{C})$  or, equivalently,  $\mathfrak{v}(\mathbb{C}\mathbb{P}^1, \mathcal{O}_{\text{gr}})_0 = \text{End } \mathbf{E} \oplus \mathfrak{a}$  is a direct sum of Lie algebras. In [BuO1], it was shown that the supermanifold with retract  $(\mathbb{C}\mathbb{P}^1, \mathcal{O}_{\text{gr}})$  is even-homogeneous if and only if a subalgebra  $\mathfrak{a}$  splitting (1) can be lifted to it.

If this is the case, we will say that the  $(\mathbb{C}\mathbb{P}^1, \mathcal{O})$  is *even-homogeneous relative*  $\mathfrak{a}$ . Then, there exist only the following (up to an automorphism in  $\text{Aut } \mathbf{E}$ ) splitting subalgebras  $\mathfrak{a}_i \simeq \mathfrak{sl}_2(\mathbb{C})$  (see [BuO1]) for the supermanifold  $\mathcal{CP}_{kk20}^{1|4}$  (spanned by  $\mathbf{e}$ ,  $\mathbf{f}$ , and  $\mathbf{h} = [\mathbf{e}, \mathbf{f}]$ ) for  $k = 2$

$$\begin{aligned} \mathfrak{a}_1 : & \mathbf{e} = \partial_x, \quad \mathbf{f} = -x^2\partial_x - 2x(\xi_1\partial_{\xi_1} + \xi_2\partial_{\xi_2} + \xi_3\partial_{\xi_3}); \\ \mathfrak{a}_2 : & \mathbf{e} = \xi_2\partial_{\xi_1} + \partial_x, \quad \mathbf{f} = \xi_1\partial_{\xi_2} - x^2\partial_x - 2x(\xi_1\partial_{\xi_1} + \xi_2\partial_{\xi_2} + \xi_3\partial_{\xi_3}); \\ \mathfrak{a}_3 : & \mathbf{e} = \xi_3\partial_{\xi_2} + \xi_2\partial_{\xi_1} + \partial_x, \\ & \mathbf{f} = 2\xi_2\partial_{\xi_3} + 2\xi_1\partial_{\xi_2} - x^2\partial_x - 2x(\xi_1\partial_{\xi_1} + \xi_2\partial_{\xi_2} + \xi_3\partial_{\xi_3}); \end{aligned}$$

for  $k \geq 3$  and  $\nabla = k\xi_1\partial_{\xi_1} + k\xi_2\partial_{\xi_2} + 2\xi_3\partial_{\xi_3}$

$\mathbf{a}_1$  :  $\mathbf{e} = \partial_x$ ,  $\mathbf{f} = -x^2\partial_x - x\nabla$ ;

$\mathbf{a}_2$  :  $\mathbf{e} = \xi_2\partial_{\xi_1} + \partial_x$ ,  $\mathbf{f} = \xi_1\partial_{\xi_2} - x^2\partial_x - x\nabla$ .

Designate by  $H^1(\mathbb{CP}^1, (\mathcal{T}_{\text{gr}}))^{\mathbf{a}}$  the set of  $\mathbf{a}$ -invariants.

**2.4 Lemma ([B1]).** *Let  $n = 4$ ,  $k_1 \geq k_2 \geq k_3 \geq k_4 \geq 0$  and  $H^0(\mathbb{CP}^1, (\mathcal{T}_{\text{gr}})_2) = \{0\}$ . Then,*

1)  $H^1(\mathbb{CP}^1, (\mathcal{T}_{\text{gr}})_2)^{\mathbf{s}} \neq \{0\}$  if and only if  $(k_1, k_2, k_3, k_4)$  is one of the following

$$\begin{aligned} & (1, 1, 1, 0), \quad (2, 2, 1, 0), \quad (3, 3, 1, 0), \quad (4, 4, 1, 0), \quad (4, 4, 3, 0), \\ & (6, 4, 3, 0), \quad (2, 2, 2, 1), \quad (3, 3, 2, 1), \quad (5, 3, 2, 1), \quad (k+1, k, 1, 1)_{k \geq 1}, \\ & (k+1, k, 2, 0)_{k \geq 2}, \quad (k, k, 2, 0)_{k \geq 2}, \quad (k+3, k, 2, 2)_{k \geq 2}, \quad (k+1, k, 2, 2)_{k \geq 2}, \\ & (k, k, 2, 2)_{k \geq 2}, \quad (k+3, k, 3, 1)_{k \geq 3}, \quad (k+1, k, 3, 1)_{k \geq 3}, \quad (k, k, 3, 1)_{k \geq 3}, \\ & (k+3, k, 4, 0)_{k \geq 4}, \quad (k+2, k, 4, 0)_{k \geq 4}, \quad (k+1, k, 4, 0)_{k \geq 4}, \quad (k, k, 4, 0)_{k \geq 4}, \\ & (k_2 + k_3 + k_4 - 2, k_2, k_3, k_4)_{k_3 \geq 1}. \end{aligned}$$

2)  $H^1(\mathbb{CP}^1, (\mathcal{T}_{\text{gr}})_4)^{\mathbf{s}} \neq \{0\}$  if and only if  $(k_1, k_2, k_3, k_4) = (1, 1, 1, 1)$ .

**2.5. Lemma.** *For a basis of  $H^1(\mathbb{CP}^1, (\mathcal{T}_{\text{gr}})_2)^{\mathbf{a}_i}$  we can take (the classes of) the following cocycles for the supermanifold  $\mathcal{CP}_{kk20}^{1|4}$ .*

for  $k = 2$

1)  $i = 1$ :

$$\begin{aligned} & x^{-1}\xi_1\xi_2\partial_x + x^{-2}(\xi_1\xi_2\xi_3\partial_{\xi_3} + \xi_1\xi_2\xi_4\partial_{\xi_4}), \quad x^{-1}\xi_1\xi_4\xi_2\partial_{\xi_2}, \quad x^{-1}\xi_2\xi_4\xi_1\partial_{\xi_1}, \\ & x^{-1}\xi_1\xi_3\partial_x + x^{-2}(\xi_1\xi_3\xi_2\partial_{\xi_2} + \xi_1\xi_3\xi_4\partial_{\xi_4}), \quad x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_1\xi_2\xi_4\partial_{\xi_3}, \\ & x^{-1}\xi_2\xi_3\partial_x + x^{-2}(\xi_2\xi_3\xi_1\partial_{\xi_1} + \xi_2\xi_3\xi_4\partial_{\xi_4}), \quad x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}, \quad x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}; \end{aligned}$$

2)  $i = 2$  :  $x^{-1}\xi_1\xi_2\xi_4\partial_{\xi_3}$ ;

3)  $i = 3$  :  $x^{-1}\xi_1\xi_2\xi_4\partial_{\xi_3}$ ,  $2x^{-3}\xi_1\xi_2\xi_3\partial_{\xi_3} + x^{-2}\xi_1\xi_3\xi_2\partial_{\xi_2} + x^{-1}\xi_2\xi_3\xi_1\partial_{\xi_1}$ ,  
 $2x^{-3}\xi_1\xi_2\xi_4\partial_{\xi_4} + x^{-2}\xi_1\xi_3\xi_4\partial_{\xi_4} + x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_4}$ ;

for  $k = 3$  and  $k \geq 5$

1)  $i = 1$  :  $x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}$ ,  $x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}$ ,  $x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}$ ;

2)  $i = 2$  :  $x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}$ ,  $x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}$ ;

for  $k = 4$

1)  $i = 1$ :

$$\begin{aligned} & x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}, \quad x^{-1}\xi_1\xi_4\partial_x + 2x^{-2}\xi_1\xi_4\xi_2\partial_{\xi_2} + x^{-2}\xi_1\xi_4\xi_3\partial_{\xi_3}, \\ & x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}, \quad x^{-1}\xi_2\xi_4\partial_x + 2x^{-2}\xi_2\xi_4\xi_1\partial_{\xi_1} + x^{-2}\xi_2\xi_4\xi_3\partial_{\xi_3}; \end{aligned}$$

2)  $i = 2$  :  $x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}$ ,  $x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}$ .

*Proof.* follows from Theorem 15 in [BaO2]. □

The proof of the following Lemma is similar.

**2.6. Lemma.**  $H^1(\mathbb{CP}^1, (\mathcal{T}_{\text{gr}})_4)^{\mathfrak{a}} = \{0\}$  for any splitting subalgebra  $\mathfrak{a}$  and supermanifold  $\mathcal{CP}_{kk20}^{1|4}$ .

Let  $\lambda_2 : \mathcal{A}ut_{(2)}\mathcal{O}_{\text{gr}} \rightarrow (\mathcal{T}_{\text{gr}})_2$  be a sheaf homomorphism which to any germ of the automorphism  $a$  assigns the 2-component of  $\log a$  in  $(\mathcal{T}_{\text{gr}})_2 \oplus (\mathcal{T}_{\text{gr}})_4$ .

Denote by  $H^1(\mathbb{CP}^1, \mathcal{A}ut_{(2)}\mathcal{O}_{\text{gr}})^{\mathfrak{a}}$  the set of classes that determine supermanifolds even-homogeneous relative  $\mathfrak{a}$ .

**2.7. Proposition.** If  $\mathfrak{a}$  splits the sequence (1), then  $\lambda_2^*$  maps  $H^1(\mathbb{CP}^1, \mathcal{A}ut_{(2)}\mathcal{O}_{\text{gr}})^{\mathfrak{a}}$  bijectively to  $H^1(\mathbb{CP}^1, (\mathcal{T}_{\text{gr}})_2)^{\mathfrak{a}}$ .

*Proof.* follows from Statement 2.1 and Lemma 2.6. □

Using the lifting condition of the vector field on non-split supermanifold described in [O2], and Lemma 2.6, we deduce that any supermanifold  $\bar{0}$ -homogeneous relative  $\mathfrak{a}$  is determined by cocycles  $u^2$  such that the class  $[u^2]$  is  $\mathfrak{a}$ -invariant and  $[u^2, u^2] = 0$ . Since  $[u^2, u^2] = 0$  is true for all cocycles in Lemma 2.5, the following Theorem holds.

**2.8. Theorem.** For any splitting subalgebra  $\mathfrak{a}$ , the even-homogeneous relative  $\mathfrak{a}$  supermanifolds are determined by the cocycles listed in Lemma 2.5.

Thus, all even-homogeneous relative  $\mathfrak{a}$  non-split supermanifolds with retract  $\mathcal{CP}_{kk20}^{1|4}$  are described for any  $k \geq 2$ .

Let us find out if any of these even-homogeneous relative  $\mathfrak{a}$  non-split supermanifolds are homogeneous. For this we use the following Proposition analogous to Proposition 15 in [BuO1]:

**2.9. Proposition.** Under conditions of Proposition 2.7, let  $(\mathbb{CP}^1, \mathcal{O})$  be even-homogeneous relative  $\mathfrak{a}_i$  for one of  $i = 1, 2, 3$ .

If  $i = 1$ , then  $(\mathbb{CP}^1, \mathcal{O})$  is homogeneous if and only if the vector fields  $\partial_{\xi_j}$ , where  $j = 1, \dots, 4$  can be lifted to  $(\mathbb{CP}^1, \mathcal{O})$ ;

If  $i = 2$ , then  $(\mathbb{CP}^1, \mathcal{O})$  is homogeneous if and only if the vector fields  $\partial_{\xi_j}$ , where  $j = 1, 3, 4$ , can be lifted to  $(\mathbb{CP}^1, \mathcal{O})$ .

If  $i = 3$ , then  $(\mathbb{CP}^1, \mathcal{O})$  is homogeneous if and only if the vector fields  $\partial_{\xi_1}$  and  $\partial_{\xi_4}$  can be lifted to  $(\mathbb{CP}^1, \mathcal{O})$ .

Proposition 2.9 applied to the cocycles of Lemma 2.5 implies the classification (Theorems 2.10–2.20). For other 4-tuples, homogeneous non-split supermanifolds do not exist.

**2.10. Theorem.** There does not exist homogeneous non-split supermanifolds with retract  $\mathcal{CP}_{kk20}^{1|4}$  for any  $k \geq 2$ .

**2.11. Theorem.** *For the following tuples  $(k_1 k_2 k_3 k_4)$ , there does not exist homogeneous non-split supermanifolds  $\mathcal{CP}_{k_1 k_2 k_3 k_4}^{1|4}$ .*

$$(1, 1, 1, 0), (3, 3, 1, 0), (4, 4, 1, 0), (4, 4, 3, 0), (6, 4, 3, 0), (5, 4, 3, 0), (k+2, k, 4, 0)_{k \geq 4}, \\ (3, 3, 2, 1), (5, 3, 2, 1), (k+3, k, 4, 0)_{k \geq 4}, (k+3, k, 3, 1)_{k \geq 3}, (k+1, k, 4, 0)_{k \geq 4}, \\ (k+1, k, 3, 1)_{k \geq 3}, (k+1, k, 2, 0)_{k > 2}, (k, k, 4, 0)_{k \geq 4}, (k, k, 3, 1)_{k \geq 3}.$$

**2.12. Theorem.** *For every of the following tuples  $(k_1 k_2 k_3 k_4)$ , there exists one homogeneous non-split supermanifold with retract  $\mathcal{CP}_{k_1 k_2 k_3 k_4}^{1|4}$  which can be represented, up to an isomorphism, by (the classes of) the following cocycles.*

$$(2, 2, 1, 0) \quad x^{-1} \xi_1 \xi_4 \xi_3 \partial_{\xi_3}; \\ (2, 2, 2, 1) \quad x^{-1} \xi_1 \xi_2 \partial_x; \\ (3, 2, 2, 0) \quad 2x^{-1} \xi_1 \xi_3 \partial_x + 3x^{-2} \xi_2 \xi_3 \xi_1 \partial_{\xi_1}; \\ (4, 3, 2, 1) \quad x^{-1} \xi_2 \xi_3 \xi_4 \partial_{\xi_1}; \\ (k+1, k, 1, 1)_{k > 1} \quad x^{-1} \xi_3 \xi_4 \xi_1 \partial_{\xi_1}; \\ (k+3, k, 2, 2)_{k \geq 2} \quad 2x^{-1} \xi_3 \xi_4 \partial_x + (k+3)x^{-2} \xi_3 \xi_4 \xi_1 \partial_{\xi_1} + kx^{-2} \xi_3 \xi_4 \xi_2 \partial_{\xi_2}; \\ (k+1, k, 2, 2)_{k > 2} \quad 2x^{-1} \xi_3 \xi_4 \partial_x + (k+1)x^{-2} \xi_3 \xi_4 \xi_1 \partial_{\xi_1} + kx^{-2} \xi_3 \xi_4 \xi_2 \partial_{\xi_2}; \\ (k, k, 2, 2)_{k \geq 4} \quad x^{-1} \xi_3 \xi_4 \partial_x.$$

**2.13. Theorem.** *If  $k_4 \neq 0$ , then for every  $(k_2 + k_3 + k_4 - 2, k_2, k_3, k_4)_{k_3 \geq 1}$  which is different from  $(k, k, 1, 1)_{k \geq 1}$ , there exists one homogeneous non-split supermanifold which can be represented, up to an isomorphism, by the (class of) cocycle*

$$x^{-1} \xi_2 \xi_3 \xi_4 \partial_{\xi_1}.$$

**2.14. Theorem.** *There exist two homogeneous non-split supermanifolds with their retracts being  $\mathcal{CP}_{2222}^{1|4}$ , and which can be represented, up to an isomorphism, by (the classes of) the following cocycles.*

$$x^{-1} \xi_1 \xi_2 \partial_x + x^{-2} \xi_1 \xi_2 \xi_3 \partial_{\xi_3} + x^{-2} \xi_1 \xi_2 \xi_4 \partial_{\xi_4}, \\ x^{-1} \xi_1 \xi_2 \partial_x + x^{-2} \xi_1 \xi_2 \xi_3 \partial_{\xi_3} + x^{-2} \xi_1 \xi_2 \xi_4 \partial_{\xi_4} + x^{-1} \xi_3 \xi_4 \partial_x + \\ + x^{-2} \xi_3 \xi_4 \xi_1 \partial_{\xi_1} + x^{-2} \xi_3 \xi_4 \xi_2 \partial_{\xi_2}.$$

**2.15. Theorem.** *There exist two homogeneous non-split supermanifolds with their retracts being  $\mathcal{CP}_{3222}^{1|4}$ , and which can be represented, up to an isomorphism, by (the classes of) the cocycles*

$$x^{-1} \xi_2 \xi_3 \partial_x + x^{-2} \xi_2 \xi_3 \xi_1 \partial_{\xi_1} + x^{-2} \xi_2 \xi_3 \xi_4 \partial_{\xi_4}, \\ 2x^{-3} \xi_2 \xi_3 \xi_1 \partial_{\xi_1} + x^{-2} \xi_2 \xi_4 \xi_1 \partial_{\xi_1} + x^{-1} \xi_3 \xi_4 \xi_1 \partial_{\xi_1}.$$

**2.16. Theorem.** *There exist three homogeneous non-split supermanifolds with their retracts being  $\mathcal{CP}_{3322}^{1|4}$ , and which can be represented, up to an isomorphism, by (the classes of) the following cocycles.*

$$x^{-1} \xi_3 \xi_4 \partial_x, \quad x^{-2} \xi_1 \xi_3 \partial_x + x^{-1} \xi_2 \xi_3 \partial_x, \\ x^{-1} \xi_3 \xi_4 \partial_x + x^{-2} \xi_1 \xi_3 \partial_x + x^{-1} \xi_2 \xi_3 \partial_x.$$



**2.17. Theorem.** *There exist five homogeneous non-split supermanifolds with their retracts being  $\mathcal{CP}_{kk11}^{1|4}$ , where  $k > 2$ , which can be represented, up to an isomorphism, by (the classes of) the following cocycles.*

$$\begin{aligned} & x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}, \quad x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1} + x^{-1}\xi_3\xi_4\xi_2\partial_{\xi_2}, \\ & x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2} + x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}, \quad x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1} + x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2} + x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}. \end{aligned}$$

**2.18. Theorem.** *There exist eight homogeneous non-split supermanifolds with their retracts being  $\mathcal{CP}_{2111}^{1|4}$ , which can be represented, up to an isomorphism, by (the classes of) the following cocycles.*

$$\begin{aligned} & x^{-1}\xi_2\xi_3\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_4}, \quad x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}, \\ & x^{-1}\xi_2\xi_3\xi_1\partial_{\xi_1} + x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_4}, \quad x^{-1}\xi_2\xi_3\xi_1\partial_{\xi_1} + x^{-1}\xi_3\xi_4\xi_2\partial_{\xi_2}, \\ & x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_4} + x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1} + x^{-1}\xi_3\xi_4\xi_2\partial_{\xi_2}, \\ & x^{-1}\xi_2\xi_3\xi_1\partial_{\xi_1} + x^{-1}\xi_2\xi_4\xi_3\partial_{\xi_3} + x^{-1}\xi_3\xi_4\xi_2\partial_{\xi_2}. \end{aligned}$$

**2.19. Theorem.** *There exist nine homogeneous non-split supermanifolds with their retracts being  $\mathcal{CP}_{2211}^{1|4}$ , which can be represented, up to an isomorphism, by (the classes of) the following cocycles.*

$$\begin{aligned} & x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}, \\ & x^{-2}\xi_2\xi_3\xi_1\partial_{\xi_1} - x^{-1}\xi_2\xi_4\xi_1\partial_{\xi_1}, \quad x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1} + x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}, \\ & x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1} + x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1} + x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}, \\ & x^{-2}\xi_2\xi_3\xi_1\partial_{\xi_1} - x^{-1}\xi_2\xi_4\xi_1\partial_{\xi_1} + x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1}, \\ & x^{-2}\xi_2\xi_3\xi_1\partial_{\xi_1} - x^{-1}\xi_2\xi_4\xi_1\partial_{\xi_1} + x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}, \\ & x^{-2}\xi_2\xi_3\xi_1\partial_{\xi_1} - x^{-1}\xi_2\xi_4\xi_1\partial_{\xi_1} + x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1} + x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}, \\ & x^{-2}\xi_2\xi_3\xi_1\partial_{\xi_1} - x^{-1}\xi_2\xi_4\xi_1\partial_{\xi_1} + x^{-1}\xi_3\xi_4\xi_1\partial_{\xi_1} + \\ & \quad + x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1} + x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}. \end{aligned}$$

**2.20 Theorem ([B5]).** *There exist four homogeneous non-split superstrings and one 1-parameter family of homogeneous non-split superstrings with retract  $\mathcal{CP}_{1111}^{1|4}$ , which can be represented, up to an isomorphism, by (the classes of) the following cocycles.*

$$\begin{aligned} & x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1}, \quad x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1} - x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2}, \\ & x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1} - x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2} + x^{-1}\xi_1\xi_2\xi_4\partial_{\xi_3}, \\ & x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1} - x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2} + x^{-1}\xi_1\xi_2\xi_4\partial_{\xi_3} - x^{-1}\xi_1\xi_2\xi_3\partial_{\xi_4}, \\ & t(x^{-1}\xi_2\xi_3\xi_4\partial_{\xi_1} - x^{-1}\xi_1\xi_3\xi_4\partial_{\xi_2} + x^{-1}\xi_1\xi_2\xi_4\partial_{\xi_3} - x^{-1}\xi_1\xi_2\xi_3\partial_{\xi_4}) + \\ & \quad + x^{-1}\xi_1\xi_2\xi_3\xi_4\partial_x, \text{ where } t \in \mathbb{C}^\times. \end{aligned}$$

**Acknowledgements.** Thanks are due to A. Onishchik for his help and to D. Leites for his comments and help.

## References

- [B1] Bashkin M. A., Homogeneous non-split supermanifolds of odd dimension 4 over a complex projective line. In: Nevsky M. V. (ed.) Mathematics in Yaroslavl U., Yaroslavl U., Yaroslavl, (2011) 7–12.
- [B2] Bashkin M. A., Homogeneous and even-homogeneous supermanifolds with retract  $\mathcal{CP}_{k,k,2,0}^{1|4}$  for  $k \geq 2$ . Models and analysis of inform. systems (2009) **16**(3), 14–21.
- [B3] Bashkin M. A., Homogeneous non-split supermanifolds with retract  $\mathcal{CP}_{k,k,2,2}^{1|4}$ . In: Abstracts of talks at the International algebraic conference dedicated to A. V. Yakovlev's 70th birthday. St. Petersburg State U., St. Petersburg. (2010) 6–8.
- [B4] Bashkin M. A., Homogeneous non-split supermanifolds with retract  $\mathcal{CP}_{6,4,2,2}^{1|4}$ . In: Mathematics and natural sciences. Theory and practice. Interuniversity collection of scientific papers. V.11, Yaroslavl State U., Yaroslavl (2016), 13–19.
- [B5] Bashkin M. A., Homogeneous superstrings with retract  $\mathcal{CP}^{1|4}$ . Communications in Mathematics (2022) **30**(3), 209–236.
- [BaO1] Bashkin M. A., Onishchik A. L., Homogeneous non-split supermanifolds of dimension  $1|4$  over complex projective line. In: Mathematics in Yaroslavl University, Collection of review papers to the 30th anniversary of the Mathematical Department, Yaroslavl State U., Yaroslavl (2006) 17–32.
- [BaO2] Bashkin M. A., Onishchik A. L., Homogeneous non-split supermanifolds over complex projective line. In: Mathematics, cybernetics, informatics: proceedings of the International scientific conference to the memory of A. Yu. Levin. Yaroslavl State U., Yaroslavl (2008) 40–57.
- [BG\*] Bernstein J., Gelfand S.,  $\Lambda(\Theta)$ -modules and algebraic vector bundles on  $\mathbb{P}(\Theta)$ . In: D. Leites (ed.) Seminar on supersymmetries. Preprints of Stockholm University 10/1987-14, <http://staff.math.su.se/mleites/sos/bernstein-gelfand-1987.pdf> For an abridged version, see [Ge\*].
- [BGLS\*] Bouarroudj S., Grozman P., Leites D., Shchepochkina I., Minkowski superspaces and superstrings as almost real-complex supermanifolds. Theor. and Mathem. Physics (2012) **173**(3), 1687–1708.
- [BuO1] Bunegina V. A., Onishchik A. L., Homogeneous supermanifolds associated with the complex projective line. J. Math. Sci. **82** (1996) 3503–3527.
- [BuO2] Bunegina V. A., Onishchik A. L., Two families of flag supermanifolds.<sup>1</sup> Different. Geom. and its Appl. **4** (1994), 329–360.
- [CCLL\*] Chapovalov D., Chapovalov M., Lebedev A., Leites D. The classification of almost affine (hyperbolic) Lie superalgebras. Journal of Nonlinear Mathematical Physics **17**, Special issue in memory of F. Berezin, (2010) 103–161.

<sup>1</sup>Certain attributions in this paper are wrong. For example, the Lie superalgebra  $\Gamma(\sigma_1, \sigma_2, \sigma_3)$  — the deformation of  $\mathfrak{osp}(4|2)$  — was discovered by Kaplansky [Kap\*], [Kapp\*], see [KIE\*]; later Kac redenoted  $\Gamma(\sigma_1, \sigma_2, \sigma_3)$  by  $D(2, 1; \alpha)$ ; for reasons given in [CCLL\*], this is an ill-chosen notation. For a reasonable and meaningful notation which gradually started to replace the initial *ad hoc* names of the exceptional Lie superalgebras given by Kaplansky and later ones, due to Kac, see [Sr\*], [CCLL\*].

- [Ge\*] Gelfand S. Sheaves on  $\mathbb{P}_n$  and problems of linear algebra. Appendix in the book Okonek Ch., Schneider M., Spindler H. [Vector bundles on complex projective spaces] (Russian) Translated from the English by V. Ya. Lin. With an appendix by S. I. Gelfand. Translation edited and with a preface by Yu. I. Manin. Mir, Moscow, 1984. 312 pp. Okonek Ch., Schneider M., Spindler H., Vector bundles on complex projective spaces. Corrected reprint of the 1988 edition. With an appendix by S. I. Gelfand. Modern Birkhäuser Classics. Birkhäuser/Springer Basel AG, Basel, 2011. viii+239 pp.
- [G] Green P., On holomorphic graded manifolds. Proc. Amer. Math. Soc. **85** (1982), 587–590.
- [HM\*] Hazewinkel M., Martin C., A short elementary proof of Grothendieck’s theorem on algebraic vectorbundles over the projective line. Journal of Pure and Applied Algebra **25** (1982), 207 – 211.
- [KIE\*] Kac V. G., Letter to the editors: “Classification of simple Lie superalgebras” (Funkcional. Anal. i Priložen. (1975) **9**(3), 91–92. (in Russian) Funkcional. Anal. i Priložen. (1976) **10**(2), 93.
- [Kap\*] Kaplansky I., Superalgebras. Pacific J. Math. (1980) **86**(1), 93–98
- [Kapp\*] Kaplansky I., Graded Lie algebras I, II, preprints, Univ. Chicago, Chicago, Ill., 1975, see <http://www1.osu.cz/~Zusmanovich/links/files/kaplansky/>
- [O1] Onishchik A.L. On complex homogeneous supermanifolds. In: Mathematics in Yaroslavl University. Collection of review papers. To 20th anniversary of Mathematical Department. Yaroslavl State U., Yaroslavl (1996) 133–153.
- [O2] Onishchik A.L., A Construction of Non-Split Supermanifolds. Annals of Global Analysis and Geometry **16**, (1998) 309–333.
- [O3] Onishchik A.L., Non-Abelian Cohomology and Supermanifolds. SFB 288. Preprint no 360. Berlin, 1998.
- [O4] Onishchik A.L. Classification problems of complex supermanifolds. In: Mathematics in Yaroslavl University. Collection of review papers. To 25th anniversary of Mathematical Department. Yaroslavl State U., Yaroslavl (2001) 7–34.
- [Sr\*] Serganova V., On generalizations of root systems, Comm. in Algebra (1996) **24**(13), 4281–4299.
- [Vi\*] Vishnyakova E., Vector fields on flag supermanifolds. In: Modern problems of mathematics and informatics **8**. Yaroslavl State U., Yaroslavl (2006) 11–23.
- [Vi1\*] Vishnyakova E., Even-homogeneous supermanifolds on the complex projective line. Differential Geometry and its Applications (2013) **31**(5), 698–706.
- [Vi2\*] Vishnyakova E., Rigidity of flag supermanifolds; [arXiv:1908.11753](https://arxiv.org/abs/1908.11753).