

Reduced Order Modeling of Cyclically Symmetric Bladed Disks with Geometric and Contact Nonlinearities

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Outline

Context

Single blade

Full bladed disk

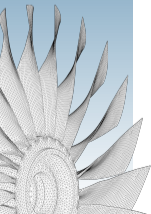
Conclusion

1 Context

2 Single blade

3 Full bladed disk

4 Conclusion





Environmental constraints

Context

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Full bladed disk

Conclusion



By 2050...

- ▶ 75% reduction in CO₂
- ▶ 90% reduction in NO_x
- ▶ 65% reduction of noise



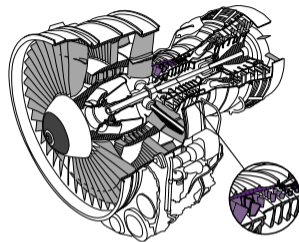
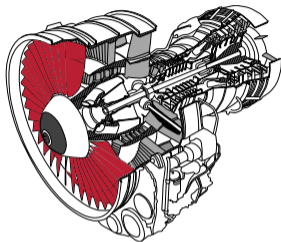
Consequences on bladed disks design

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- ▶ Designing lighter and more flexible blades
- ▶ Geometric nonlinearities
- ▶ Reducing clearances between the rotating blades and the casing
- ▶ Contact nonlinearities

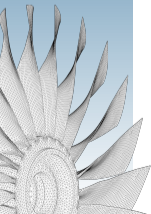
Bladed disks dynamics fundamentally nonlinear



Numerical modeling

Full order model

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} + \mathbf{g}_{nl}(\mathbf{u}) = \mathbf{f}_e(t) + \mathbf{f}_c(\mathbf{u}, \dot{\mathbf{u}})$$





Numerical modeling

Full order model

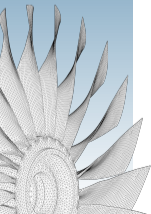
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Reduced order model

$$\tilde{\mathbf{M}}\ddot{\mathbf{q}} + \tilde{\mathbf{C}}\dot{\mathbf{q}} + \tilde{\mathbf{K}}\mathbf{q} + \tilde{\mathbf{g}}_{nl}(\mathbf{q}) = \tilde{\mathbf{f}}_e(t) + \tilde{\mathbf{f}}_c(\mathbf{q}, \dot{\mathbf{q}})$$

$$\mathbf{u} = \Phi \mathbf{q}$$

- ▶ Projection basis Φ ?
- ▶ Reduced nonlinear internal forces $\tilde{\mathbf{g}}_{nl}$?
- ▶ Treatment of contact in the reduced space $\tilde{\mathbf{f}}_c(\mathbf{q}, \dot{\mathbf{q}})$?

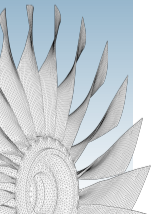




Objectives

Previous work¹

- ▶ Development of a methodology to study the contact interactions of a single rotating blade with geometric nonlinearities
- ▶ Validation on an industrial compressor blade model



¹E. Delhez et al. *Journal of Sound and Vibration* (2021). doi: 10.1016/j.jsv.2021.116037.



Objectives

Previous work¹

- ▶ Development of a methodology to study the contact interactions of a single rotating blade with geometric nonlinearities
- ▶ Validation on an industrial compressor blade model

This presentation

- ▶ In-depth contact analyses to characterize the influence of geometric nonlinearities
- ▶ Generalization of the methodology to full bladed disks

¹E. Delhez et al. *Journal of Sound and Vibration* (2021). doi: 10.1016/j.jsv.2021.116037.



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Contact simulations

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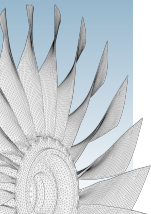
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$$\mathbf{u} = \Phi \mathbf{q}$$

- ▶ **Projection basis:** Craig-Bampton modes and a selection of their modal derivatives²
- ▶ **Reduced nonlinear internal forces:** evaluation with the stiffness evaluation procedure (STEP)³
- ▶ **Contact:** explicit central finite difference time integration scheme with Lagrange multipliers⁴

²L. Wu et al. *Proceedings of the 27th International Conference on Noise and Vibration Engineering*. Leuven (Belgium), 2016.

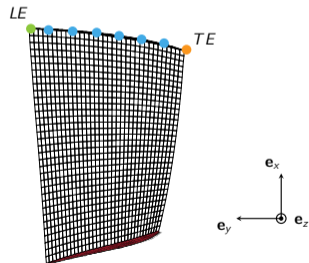
³A. Muravyov et al. *Computers & Structures* (2003). doi: 10.1016/s0045-7949(03)00145-7.

⁴N. J. Carpenter et al. *International Journal for Numerical Methods in Engineering* (1991). doi: 10.1002/nme.1620320107.



Test case

- ▶ NASA rotor 37 blade (transonic compressor blade) clamped at its root⁵
- ▶ Open and industrial test case
- ▶ 8 boundary nodes distributed between *LE* and *TE* (contact interface)
- ▶ Reduction basis: 189 modes = 24 static modes + 15 fixed interface linear normal modes + 150 modal derivatives

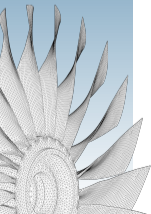
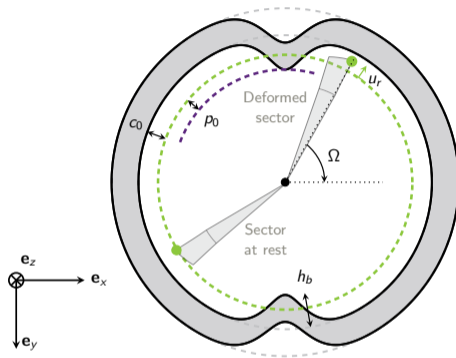


⁵L. Reid et al. Technical report. NASA TP 1337, 1978. doi: 10.1049/iet-gtd.2015.0403.



Contact scenario

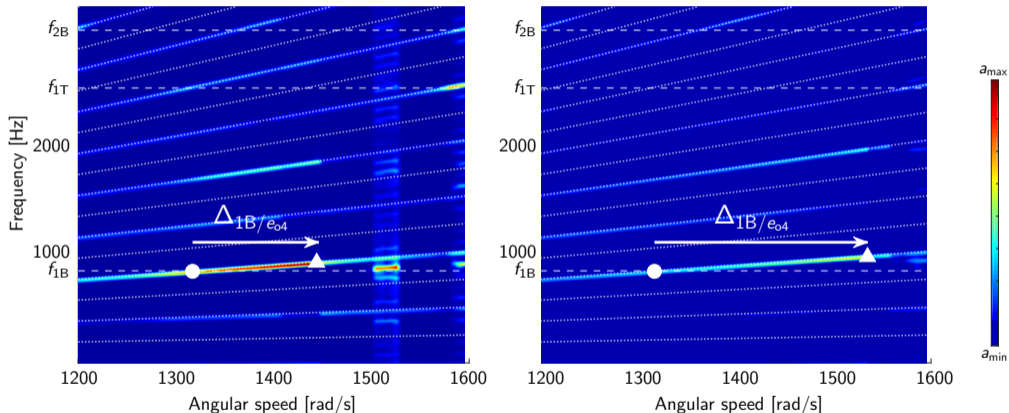
- ▶ Blade rotating at a constant speed Ω around \mathbf{e}_z
- ▶ Direct contact with rigid casing – sliding friction
- ▶ Contact initiated by deformation of the casing with two lobes
- ▶ No aerodynamic loading, no gyroscopic or centrifugal effects, no thermal effects





Interaction maps of the radial displacement at LE

Interaction between the first bending mode (1B) and the fourth engine order (e_{o4})



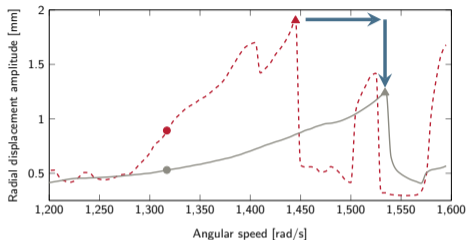
(a) Without geometric nonlinearities.

(b) With geometric nonlinearities.

► Interaction maps, predicted linear (●) and nonlinear (▲) resonances.



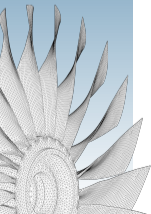
Nonlinear frequency response curve



- ▶ NFRC without (- -) and with (—) geometric nonlinearities, predicted linear (●) and nonlinear (▲) resonances.

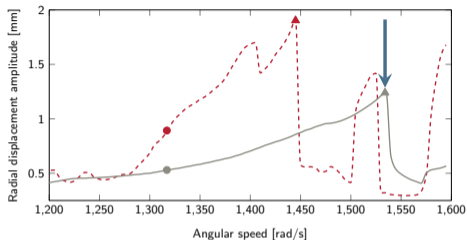
Observations

- ▶ Contact stiffening
- ▶ Amplitude jumps
- ▶ Influence of geometric nonlinearities
 - Smoother interactions
 - Additional contact stiffening





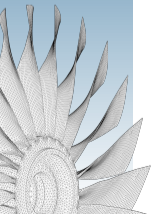
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- ▶ Contact stiffening
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- ▶ Influence of geometric nonlinearities
 - Smoother interactions?
 - Additional contact stiffening





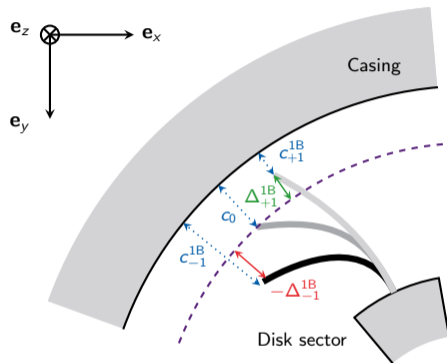
Clearance consumption

Definition

- ▶ Evolution of the clearance between the blade and the casing when the blade vibrates along 1 mode

$$\Delta(\delta) = c_0 - c(\delta)$$

- ▶ Possible key parameter for the design of blades subjected to contact interactions⁶



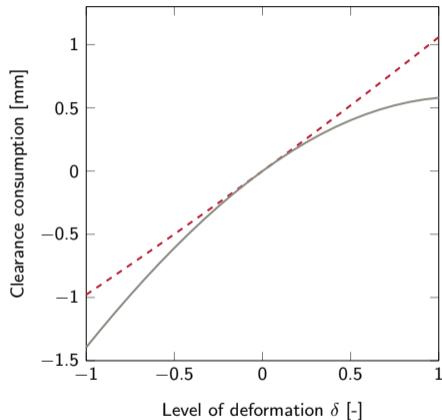
⁶A. Batailly et al. *Proceedings of the ASME Turbo Expo* (2016). doi: 10.1115/gt2016-56721.



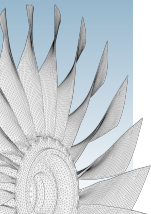
Clearance consumption

Observations

- ▶ Reduced clearance consumption with geometric nonlinearities
- ▶ Justify that the blade with geometric nonlinearities features lower vibration response to contact
- ▶ Linear model valid for $\delta \in [-0.25, 0.2]$

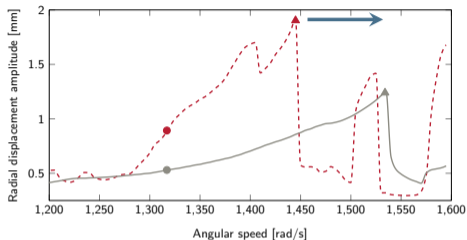


- ▶ Clearance consumption at *LE* without (- -) and with (—) geometric nonlinearities.





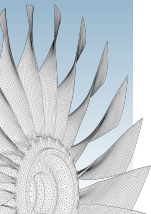
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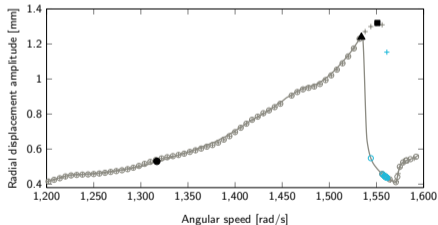
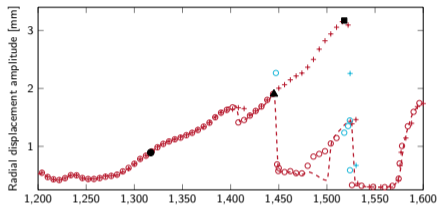
Observations

- ▶ Contact stiffening
- ▶ Amplitude jumps
- ▶ Influence of geometric nonlinearities
 - Smoother interactions
 - **Additional contact stiffening?**





Nonlinear frequency response curve with continuation



- ▶ NFRC without (- - / —) and with continuation (+ / ○), without geometric nonlinearities (above) and with geometric nonlinearities (below).

Numerical procedure

- ▶ NFRC built with a sequential continuation procedure
- ▶ Upward (+) and downward (○) angular speed sweeps

Observations

- ▶ Without continuation, nonlinear resonance (■) not correctly captured
- ▶ Contact stiffening similar with and without geometric nonlinearities



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Full bladed disk

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Verification without
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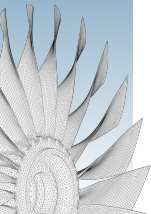
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Generalization of the methodology with CMS techniques

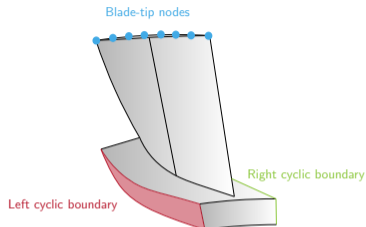
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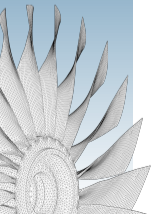
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- **Projection basis:** for each sector, Craig-Bampton modes and a selection of their modal derivatives + second reduction of the cyclic boundary





Generalization of the methodology with CMS techniques

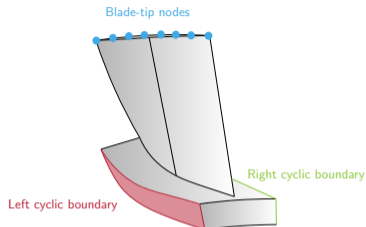
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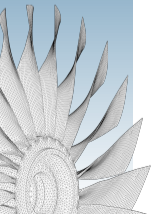
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- ▶ **Projection basis:** for each sector, Craig-Bampton modes and a selection of their modal derivatives + second reduction of the cyclic boundary
- ▶ **Reduced nonlinear internal forces:** STEP, assumption of linear coupling between the sectors





Generalization of the methodology with CMS techniques

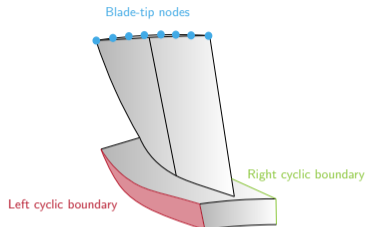
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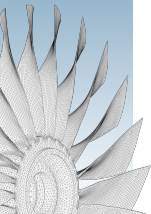
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- ▶ **Contact:** explicit central finite difference time integration scheme with Lagrange multipliers





Test case

Context

Single blade

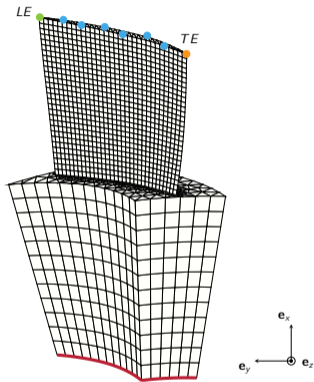
Full bladed disk

Methodology

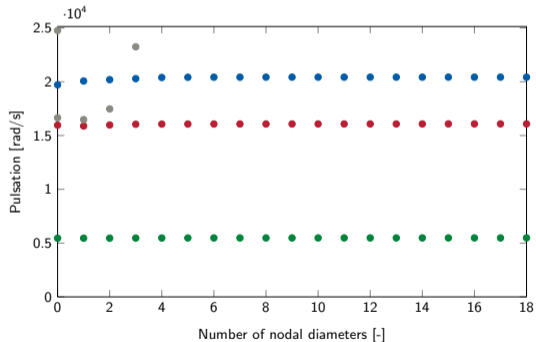
Verification without contact

Contact simulations

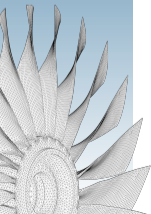
Conclusion



- ▶ NASA rotor 37 bladed disk with 36 sectors
- ▶ 133,605 degrees-of-freedom per sector
- ▶ Sectors clamped at disk lower surface



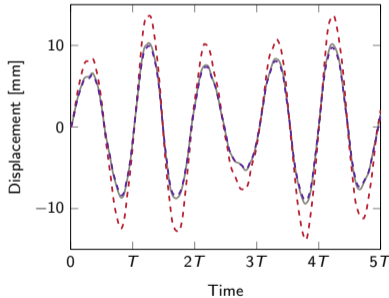
▶ SAFE diagram.



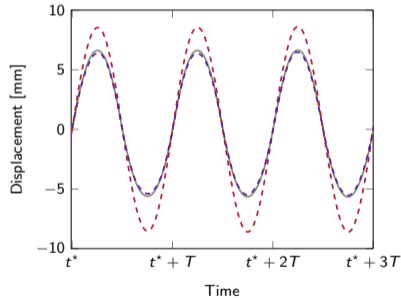


Verification without contact

- ▶ Reduction basis: per sector, 3 static modes (LE) + 10 fixed interface linear normal modes + 10 modal derivatives + 3 modes for the second projection (total: 972 modes)
- ▶ Blade excited by a harmonic excitation of amplitude $A = 400$ N and pulsation $\omega = 4,500$ rad/s in the e_z direction



(a) Transient regime.

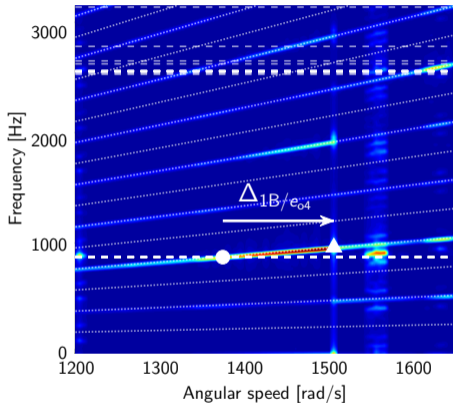


(b) Steady-state regime.

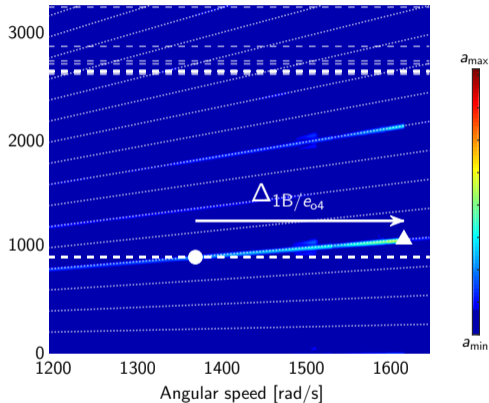
- ▶ Reference linear (---) and nonlinear (—) solutions, reduced order model nonlinear solution (- -).



Interaction maps of the radial displacement at LE



(a) Without geometric nonlinearities.

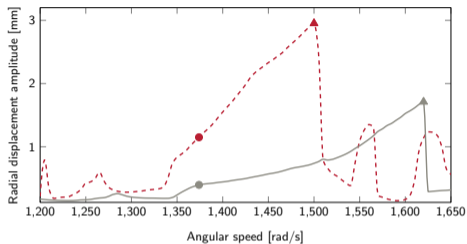


(b) With geometric nonlinearities.

► Interaction maps, predicted linear (●) and nonlinear (▲) resonances.



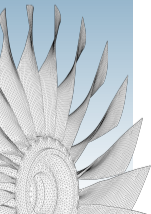
Nonlinear frequency response curve



▶ NFRC without (---) and with (—) geometric nonlinearities, predicted linear (●) and nonlinear (▲) resonances.

Observations

- ▶ Contact stiffening
- ▶ Amplitude jumps
- ▶ Influence of geometric nonlinearities
 - Smoother interactions (see clearance consumption analysis)
 - 'Additional contact stiffening' (continuation procedure required for accurate quantification)





Von Mises stress fields

Context

Single blade

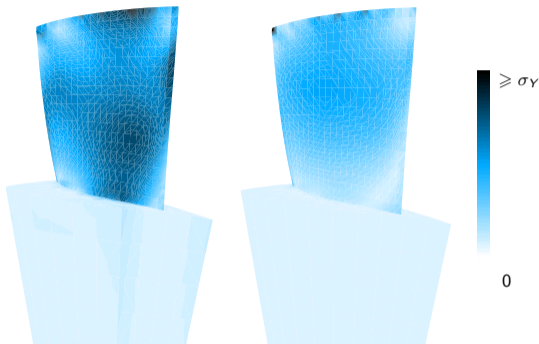
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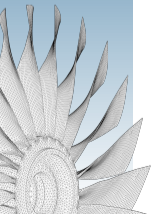


(a) Without geometric nonlinearities. (b) With geometric nonlinearities.

► Von Mises stress fields at the resonance.

Comparison

- Zones of maximal stresses not at the same locations
- Non-negligible stresses in the disk for the case without geometric nonlinearities
- Smaller stresses predicted with geometric nonlinearities (in line with predicted displacements)





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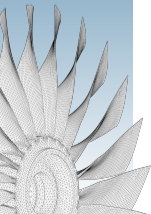
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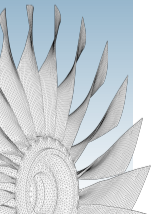
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Conclusion

- ▶ Methodology to study the rubbing interactions of full bladed disks with geometric nonlinearities
 - Projection basis: Craig-Bampton reduction basis and selection of their modal derivatives + second reduction of cyclic boundary
 - Geometric nonlinearities: STEP
 - Contact nonlinearities: Lagrange multipliers
- ▶ Reduced order models are an efficient alternative to full order models
- ▶ Influence of geometric nonlinearities not negligible
- ▶ Parametric reduced order models can be built to account for gyroscopic and centrifugal effects
- ▶ Methodology also compatible with the introduction of mistuning





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Thank you for your attention

