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# On the Classification of n-Refined Neutrosophic Rings and Its Applications in Matrix Computing Algorithms and Linear Systems

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## Abstract

This work is dedicated to classifying the general n-refined neutrosophic ring by building a ring isomorphism and the direct product of the corresponding classical rings with itself. On the other hand, we use the classification property to solve the problem of n-refined neutrosophic computing of Eigen values and Eigen vectors of an n-refined neutrosophic matrix. Also, it will be applied to solve n-refined linear systems and models.

**Keywords:** n-refined neutrosophic ring; n-refined neutrosophic matrix; neutrosophic eigen value; neutrosophic eigen vector.

## 1. Introduction

Neutrosophic logic as a new branch of mathematical philosophy has an impact in many fields of human knowledge. We see a lot of applications of Smarandache's work [7] in algebra, analysis, matrix computing, and geometry [1-6,8-15].

The structures of n-refined sets were studied by many researchers around the world, especially in n-refined matrix theory and computing [16].

In this work, we build a ring isomorphism between an n-refined neutrosophic ring and (n+1) copies of the direct product of the classical corresponding ring itself. This classification will lead to many strong solutions for computing eigen values and eigen vectors for an n-refined neutrosophic matrix, which may be very useful in engineering, and statistics [13].

Also, we apply this isomorphisms to get a solution for the linear system\model of n-refined neutrosophic equations.

For definitions and properties of n-refined neutrosophic rings see [19].

## 2. Main Discussion

### Definition

Let  $R_n(I) = \{a_0 + a_1I_1 + \dots + a_nI_n; a_i \in R\} = \{a_0 + \sum_{i=1}^n a_iI_i; a_i \in R\}$  be an n-refined neutrosophic ring, we define the n-refined AH-isometry as follows:

$$f: R_n(I) \rightarrow R^{n+1} = R \times R \dots \times R \text{ (n + 1 times)};$$

$$f\left(a_0 + \sum_{i=1}^n a_iI_i\right) = (a_0, a_0 + a_1 + \dots + a_n, a_0 + a_2 + a_3 + \dots + a_n, \dots, a_0 + a_n)$$

$$=(a_0, \sum_{i=0}^n a_i, \sum_{i \neq 1}^n a_i, \sum_{i \neq 1,2}^n a_i, \dots, \sum_{i \neq 1, \dots, n-1}^n a_i)$$

### Theorem

Let  $f$  be the n-refined neutrosophic AH-isometry defined above, then:

- $f$  is well defined mapping.
- $f$  is a bijection.
- $f$  is a ring isomorphism.
- $R_n(I) \cong R^{n+1}$ .

### proof.

- Assume that  $x = x_0 + \sum_{i=1}^n x_iI_i = y = y_0 + \sum_{i=1}^n y_iI_i \in R_n(I)$ , hence  $x_i = y_i$  for all  $i = 1, 2, \dots, n$

This implies  $x_0 = y_0, \sum_{i=0}^n x_i = \sum_{i=0}^n y_i, \sum_{i \neq 1}^n x_i = \sum_{i \neq 1}^n y_i, \dots, \sum_{i \neq 1, \dots, n-1}^n x_i = \sum_{i \neq 1, \dots, n-1}^n y_i$ .

thus  $f(x) = f(y)$ .

- First of all,  $f$  is injective that is because:

Let  $x = x_0 + \sum_{i=1}^n x_iI_i \in \ker f$ , hence  $f(x) = 0_{R^{n+1}} = (0, 0, \dots, 0)$ , so that  $x_0 = 0, \sum_{i=0}^n x_i = 0, \sum_{i \neq 1}^n x_i = 0, \dots, \sum_{i \neq 1, \dots, n-1}^n x_i = 0$ , thus  $x_i = 0$  for all  $i = 1, 2, \dots, n$  and  $\ker f = \{0\}$ .

Now, we must prove that  $f$  is surjective. For this purpose, we take an arbitrary element  $(x_0, x_1, \dots, x_n) \in R_n(I)$ .

There exists  $x = x_0 + (x_1 - x_2)I_1 + (x_2 - x_3)I_2 + \dots + (x_{n-1} - x_n)I_{n-1} + (x_n - x_0)I_n$ .

Such that  $f(x) = (x_0, x_1, x_2, \dots, x_n)$ , hence  $f$  is surjective.

- $f$  preserves addition clearly. It remains to prove that  $f$  preserves multiplication.

We use induction on  $n$  the index of  $R_n(I)$ .

If  $n = 0$ , the assumption is true easily.

Suppose that it holds for  $n = k$ , we must prove it for  $n = k + 1$ .

Let

$$x = x_0 + x_1I_1 + \dots + x_{k+1}I_{k+1} = x_0 + \sum_{i=1}^k x_iI_i + x_{k+1}I_{k+1}.$$

$$y = y_0 + y_1I_1 + \dots + y_{k+1}I_{k+1} = y_0 + \sum_{i=1}^k y_iI_i + y_{k+1}I_{k+1}.$$

It is sufficient to check the coefficients of  $I_{k+1}$  in the product  $(x, y)$ .

The coefficient of  $I_{k+1}$  is:  $x_0y_{k+1} + x_{k+1}y_0 + x_{k+1}y_{k+1}$ .

This implies that the  $(k + 1)$  component in  $f(xy)$  is equal to:

$x_0y_0 + x_0y_{k+1} + x_{k+1}y_0 + x_{k+1}y_{k+1} = (x_0 + x_{k+1}) \cdot (y_0 + y_{k+1})$  which is exactly equal to the  $k + 1$ . Component of  $f(x)$  multiplied by the corresponding  $(k + 1)$  Component of  $f(y)$ , thus by induction we get  $f(xy) = f(x) \cdot f(y)$  and  $f$  is a ring isomorphism.

d) It holds directly according to (c).

### Example

Let  $Z_4(I) = \{a + bI_1 + cI_2 + dI_3 + eI_4, a, b, c, d, e \in Z\}$  be the 4-refined neutrosophic of integers, hence  $Z_4(I) \cong Z \times Z \times Z \times Z \times Z$ .

### Definition

$A = (x_{11} \cdots x_{1n} \quad \vdots \quad x_{n1} \cdots x_{nn})$  is called an  $n$ -refined neutrosophic square matrix if  $x_{ij} \in R_n(I)$ ;  $R_n(I)$  is an  $n$ -refined neutrosophic ring.

### Remark

Any  $n$ -refined neutrosophic matrix  $A$  can be splat as follows:  $A = A_0 + A_1I_1 + \cdots + A_nI_n$ ;  $A_i$  are square matrices defined over the classical ring  $R$ .

### Example

Consider the following real, 3-refined neutrosophic matrix:  $A = (1 + I_1 \ 2 - I_2 + I_3 \ 3 + I_1 + I_2 + I_3 \ 2I_1 - I_3)$ .  
 $A$  is equal to:  $(1 \ 2 \ 3 \ 0) + (1 \ 0 \ 1 \ 2)I_1 + (0 \ -1 \ 1 \ 0)I_2 + (0 \ 1 \ 1 \ -1)I_3$ .

## 3. The applications of $n$ -refined AH-isometry in matrix computing

In applied mathematics, it is important to compute eigen values and vectors, that is because these values can be used in statistics [ 13], and diagonalization [ 3 ].

In the following, we describe an algorithm to compute the eigen values and vectors of an  $n$ -refined neutrosophic real square matrix.

The description of the method:

Let  $A = A_0 + A_1I_1 + \cdots + A_nI_n$  be an  $m \times m$  square  $n$ -refined neutrosophic matrix, to compute its eigen values follow these steps.

Step 1: compute the AH-isometric image of  $A$ , i.e.

$$f(A) = (A_0, \sum_{i=0}^n A_i, \sum_{i \neq 1}^n A_i, \dots, \sum_{i \neq 1, \dots, n-1}^n A_i).$$

Step 2: compute the eigen values and vectors of each component by using classical matrix theory.

Step 3: Go back with the inverse isomorphism to get the corresponding  $n$ -refined neutrosophic eigen values and vectors.

### Example

Consider the following  $2 \times 2$  2-refined neutrosophic real matrix:

$$A = (1 \ 2 \ 0 \ 2) + (3 \ -3 \ 1 \ 1)I_1 + (1 \ 1 \ 0 \ 1)I_2 = (1 + 3I_1 + I_2 \ 2 - 3I_1 + I_2 \ 1 + I_1 + I_2)$$

The AH-isometric image is:

$$f(A) = ((1\ 2\ 0\ 2), (5\ 0\ 1\ 4), (2\ 3\ 0\ 3))$$

The eigen values of  $(1\ 2\ 0\ 2)$  is  $\{1,2\}$ .

The eigen values of  $(5\ 0\ 1\ 4)$  is  $\{5,4\}$ .

The eigen values of  $(2\ 3\ 0\ 3)$  is  $\{2,3\}$ .

Now, we get the following triples:

$$x_1 = (1,5,2), x_2 = (1,5,3), x_3 = (1,4,2), x_4 = (1,4,3), x_5 = (2,5,2), x_6 = (2,5,3), x_7 = (2,4,3), \\ x_8 = (2,4,2).$$

Now, we get the corresponding n-refined neutrosophic eigen values by taking the inverse image of each  $x_i; i = 1, \dots, 8$ .

$$y_1 = f^{-1}(x_1) = 1 + 3I_1 + I_2, y_2 = f^{-1}(x_2) = 1 + 2I_1 + 2I_2, y_3 = f^{-1}(x_3) = 1 + 2I_1 + I_2, \\ y_4 = f^{-1}(x_4) = 1 + 1I_1 + 2I_2, y_5 = f^{-1}(x_5) = 2 + 3I_1, y_6 = f^{-1}(x_6) = 2 + 2I_1 + I_2, \\ y_7 = f^{-1}(x_7) = 2 + I_1 + I_2, y_8 = f^{-1}(x_8) = 2 + 2I_1.$$

#### 4. Algorithm to Solve n-Refined Linear Systems of Equations

To solve a linear system of n-refined neutrosophic equations  $A_i X_i = B_i; i = 1, \dots, m$ , follow these steps:

Step1: Transform  $A_i X_i = B_i$  to a classical linear system by taking the direct AH-isometric image  $f(A_i)f(X_i) = f(B_i)$ .

Step2: solve the classical corresponding system and get  $X, Y$ .

#### Example

Consider the following 3-refined neutrosophic system of linear equations:

$$(1 + 2I_1 - I_2)X + (I_2 + I_3)Y = 1 \dots (1)$$

$$(2I_1 - I_3)X + (1 + I_3)Y = I_1 + I_2 \dots (2)$$

Where  $X = x_0 + x_1 I_1 + x_2 I_2 + x_3 I_3, Y = y_0 + y_1 I_1 + y_2 I_2 + y_3 I_3; x_i, y_i \in R$ .

The direct AH-isometry image of equation(1)is:

$$(1,2,0,2). (x_0, x_0 + x_1 + x_2 + x_3, x_0 + x_2 + x_3, x_0 + x_3) \\ + (0,2,2,1). (y_0, y_0 + y_1 + y_2 + y_3, y_0 + y_2 + y_3, y_0 + y_3) = (1,1,1,1)$$

Which implies:  $\{x_0 = 1 \dots (a) 2(x_0 + x_1 + x_2 + x_3) + 2(y_0 + y_1 + y_2 + y_3) = 1 \dots (b) 2(y_0 + y_2 + y_3) = 1 \dots (c) (x_0 + x_3) + (y_0 + y_3) = 1 \dots (d)$

the direct AH-isometry image of equation (2) is:

$$(0,1,1,-1). (x_0, x_0 + x_1 + x_2 + x_3, x_0 + x_2 + x_3, x_0 + x_3) \\ + (1,2,2,2). (y_0, y_0 + y_1 + y_2 + y_3, y_0 + y_2 + y_3, y_0 + y_3) = (0,2,1,0)$$

Which implies:  $\{y_0 = 0 \dots (e) (x_0 + x_1 + x_2 + x_3) + 2(y_0 + y_1 + y_2 + y_3) = 2 \dots (f) (x_0 + x_2 + x_3) + 2(y_0 + y_2 + y_3) = 1 \dots (g) -(x_0 + x_3) + 2(y_0 + y_3) = 0 \dots (h)$

From (a) and (e), we get  $x_0 = 1, y_0 = 0$ .

From (b) and (f), we get  $x_0 + x_1 + x_2 + x_3 = -1, y_0 + y_1 + y_2 + y_3 = \frac{3}{2}$ .

From (c) and (g), we get  $x_0 + x_2 + x_3 = 0, y_0 + y_2 + y_3 = \frac{1}{2}$ .

From (d) and (h), we get  $x_0 + x_3 = \frac{1}{3}, y_0 + y_3 = \frac{2}{3}$ .

This implies:  $\{x_0 = 1, y_0 = 0, x_1 = -1, y_1 = 1, x_2 = -\frac{2}{3}, y_2 = \frac{1}{6}, x_3 = -\frac{1}{3}, y_3 = \frac{1}{3}\}$

The corresponding 3-refined solutions are:

$$X = 1 - I_1 \frac{2}{3} I_2 - \frac{1}{3} I_3, Y = y_1 I_1 + \frac{1}{6} I_2 + \frac{1}{3} I_3$$

Now, we examine the matrix method to solve the previous linear system.

#### Example:

Consider the linear system of 3-refined neutrosophic equations in the previous example. We write it by using matrix representation as follows:

$$(1 + 2I_1 - I_2, I_2 + I_3, I_2 - I_3, 1 + I_3)(XY) = (1, I_1 + I_2) \Leftrightarrow AZ = B \Rightarrow Z = A^{-1} \times B$$

Thus, we must compute  $A^{-1}$  by using n-refined AH-isometry.

$$A = (1 + 2I_1 - I_2, I_2 + I_3, I_2 - I_3, 1 + I_3) = (1, 0, 0, 1) + (2, 0, 0, 0)I_1 + (-1, 1, 2, 0)I_2 + (0, 1, -1, 1)I_3$$

$$f(A) = ((1, 0, 0, 1), (2, 2, 1, 2), (0, 2, 1, 2) + (1, 1, -1, 2))$$

The inverse of  $f(A)$  is:

$$[f(A)]^{-1} = \left( (1, 0, 0, 1), \left(1 - 1, \frac{-1}{2}, 1\right), \left(-1, 1, \frac{1}{2}, 0\right) + \left(\frac{2}{3}, \frac{-1}{3}, \frac{1}{3}, \frac{1}{3}\right) \right)$$

Hence,  $f^{-1}[(f(A))^{-1}] = (1, 0, 0, 1) + (2, -2, -1, 1)I_1 + \left(-\frac{5}{3}, \frac{4}{3}, \frac{1}{6}, -\frac{1}{3}\right)I_2 + \left(-\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\right)I_3$

$$= \left(1 + 2I_1 - \frac{5}{3}I_2 - \frac{1}{3}I_3, -2I_1 + \frac{4}{3}I_2 - \frac{1}{3}I_3, -I_1 + \frac{1}{6}I_2 + \frac{1}{3}I_3, 1 + I_1 - \frac{1}{3}I_2 - \frac{2}{3}I_3\right)$$

$$A^{-1} \times B = \left(1 + 2I_1 - \frac{5}{3}I_2 - \frac{1}{3}I_3, -2I_1 + \frac{4}{3}I_2 - \frac{1}{3}I_3, -I_1 + \frac{1}{6}I_2 + \frac{1}{3}I_3, 1 + I_1 - \frac{1}{3}I_2 - \frac{2}{3}I_3\right) (1, I_1 + I_2)$$

$$\Rightarrow A^{-1} \times B = \left(1 - I_1 - \frac{2}{3}I_2 - \frac{1}{3}I_3, I_1 + \frac{1}{6}I_2 + \frac{1}{3}I_3\right) \Rightarrow X = 1 - I_1 - \frac{2}{3}I_2 - \frac{1}{3}I_3, Y = I_1 + \frac{1}{6}I_2 + \frac{1}{3}I_3.$$

## 5. Solving n-refined quadratic equations.

#### Example:

Consider the linear system of 3-refined neutrosophic quadratic equation:

$$I_2 X^2 + (1 - I_3)X = 0; X = x_0 + x_1 I_1 + x_2 I_2 + x_3 I_3 \in R_3(I)$$

To solve this equation, we can use the AH-isometry one more time as follows:

$$f(I_2)f(X^2) + f(1 - I_3).f(X) = f(0)$$

$$\Leftrightarrow (0, 1, 1, 0). (x_0^2, (x_0 + x_1 + x_2 + x_3)^2, (x_0 + x_2 + x_3)^2, (x_0 + x_3)^2) + (1, 0, 0, 0). (x_0, x_0 + x_1 + x_2 + x_3, x_0 + x_2 + x_3, x_0 + x_3) = (0, 0, 0, 0)$$

$$\Leftrightarrow \{(x_0 + x_1 + x_2 + x_3)^2 = 0 \Rightarrow x_1(x_0 + x_2 + x_3)^2 = 0 \Rightarrow x_2 = -x_3(x_0 + x_3) = 0 \Rightarrow x_3 \text{ is arbitrary } x_0 = 0$$

Thus the equation has infinite solutions with form:

$$X = -x_3 I_2 + x_3 I_3 = x_3(-I_2 + I_3); x_3 \in R.$$

#### Theorem.

Let  $R_n(I)$  be an n-refined neutrosophic ring,  $U(R_n(I))$  be the group of units  $R_n(I)$ , then

$$U(R_n(I)) \cong [U(R)]^{n+1} = U(R) \times U(R) \times \dots \times U(R) (n + 1. \text{ times}).$$

#### proof.

According to the previous results, we have  $R_n(I) \cong R^{n+1}$ , thus  $U(R_n(I)) \cong [U(R)]^{n+1}$ .

**Remark.**

To compute the units in  $R_n(I)$  follow these steps.

Step1. Find the units in the AH-isometric image  $f(R_n(I)) = R^{n+1}$ .

Step2. Use the inverse AH-isometry to find the corresponding n-refined neutrosophic units.

**Example.**

Let  $(Z_4)_2(I)$  be the 2-refined neutrosophic ring of integers module 4, we will find its units.

$$f((Z_4)_2(I)) = Z_4 \times Z_4 \times Z_4 = (Z_4)^3$$

The unit group of  $Z_4$  is  $\{1,3\}$ .

The units of  $(Z_4)^3$  are:

$$e_1 = (1,1,1), e_2 = (1,3,1), e_3 = (3,1,1), e_4 = (1,1,3), e_5 = (3,3,3), e_6 = (3,3,1), e_7 = (3,1,3),$$

$$e_8 = (1,3,3).$$

The units of  $R_2(I)$  are:

$$f^{-1}(e_1) = 1, f^{-1}(e_2) = 1 + 2I_1, f^{-1}(e_3) = 3 - 2I_2 = 3 + 2I_2, f^{-1}(e_4) = 1 - 2I_1 + 2I_2,$$

$$f^{-1}(e_5) = 3, f^{-1}(e_6) = 3 + 2I_1 + 2I_2, f^{-1}(e_7) = 3 + 2I_1, f^{-1}(e_8) = 1 + 2I_2.$$

**Example.**

Let  $Z_3(I)$  be the 3-refined neutrosophic ring of integers,  $Z_3(I) \cong (Z)^5$ ,

The units in  $Z$  are  $\{1, -1\}$ , thus the units of  $(Z)^5$  are:

$$(1,1,1,1), (1,1,1, -1), (1,1, -1,1), (1, -1,1,1), (-1,1,1,1), (1,1, -1, -1), (-1, -1,1,1), (-1,1,1, -1), (-1,1, -1,1),$$

$$(-1, -1, -1,1), (-1, -1, -1, -1), (-1, -1, -1, -1), (-1, -1, -1, -1), (1, -1, -1, -1), (1, -1,1, -1), (1, -1, -1,1)$$

The corresponding 3-refined neutrosophic units are:

$$1, 1 + 2I_2 - 2I_3, 1 + 2I_1 - 2I_2, 1 - 2I_1, -1 + 2I_3, 1 + 2I_1 - 2I_3, -1 - 2I_1 + 2I_3, -1 + 2I_2, -1 + 2I_2, -1 +$$

$$2I_1 - 2I_2 + 2I_3, -1 - 2I_2 + 2I_3, -1 - 2I_1 + 2I_2, -1 + 2I_1, -1, -1 - 2I_2 + 2I_3, -1 - 2I_1 + 2I_2, -1 + 2I_1, 1 -$$

$$2I_3, 1 - 2I_1 + 2I_2 - 2I_3, 1 - 2I_2.$$

**6. Applications in computing powers**

Now, we are able to compute any power  $x^m$  for all  $x \in R_n(I)$ . By using the AH-isometry.

**Example.**

Consider  $x = 2 + I_1 - 3I_2 + I_3 + 2I_4 \in R_4(I)$  the 4-refined neutrosophic field of real numbers.

Let's compute  $x^3$ .

The AH-isometric image of  $x$  is:

$$f(x) = (2,3,2,5,4), (f(x))^3 = (8,27,8,125,64), \text{ thus } x^3 = f^{-1}[(f(x))^3] = 8 + 19I_1 - 117I_2 + 61I_3 + 56I_4.$$

**Example.**

Consider  $x = 2 + I_1 - 4I_2 \in R_2(I), y = 1 - I_1 + I_2 \in R_2(I)$ , we must compute  $x^y$ .

$$z = [f(x)]^{f(y)} = (2, -1, -2)^{(1,1,2)} \Rightarrow x^y = f^{-1}(z) = 2 - 5I_1 + 2I_2.$$

**7. Conclusion**

In this paper, we have founded a novel algorithm to compute n-refined neutrosophic eigen values\ vectors of an n-refined neutrosophic matrix by building a classification isomorphism. Also, we have applied this classification

property to solve the group of units' problem and  $n$ -refined linear system of equations. In the future, we aim that the previous problems will be discussed for  $n$ -cyclic refined neutrosophic matrices.

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