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# Three-dimensional unsteady model of arc heater plasma flow

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## Abstract

Arc jets are unique facilities used to evaluate the performance of Thermal Protection Systems (TPS) for hypersonic vehicles. They produce high pressure and high enthalpy plasma flow to simulate the extreme heat encountered during atmospheric entry. The constricted arc heater part of an arc jet increases the test gas temperature by Joule heating. This study details the development of the three-dimensional unsteady plasma flow analysis tool, ARChES (ARC Heater Simulator). Coupled Navier-Stokes, radiative transfer and Maxwell equations yield current density, magnetism, radiation, and flow field solutions. The present work constitutes the first demonstration of an unsteady three-dimensional plasma flow simulation of high pressure and high enthalpy arc heater that captures kink instabilities of the electric arc. It is found that the arc attachment is mainly driven by upstream arc instabilities.

*Keywords:* Arc jet, Arc heater, Radiative transfer, Joule heating, Plasma, Arc instability, Arc attachment.

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## Nomenclature

$\mathcal{I}_\nu^d$	Discrete radiative intensity in direction $\Omega^d$ , $\text{W} \cdot \text{m}^{-2}$
$\mathcal{I}_\nu$	Radiative intensity, $\text{W} \cdot \text{m}^{-2} \cdot \text{sr}^{-1}$
$\mathbf{A}$	Magnetic vector potential, $\text{T} \cdot \text{m}$
$\mathbf{B}$	Magnetic field, $\text{T}$
$\mathbf{E}$	Electric field, $\text{V} \cdot \text{m}^{-1}$
$\mathbf{J}$	Current density, $\text{A} \cdot \text{m}^{-2}$
$\mathbf{n}$	Normal direction
$\mathbf{q}$	Heat flux, $\text{W} \cdot \text{m}^{-2}$
$\mathbf{t}$	Tangential direction
$\mathbf{u}$	Velocity, $\text{m} \cdot \text{s}^{-1}$
$\mathcal{H}$	Planck's constant, $\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-1}$
$\dot{m}$	Mass flux, $\text{kg} \cdot \text{s}^{-1}$
$\mathbf{S}$	Outgoing surface normal, $\text{m}^2$
$a_d$	Gaussian quadrature weight
$B_\nu$	Banded black body spectral radiative intensity, $\text{W} \cdot \text{m}^{-2}$
$c$	Speed of light in the medium., $\text{m} \cdot \text{s}^{-1}$
$e$	Internal energy, $\text{J} \cdot \text{kg}^{-1}$
$E_0$	Stagnation energy, $\text{J} \cdot \text{kg}^{-1}$
$h$	Enthalpy, $\text{J} \cdot \text{kg}^{-1}$
$H_0$	Stagnation enthalpy, $\text{J} \cdot \text{kg}^{-1}$
$I$	Electric current, $\text{A}$
$k_B$	Boltzmann's constant, $\text{J} \cdot \text{K}^{-1}$
$k_i$	Incoming part of the radiative directional split
$k_o$	Outgoing part of the radiative directional split
$L_0$	Characteristic length, $\text{m}$
$M$	Mach number
$N_a$	Number of anodes
$N_b$	Number of spectral bands
$N_d$	Number of directions

$p$	Pressure, Pa
$Q_J$	Joule heating, $\text{W} \cdot \text{m}^{-3}$
$R_b$	Resistance of the ballast configuration, $\Omega$
$Re_m$	Magnetic Reynolds number
$S$	Surface area, $\text{m}^2$
$T$	Temperature, K
$U_0$	Characteristic velocity, $\text{m} \cdot \text{s}^{-1}$
$V$	Volume, $\text{m}^3$

*Greek*

$\Omega^d$	Discrete direction
$\alpha$	Diffusivity, $\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$
$\omega$	Vorticity, $\text{s}^{-1}$
$\underline{\underline{\tau}}$	Viscous stress tensor, $\text{kg} \cdot \text{m} \cdot \text{s}^{-2}$
$\epsilon_\nu$	Emissivity
$\eta$	Magnetic diffusivity, $\text{m}^2 \cdot \text{s}^{-1}$
$\gamma$	Heat capacity ratio
$\kappa_\nu$	Linear absorption coefficient, $\text{m}^{-1}$
$\mu$	Mixture viscosity, $\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$
$\mu_0$	Vacuum magnetic permeability, $\text{H} \cdot \text{m}^{-1}$
$\phi$	Electric potential, V
$\psi$	Compressibility, $\text{s}^2 \cdot \text{m}^{-2}$
$\rho$	Mass density, $\text{kg} \cdot \text{m}^{-3}$
$\sigma$	Electrical conductivity, $\text{S} \cdot \text{m}^{-1}$

*Subscripts*

$w$	Wall
$e$	External
$imp$	Imposed
$ind$	Induced
$a$	Anodes
$b$	Spectral band

c Cathodes  
nc Neighbor cell  
tot Total

*Superscripts*

cond Conductive  
d Direction  
rad Radiative

## 1. Introduction

High enthalpy arc jet wind tunnels are the established ground-test technology used to evaluate the performance of Thermal Protection Systems (TPS) materials for human and robotic exploration missions. By providing a highly energetic flow over prolonged test times, arc jet facilities enable testing materials under extreme aerothermal heating experienced by spacecraft during planetary entry.

Arc jets are usually composed of an anode chamber, a long constrictor, a cathode chamber, a converging/diverging nozzle, and a test chamber. The number of electrodes, commonly made of copper, around the anode and cathode chambers is different among arc jet facilities. The large difference in electric potential between the anodes and the cathodes generates the extreme current necessary to heat the test gas via Joule heating. The electrodes and the walls have an internal water-cooling system to remove the heat created by the electric arc. The constrictor, where most of the heating occurs, is usually built as a succession of gas-injecting disks that are insulated from each other by a ceramic material. The mixture of the cold gas injected through the disks is representative of the planetary atmosphere of interest to a test campaign. This study focuses on the arc heater part of an arc jet from the anode chamber to the nozzle throat.

Although the supersonic aerothermal modeling technology from the nozzle throat to the test chamber is well established and routinely used, the inlet conditions from the arc heater section have always been highly approximated in the literature [28, 6, 15]. An accurate model of the plasma physics that occur in the arc heater is needed to understand the gas conditions at the nozzle throat. High fidelity heat and mass transfer analysis tools of the sample material response in the test chamber are an active area of research [26, 3, 23, 8].

The importance of understanding the arc attachment and arc instabilities in arc jets was studied by [2, 36]. The interaction between the electric arc and the surrounding medium results in strong gradients in both the fluid and the electromagnetic fields, which can lead to various types of instabilities [5].

31 Numerical methods to compute the radiative transfer are available in the lit-  
32 erature, such as ray tracing [48, 20, 22] and spatial and spectral discretization  
33 [4, 25, 24, 1]. Research by Trelles et al. [40, 37, 38, 39] investigated finite element  
34 modeling of industrial arc plasma torches using Argon gas. The collection of  
35 work by Trelles et al. includes detailed models of the plasma. However, given  
36 the empirical nature of the radiation model used, this work is more suitable  
37 for smaller geometries, and lower total enthalpy flows. Park et al. [27], Sakai  
38 et al. [32, 31, 33, 30], and Lee et al. [19, 18] have studied numerical models of  
39 plasma flow in arc heaters. Wang et al. [44, 42, 43] have worked on modeling  
40 3D vacuum arcs used in industrial applications, such as circuit breakers. Some  
41 of their results have been compared to anode jet experimental data. A three-  
42 dimensional unsteady simulation of NASA’s Aerodynamic Heating Facility was  
43 performed by Sahai et al. [29] using the Fully-Implicit Navier-Stokes (FIN-S)  
44 solver [16, 17]. Their finite element solver included Joule heating, 3D radiative  
45 transfer, and non-equilibrium model for the plasma. A simplified geometry was  
46 used for the anode and cathode chambers in their study. The computational  
47 cost of the solver limited the integration in time.

48 The objective of this study is to develop a better understanding of the multi-  
49 physics of the plasma flow inside arc heaters which will ultimately lead to a  
50 more accurate characterization of the use of an arc jet in testing TPS materials.  
51 The state-of-the-art ARC Heater Simulator (ARChES) analysis tool provides a  
52 parallel, unstructured, finite volume solver based on the OpenFOAM library.  
53 ARChES includes the solution of three-dimensional radiative transfer, imposed  
54 current density, imposed magnetic field, external magnetic field, and turbulence.  
55 Realistic boundary conditions are added to the model. The open-source third  
56 party library Mutation++, developed at the von Karman Institute for Fluid  
57 Dynamics, is used to compute the equilibrium gas mixture composition as well  
58 as thermodynamic and transport properties [35]. Three-dimensional and time-  
59 dependent simulations presented in this work show plasma instabilities in the  
60 constrictor of the arc heater. Analysis of the electric arc dynamics will provide a  
61 better intuitive understanding of the complex behavior of plasma flow observed

62 in arc jets. Massively parallel and efficient simulation capability in ARCHEs  
 63 allows long-time integration, necessary for the development of the plasma insta-  
 64 bilities.

65 The paper is organized as follows. In section 2, the governing equations and  
 66 the boundary conditions used in the plasma flow model are presented. Section  
 67 3 details the numerical model applied to solve the governing equations. In sec-  
 68 tion 4, overall results are presented, including arc instabilities and attachment.  
 69 Finally, section 5 summarizes the conclusions drawn from this study.

## 70 2. Governing Equations

### 71 2.1. Mass, momentum, and energy equations

The governing equations, described in Eq. (1), are the mass, momentum,  
 and energy conservation laws.

$$\partial_t \rho + \boldsymbol{\partial}_x \cdot (\rho \mathbf{u}) = 0 \quad (1a)$$

$$\partial_t (\rho \mathbf{u}) + \boldsymbol{\partial}_x \cdot (\rho \mathbf{u} \mathbf{u}) = -\boldsymbol{\partial}_x p + \boldsymbol{\partial}_x \cdot \underline{\boldsymbol{\tau}} + \mathbf{J} \times \mathbf{B} \quad (1b)$$

$$\partial_t (\rho E_0) + \boldsymbol{\partial}_x \cdot (\rho H_0 \mathbf{u}) = \boldsymbol{\partial}_x \cdot (\underline{\boldsymbol{\tau}} \cdot \mathbf{u} - \mathbf{q}^{cond}) + \sigma |\mathbf{E}|^2 + \mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) - \boldsymbol{\partial}_x \cdot \mathbf{q}^{rad} \quad (1c)$$

72

$H_0$ ,  $E_0$  and  $e$  are the stagnation enthalpy, stagnation energy, and internal  
 energy.

$$H_0 = E_0 + \frac{p}{\rho} = e + \frac{|\mathbf{u}|^2}{2} + \frac{p}{\rho} \quad (2)$$

73 The Lorentz force and work, the Joule heating, and the radiative heat-  
 74 ing/cooling are detailed in the following subsections.

### 75 2.2. Electromagnetic equations

Chapter 3.4 of Goedbloed & Poedts [12] and section 5 of Giordano [10] give  
 a general review of the coupling between plasma and Maxwell's equations. The  
 governing equations, detailed in Eq. (1), remain unclosed. Maxwell's equations



result in the following dynamic equations for the current and magnetic fields. Ampere's law is given by

$$-\frac{1}{c^2}\partial_t\mathbf{E} + \partial_x \times \mathbf{B} = \mu_0\mathbf{J} \quad (3)$$

Given the non-relativistic system in the arc heater ( $U_0 \ll c^2$ ), Ampere's law, Eq. (3), reduces to

$$\partial_x \times \mathbf{B} = \mu_0\mathbf{J} \quad (4)$$

The generalized Ohm's law, as shown in Eq. (40) of Ref. [10], is given by

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \quad (5)$$

The magnetic Reynolds number, detailed in Eq. (6), compares magnetic induction to magnetic diffusion.

$$Re_m = \frac{U_0 L_0}{\eta} \quad (6a)$$

$$\eta = \frac{1}{\mu_0\sigma} \quad (6b)$$

In the considered arc heaters, the magnetic Reynolds number is about  $10^{-3}$ , which corresponds to a highly resistive plasma. This system with low  $Re_m$  is dominated by magnetic diffusion, and inductive terms are neglected. Using the quasi-static approximation, the Ohm's law, from Eq. (5), reduces to

$$\mathbf{J} = \sigma\mathbf{E} \quad (7)$$

In this work, the electromagnetic four-potential formulation [14] is used to write

$$\mathbf{E} = -\partial_x\phi - \partial_t\mathbf{A} \quad (8a)$$

$$\mathbf{B} = \partial_x \times \mathbf{A} \quad (8b)$$

In this case, the current is driven by a differential of electric potential between the electrodes, and an external magnetic field in the electrode region is added to prevent the arc attachment at one point. The total electric field is split into two parts in order to simplify the boundary conditions. The first part is an imposed electric field,  $\mathbf{E}_{imp}$ , where the fluid is assumed stationary compared to the imposed electric potential. Therefore, the fluid is treated as an electric conductor. The second part is an induced electric field,  $\mathbf{E}_{ind}$ , generated by fluctuations of the total magnetic vector potential and the gradient of the induced electric potential. The split of the total electric field follows

$$\mathbf{E}_{imp} = -\partial_{\mathbf{x}}\phi_{imp} \quad (9a)$$

$$\mathbf{E}_{ind} = -\partial_t(\mathbf{A}_{imp} + \mathbf{A}_e + \mathbf{A}_{ind}) - \partial_{\mathbf{x}}\phi_{ind} \quad (9b)$$

$$\mathbf{E} = \mathbf{E}_{imp} + \mathbf{E}_{ind} \quad (9c)$$

Ohm's law, from Eq. (7), yields

$$\mathbf{J}_{imp} = -\sigma\partial_{\mathbf{x}}\phi_{imp} \quad (10a)$$

$$\mathbf{J}_{ind} = -\sigma[\partial_t(\mathbf{A}_{imp} + \mathbf{A}_e + \mathbf{A}_{ind}) + \partial_{\mathbf{x}}\phi_{ind}] \quad (10b)$$

$$\mathbf{J} = \mathbf{J}_{imp} + \mathbf{J}_{ind} \quad (10c)$$

The induced fields generated by the movement of the charges in the plasma flow were not computed in this work. As shown in Eq. (6), the low magnetic Reynolds number indicates that advection is relatively unimportant, which means that the electric field ( $\mathbf{E}$ ) is determined by the boundary conditions ( $\mathbf{E}_{imp}$ ) rather than the flow ( $\mathbf{E}_{ind}$ ). Therefore, the induced electric field and the magnetic vector potential were neglected. Future work will compare the

induced and imposed fields.

$$\mathbf{E}_{ind} = \mathbf{A}_{ind} = \mathbf{0} \quad (11)$$

79 *2.2.1. Imposed electric field*

We define the imposed electric field as the electric field that is created by the potential difference between the electrodes. The electric field is sustained inside the plasma due to the finite conductivity, which generates a conductive current that heats the gas. Consequently, the imposed current continuity comes from the divergence of Eq. (4) that yields the current conservation relation,

$$\boldsymbol{\partial}_{\mathbf{x}} \cdot \mathbf{J}_{imp} = 0. \quad (12)$$

80 The plasma is considered to be quasi-neutral in the whole domain, hence ne-  
81 glecting the sheaths at the walls and electrodes.

Substituting Eq. (10a) in Eq. (12) gives

$$\boldsymbol{\partial}_{\mathbf{x}} \cdot (-\sigma \boldsymbol{\partial}_{\mathbf{x}} \phi_{imp}) = 0 \quad (13)$$

By solving for the electric potential using Eq. (13), the electric field is computed as

$$\mathbf{E}_{imp} = -\boldsymbol{\partial}_{\mathbf{x}} \phi_{imp} \quad (14)$$

The Joule heating is expressed as follows

$$Q_J = \sigma |\mathbf{E}_{imp}|^2 = \sigma |-\boldsymbol{\partial}_{\mathbf{x}} \phi_{imp}|^2 \quad (15)$$

82

83 *2.2.2. Imposed magnetic field*

The imposed magnetic field is defined as the magnetic field induced by the imposed current. Using charge neutrality of the plasma, the imposed vector

magnetic field must satisfy,

$$-\partial_x^2 \mathbf{A}_{imp} = -\mu_0 \sigma \partial_x \phi_{imp} \quad (16)$$

After solving for the imposed magnetic vector potential using Eq. (16), the imposed magnetic field is computed as follows,

$$\mathbf{B}_{imp} = \partial_x \times \mathbf{A}_{imp} \quad (17)$$

This ensures that  $\partial_x \cdot \mathbf{B}_{imp} = 0$ . Note that in regions where the flow is not conductive, i.e.,  $\sigma \approx 0$ , the imposed magnetic vector potential satisfies the Laplace equation.

$$\partial_x^2 \mathbf{A}_{imp} \approx 0 \quad (18)$$

84 *2.2.3. External magnetic field*

85 The external magnetic field is the magnetic field that is generated outside  
 86 the plasma by an external source. In the studied arc heaters, a magnetic field is  
 87 created within the electrode to create a force that moves the attachment point  
 88 of the current and protects the electrode from erosion [46], as represented in  
 89 Fig. 1.

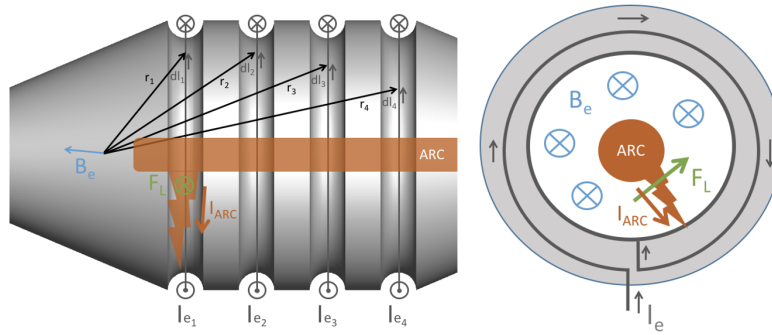


Figure 1: Representation of the magnetic spin coil in series with the arc current contained in each electrode. The external magnetic field creates a force that rotates the arc attachment point and reduces erosion.

The Biot-Savart law is used to express the external magnetic field  $\mathbf{B}_e$ , generated by the wire wrapped within the electrode assembly,

$$\partial_x^2 \mathbf{A}_e = -\mu_0 \mathbf{J}_e \quad (19)$$

The analytic solution of Eq. (19) is given by

$$\mathbf{A}_e(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}_e(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3 \mathbf{r}' \quad (20)$$

For a thin wire of infinitesimal thickness, this results in

$$\mathbf{A}_e(\mathbf{r}) = \frac{\mu_0 I_e}{4\pi} \oint \frac{d\mathbf{l}}{|\mathbf{r} - \mathbf{r}'|} \quad (21)$$

90 where  $\oint$  is the integral along the length of the wire. The external magnetic  
 91 vector potential from Eq. (21) was numerically solved using 60 points for the  
 92 contour integral. The external magnetic field was then computed as follows

$$\mathbf{B}_e = \partial_x \times \mathbf{A}_e \quad (22)$$

93 This formulation ensures that  $\partial_x \cdot \mathbf{B}_e = 0$  in the computational domain.

### 94 2.3. Radiative heat flux

95 Radiative Transfer Equations (RTE) are solved in this work to characterize  
 96 the radiative heat flux. In the literature on arc plasma torches modeling, sim-  
 97 pler approaches are found to tackle radiation. The most common one considers  
 98 the radiative flux as a function of the local properties through a Net Emission  
 99 Coefficient (NEC) [21, 11]. Alternatively, the tangent slab approximation that  
 100 reduces the RTE to one spatial dimension is often used for cylindrical geometries  
 101 [34]. However, both approximations cannot be used in the presence of 3D insta-  
 102 bilities with strong temperature gradients, as the radiation no longer depends  
 103 on the local properties nor the physics can be reduced to cylindrical symmetry.

The total radiative heat flux is given by integrating the spectral radiative heat flux over the entire spectrum. The integral is approximated by a sum of

discrete spectral radiative heat fluxes with a number of bands  $N_b$ , as follows,

$$\mathbf{q}^{rad} \approx \sum_{\nu=1}^{N_b} \mathbf{q}_{\nu}^{rad} \quad (23)$$

Here,  $\mathbf{q}_{\nu}^{rad}$  is given by integrating the radiative intensity in direction  $\boldsymbol{\Omega}$  over the complete solid angle. The Discrete Ordinates Method is used to discretize  $\mathbf{q}_{\nu}^{rad}$  into a set of 1-D steady state RTE. The integral over the solid angle is replaced by a weighted sum of discrete radiative intensities. Gaussian quadrature is employed to determine discrete directions  $\boldsymbol{\Omega}_d$  and weights  $a_d$  for different degrees of approximation ( $N_d$  directions) as

$$\mathbf{q}_{\nu}^{rad} = \int_{4\pi} \boldsymbol{\Omega} \mathcal{I}_{\nu}(\boldsymbol{\Omega}) d\boldsymbol{\Omega} \approx \sum_{d=1}^{N_d} a_d \mathcal{I}_{\nu}^d \boldsymbol{\Omega}_d \quad (24)$$

104

For a non-scattering plasma, the 1-D steady state RTE of direction  $\boldsymbol{\Omega}_d$  is expressed as

$$\boldsymbol{\Omega}_d \cdot \partial_{\mathbf{x}} \mathcal{I}_{\nu}^d = \kappa_{\nu} B_{\nu} - \kappa_{\nu} \mathcal{I}_{\nu}^d \quad (25)$$

105

The linear absorption coefficient  $\kappa_{\nu}$  was calculated following the Planck  
 106 Mean Absorption Coefficient band-averaging model [9]. The Line-By-Line spec-  
 107 tral properties in air plasma were computed by the Nonequilibrium Air Radi-  
 108 ation (NEQAIR) program [45] using more than 500,000 wavelengths between  
 109 0.04  $\mu\text{m}$  and 20  $\mu\text{m}$ .

The banded black body spectral radiance intensity is given by

$$B_{\nu}(T) = \int_{\nu}^{\nu+1} \frac{2\mathcal{H}\nu^3}{c^2} \frac{1}{\exp(\mathcal{H}\nu/k_B T) - 1} d\nu \quad (26)$$

110

The left-hand side of Eq. (25) integrated over the cell volume gives

$$\int_{V_c} \boldsymbol{\Omega}_d \cdot \boldsymbol{\partial}_x \mathcal{I}_\nu^d dV = \int_{V_c} \boldsymbol{\partial}_x \cdot (\boldsymbol{\Omega}_d \mathcal{I}_\nu^d) dV = \int_{S_c} \mathcal{I}_\nu^d \boldsymbol{\Omega}_d \cdot d\mathbf{S} \approx \sum_{c=1}^{N_{nc}} \mathcal{I}_{\nu,c}^d \boldsymbol{\Omega}_d \cdot \Delta \mathbf{S}_c \quad (27)$$

111 The surface integral in Eq. (27) is approximated by a summation over the  
 112 faces of face-centered values times the corresponding cell areas.

The right-hand side of Eq. (25) integrated over the cell volume gives

$$\int_{V_c} \kappa_\nu (B_\nu - \mathcal{I}_\nu^d) dV \approx \kappa_{\nu,c} (B_{\nu,c} - \mathcal{I}_{\nu,c}^d) V_c \quad (28)$$

113

The volume integral in Eq. (28) is approximated by the cell center value of the integrand times the cell volume. Eq. (27) and (28) give

$$\sum_{c=1}^{N_{nc}} \mathcal{I}_{\nu,c}^d \boldsymbol{\Omega}_d \cdot \Delta \mathbf{S}_c = \kappa_{\nu,c} (B_{\nu,c} - \mathcal{I}_{\nu,c}^d) V_c \quad (29)$$

114

As represented in Fig. 2, Eq. (29) is split into an incoming part ( $k_i \in \boldsymbol{\Omega}_d \cdot \Delta \mathbf{S}_c < 0$ ) and an outgoing part ( $k_o \in \boldsymbol{\Omega}_d \cdot \Delta \mathbf{S}_c > 0$ ). This is expressed in mathematical terms as

$$\sum_{k_i=1}^{N_{k_i}} \mathcal{I}_{\nu,k_i}^d \boldsymbol{\Omega}_d \cdot \Delta \mathbf{S}_c + \mathcal{I}_{\nu,c}^d \sum_{k_o=1}^{N_{k_o}} \boldsymbol{\Omega}_d \cdot \Delta \mathbf{S}_c = \kappa_{\nu,c} (B_{\nu,c} - \mathcal{I}_{\nu,c}^d) V_c. \quad (30)$$

115

Consequently, an explicit expression of the radiative intensity at the cell  $c$  is obtained as follows

$$\mathcal{I}_{\nu,c}^d = \frac{\sum_{k_i=1}^{N_{k_i}} \mathcal{I}_{\nu,k_i}^d \boldsymbol{\Omega}_d \cdot \Delta \mathbf{S}_c + \kappa_{\nu,c} B_{\nu,c} V_c}{\sum_{k_o=1}^{N_{k_o}} \boldsymbol{\Omega}_d \cdot \Delta \mathbf{S}_c + \kappa_{\nu,c} V_c} \quad (31)$$

116  $\mathcal{I}_{\nu,c}^d$  is solved in Eq. (31) for each band and direction following a specific order,  
 117 different for each direction, represented in Fig. 2.

The divergence of the total radiative heat flux at each cell  $c$  is given by

$$\partial_{\mathbf{x}} \cdot \mathbf{q}_c^{rad} = \sum_{\nu=1}^{N_b} \sum_{d=1}^{N_d} \frac{a_d \mathcal{I}_{\nu,c}^d \boldsymbol{\Omega}_d \cdot \Delta \mathbf{S}_c}{V_c} \quad (32)$$

118

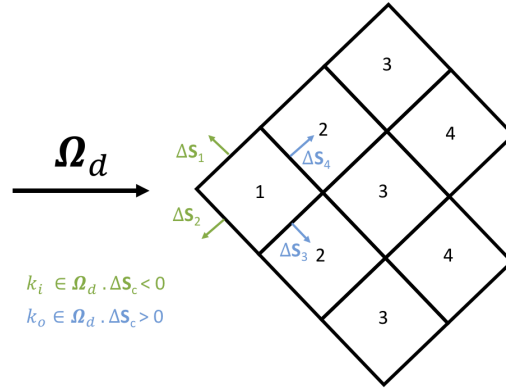


Figure 2: Representation of the directional split into an incoming and outgoing part and the specific advance order used in the computation of the radiative heat flux.

119 *2.4. Boundary Conditions*

120 The Boundary Conditions (BC) used are presented (see Fig. 3) in this section for the multiple physics phenomena involved in these arc heater simulations.

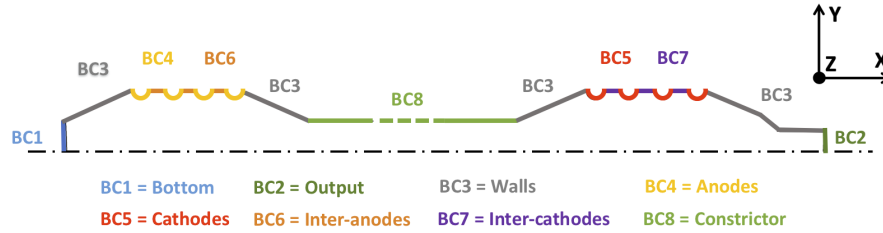


Figure 3: Representation of the boundary conditions in the studied arc heater. The flow is injected at the inter-anodes (BC6) and constrictor (BC8). BC2 is treated as an outflow boundary.

121



122 *2.4.1. Flow*

The water-cooled arc heater walls (BC3), constrictor (BC8), and inter-electrodes (BC6 and BC7) are assumed to be at a constant temperature. The electrodes (BC4 and BC5) and the outlet (BC2) are assumed adiabatic. A constant air inlet mass flux is imposed at the inter-anodes (BC6) and constrictor (BC8). The swirling inlet velocity is computed as follows

$$\mathbf{u} = \frac{-\dot{m}}{\rho |\mathbf{S}|} (\mathbf{n} + \tan(\theta) \mathbf{t}) \quad \text{with} \quad \mathbf{t} = \mathbf{n}_{axis} \times \mathbf{n}. \quad (33)$$

123 The tangential inclination angle is set at 45° to promote air mixing. Table  
124 1 gives a summary of the BC that are used for the flow in this study.

	<b>p [Pa]</b>	<b>T [K]</b>	<b>u [m/s]</b>
<i>BC1</i>	$\partial_x p \cdot \mathbf{n} = 0$	$T_w = 500 \text{ K}$	$\mathbf{u} = \mathbf{0}$
<i>BC2</i>	$\partial_x p \cdot \mathbf{n} = 0$	$\partial_x T \cdot \mathbf{n} = 0$	$\partial_x \mathbf{u} \cdot \mathbf{n} = \mathbf{0}$
<i>BC3</i>	$\partial_x p \cdot \mathbf{n} = 0$	$T_w = 500 \text{ K}$	$\mathbf{u} = \mathbf{0}$
<i>BC4</i>	$\partial_x p \cdot \mathbf{n} = 0$	$\partial_x T \cdot \mathbf{n} = 0$	$\mathbf{u} = \mathbf{0}$
<i>BC5</i>	$\partial_x p \cdot \mathbf{n} = 0$	$\partial_x T \cdot \mathbf{n} = 0$	$\mathbf{u} = \mathbf{0}$
<i>BC6</i>	$\partial_x p \cdot \mathbf{n} = 0$	$T_w = 500 \text{ K}$	$\dot{m}_{BC6} = 0.05 \text{ kg/s}$
<i>BC7</i>	$\partial_x p \cdot \mathbf{n} = 0$	$T_w = 500 \text{ K}$	$\mathbf{u} = \mathbf{0}$
<i>BC8</i>	$\partial_x p \cdot \mathbf{n} = 0$	$T_w = 500 \text{ K}$	$\dot{m}_{BC8} = 0.4 \text{ kg/s}$

Table 1: Summary of the BC for the Navier-Stokes equations.

125 *2.4.2. Electromagnetism*

The power supply to the arc heater is a constant current supply, and therefore, the electric field will adjust to enable a constant current between the anode and cathode chambers. The anode current density is considered constant in this study, as follows

$$\sigma \partial_x \phi_{imp,a_i} \cdot \mathbf{n} = -\mathbf{J}_{imp,a_i} \quad \text{and} \quad I_{imp,a_i} = \int_{S_{a_i}} \mathbf{J}_{imp,a_i} \cdot d\mathbf{S} \quad (34)$$

126

An unbalanced current at the electrodes will result in an uneven current attachment: the electrode with the highest current will experience the highest

heating near the surface, which results in the highest conductivity for the gas near the surface, thus reinforcing the location of this attachment. A stable concentrated current attachment will result in mechanical failure (erosion or melting) of the electrode. The use of a ballast configuration balances the electric potential at each electrode to force an even distribution of the total current. The same resistance is used for all electrodes. The configuration results in a mixed boundary condition at the cathodes. This reads in mathematical terms

$$-I_{imp,c_i} = \int_{S_{c_i}} \sigma \boldsymbol{\partial}_{\mathbf{x}} \phi_{imp,c_i} \cdot d\mathbf{S} = \frac{1}{R_b} (\phi_{imp,c_i} - \phi_G) \quad (35)$$

An explicit scheme is used to impose the above BC. This is done by setting the potential at the discretized time step  $n$  to

$$\phi_{imp,c_i}^n = \phi_G - R_b I_{imp,c_i}^{n-1} \quad (36)$$

with  $I_{imp,c_i}^0 = 0$  initially, otherwise

$$I_{imp,c_i}^{n-1} = \int_{S_{c_i}} -\sigma \boldsymbol{\partial}_{\mathbf{x}} \phi_{imp,c_i}^{n-1} \cdot d\mathbf{S} \quad (37)$$

i.e., the potential at the cathodes is adjusted by using the electric current from the previous time step. Note that the continuity of the electric current ensures that

$$-\sum_{i=1}^{N_{c_i}} I_{imp,c_i} = \sum_{i=1}^{N_{a_i}} I_{imp,a_i} = I_{imp,tot} \quad (38)$$

The walls of segmented arc heaters are water-cooled copper rings that are isolated from each other by thin ceramic (non-conductive) spacers. The rings are electrically isolated from the ground, and the spacers prevent current along the walls. In this configuration, electrically floating rings ensure proper adjustment of the electric potential. The walls of the constrictor are made of floating rings except at the electrodes where voltage is applied to ensure constant current. As

such, charge will accumulate in each ring so that

$$\mathbf{J}_{imp,w} = \mathbf{0} \quad (39)$$

Thus, the electrical potential boundary condition at the wall is

$$\partial_{\mathbf{x}}\phi_{imp,w} \cdot \mathbf{n} = 0 \quad (40)$$

127 Table 2 summarizes the BC that are used for the electromagnetic equations  
128 in this simulation.

	$\phi_{imp}$ [V]	$\mathbf{A}_{imp}$ [T · m]	$\mathbf{A}_e$ [T · m]
<i>BC1</i>	$\partial_{\mathbf{x}}\phi_{imp} \cdot \mathbf{n} = 0$	$\mathbf{A}_{imp} = \mathbf{0}$	$\mathbf{A}_e = \mathbf{0}$
<i>BC2</i>	$\partial_{\mathbf{x}}\phi_{imp} \cdot \mathbf{n} = 0$	$\mathbf{A}_{imp} = \mathbf{0}$	$\mathbf{A}_e = \mathbf{0}$
<i>BC3</i>	$\partial_{\mathbf{x}}\phi_{imp} \cdot \mathbf{n} = 0$	$\mathbf{A}_{imp} = \mathbf{0}$	$\mathbf{A}_e = \mathbf{0}$
<i>BC4</i>	$I_{imp,tot} = 1200$ A	$\mathbf{A}_{imp} = \mathbf{0}$	$\mathbf{A}_e = \mathbf{0}$
<i>BC5</i>	$\phi_{imp,c_i}^n = \phi_G - R_b I_{imp,c_i}^{n-1}$	$\mathbf{A}_{imp} = \mathbf{0}$	$\mathbf{A}_e = \mathbf{0}$
<i>BC6</i>	$\partial_{\mathbf{x}}\phi_{imp} \cdot \mathbf{n} = 0$	$\mathbf{A}_{imp} = \mathbf{0}$	$\mathbf{A}_e = \mathbf{0}$
<i>BC7</i>	$\partial_{\mathbf{x}}\phi_{imp} \cdot \mathbf{n} = 0$	$\mathbf{A}_{imp} = \mathbf{0}$	$\mathbf{A}_e = \mathbf{0}$
<i>BC8</i>	$\partial_{\mathbf{x}}\phi_{imp} \cdot \mathbf{n} = 0$	$\mathbf{A}_{imp} = \mathbf{0}$	$\mathbf{A}_e = \mathbf{0}$

Table 2: Summary of the BC for the Maxwell equations.

### 129 2.4.3. Radiation

The walls are made of unpolished copper and are assumed to behave as a gray body at the wall temperature. We set the emissivity of copper  $\epsilon_{\nu} = 0.6$ , constant for all wavelengths. The average radiative intensity from the wall surface, emitted by each band, is given by

$$\mathcal{I}_{\nu,w}^d = \epsilon_{\nu} B_{\nu} \quad (41)$$

130

### 131 **3. Numerical model**

132 The numerical framework is based on OpenFOAM, an open-source, modern,  
133 and modular C++ finite volume library. This framework offers a wide range  
134 of numerical schemes, interpolation schemes, integration methods, turbulence  
135 models, thermodynamic models, and flow solvers. The governing equations are  
136 sequentially solved as explained in Algorithm 1. The OpenFOAM semi-discrete  
137 finite volume solver rhoCentralFoam has been chosen for modeling the flow.  
138 The non-staggered central scheme that is used in this solver has proven to be  
139 accurate in the simulations of high-speed compressible flows [13]. Additionally,  
140 a  $k - \epsilon$  model is applied to the flow turbulence [7]. Second order accuracy in  
141 space is achieved with a total variation diminishing (TVD) scheme with van  
142 Albada’s limiter [41]. The Maxwell equations, described in Eq. (13) and (16),  
143 that link the electric and magnetic fields are computed by using a generalized  
144 Geometric-Algebraic Multi-Grid (GAMG) solver. The Mutation++ library [35],  
145 dynamically linked to the OpenFOAM thermodynamic framework, is used to  
146 compute equilibrium chemistry compositions and thermodynamic and trans-  
147 port properties, as detailed in Appendix A. The discretization of radiative heat  
148 flux uses a multi-band approximation of the non-gray spectral properties of  
149 high-temperature air. The radiative directional integration is determined by  
150 a Gaussian type quadrature. The contributions of the electric field, magnetic  
151 field, and radiative heat flux are added as source terms to the momentum and  
152 energy equations.

---

**Algorithm 1** ARCHeS

---

```
1: function MAIN
2:   create time and mesh
3:   read and initialize user configurations
4:                                     ▷ flags for radiation and electromagnetism
5:   read fields  $p^0, T^0, \mathbf{u}^0, \phi^0, \mathbf{A}^0, \partial_{\mathbf{x}} \cdot \mathbf{q}_{rad}^0$ 
6:   compute  $\alpha^0, \psi^0, \mu^0, \sigma^0, \gamma^0, e^0, \rho^0$ 
7:                                     ▷ thermodynamics in function of  $p^0, T^0$ 
8:   initialize radiation and electromagnetism
9:   while RunTime do
10:    correct boundary conditions for  $\mathbf{u}^n, \sigma^n, \phi^n$ 
11:    compute  $\phi^{n+1}, \mathbf{A}^{n+1}$ 
12:                                     ▷ solve electromagnetic equations
13:    compute  $\partial_{\mathbf{x}} \cdot \mathbf{q}_{rad}^{n+1}$ 
14:                                     ▷ solve radiation
15:    compute fluxes and source terms
16:    compute  $\rho^{n+1}$ 
17:                                     ▷ solve mass equation
18:    compute  $\mathbf{u}^{n+1}$ 
19:                                     ▷ solve momentum equations
20:    compute  $e^{n+1}$ 
21:                                     ▷ solve energy equation
22:    compute  $T^{n+1}$ 
23:    compute  $\alpha^{n+1}, \psi^{n+1}, \mu^{n+1}, \sigma^{n+1}, \gamma^{n+1}$ 
24:                                     ▷ thermodynamics in function of  $p^n, T^{n+1}$ 
25:    compute  $p^{n+1}$ 
26:    correct  $p^{n+1}, \rho^{n+1}$  boundaries
27:    compute  $\mu_{eff}^{n+1}$ , and  $\alpha_{eff}^{n+1}$ 
28:                                     ▷ solve  $k - \epsilon$  turbulence equations
29:    write fields
30:
```

---

153 **4. Results**

154 *4.1. Flow*

155 Figure 4 shows the pressure and temperature profiles along the centerline axis  
156 of the arc heater. The pressure has a maximum value of 13.3 atm, almost steady  
157 in the anode chamber. The pressure gently decreases along the constrictor from  
158 13.2 to 9.2 atm. The pressure stays constant in the cathode chamber and then  
159 drops again in the convergent part. The pressure is about 3.5 atm at the nozzle  
160 throat. The time-averaged temperature computed from 0.4640 s to 0.5759 s  
161 has a maximum value of  $\sim 8000$  K. The temperature increases in the anode  
162 chamber. The instantaneous time values of the temperature highly fluctuate in  
163 the constructor as a consequence of the arc instabilities. The temperature drops  
164 from 7500 K at the cathode chamber to 4400 K at the nozzle throat.

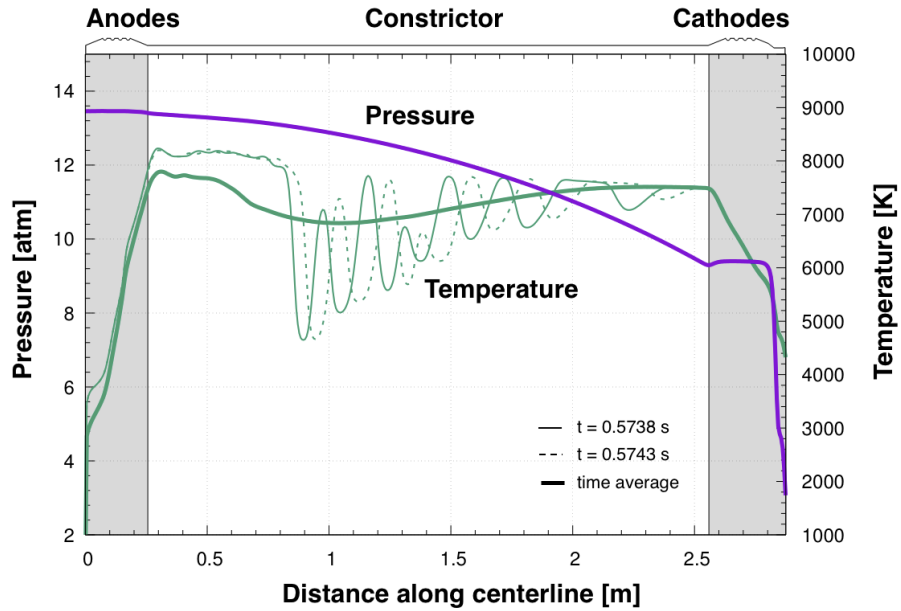


Figure 4: Pressure and temperature profiles along the centerline axis.

165 Figure 5 shows the enthalpy and velocity profiles along the centerline axis of  
166 the arc heater. The time-averaged enthalpy and velocity were computed from  
167 0.4640 s to 0.5759 s. The enthalpy follows the trend of the temperature (Fig.

168 4). The maximum of the time-averaged enthalpy is about  $24 \text{ MJ} \cdot \text{kg}^{-1}$ . For  
 169 both instantaneous and time-averaged, the gas enthalpy increases in the anode  
 170 chamber and decreases in the cathode chamber. In the constrictor region, the  
 171 averaged value drops only to increase again so that the value at the exit of the  
 172 constrictor is similar to the one at the entrance. Alternatively, instantaneous  
 173 values show high fluctuation that is indicative of arc instabilities. Instantaneous  
 174 and time-averaged velocities along the centerline follow a similar trend. The  
 175 velocity is low in the anode chamber due to the absence of air injection, which  
 176 is introduced along the inter-anodes and the constrictor (Fig. 3). The velocity  
 177 increases up to  $550 \text{ m} \cdot \text{s}^{-1}$  in the constrictor, drops in the divergent part,  
 178 and increases again in the convergent part of the cathode chamber. The flow is  
 179 subsonic in the anode chamber, the constrictor, and the cathode chamber. The  
 180 Mach number equals 1 at the nozzle throat. The flow becomes supersonic after  
 181 the nozzle throat.

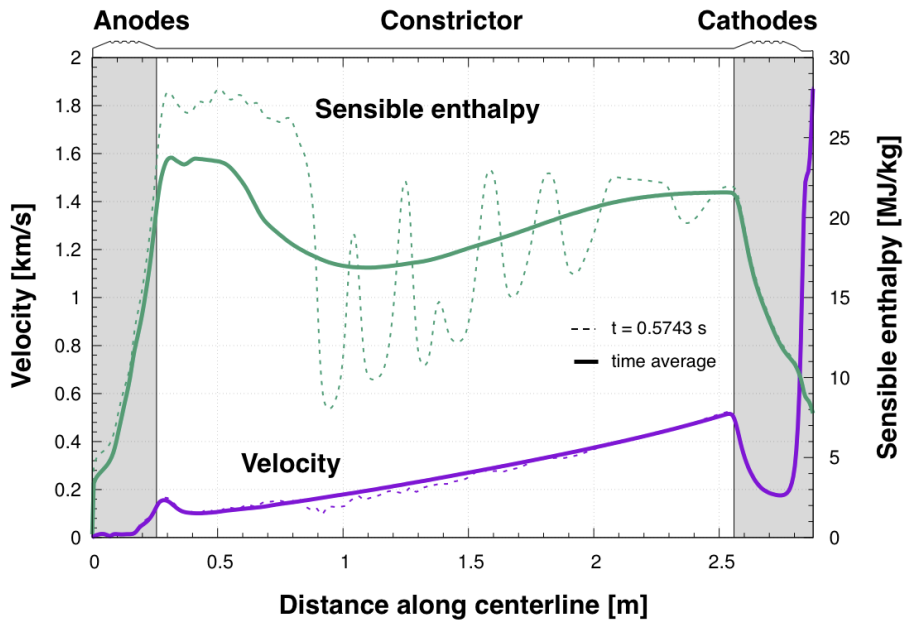


Figure 5: Velocity and enthalpy profiles along the centerline axis.

182 Figure 6 shows the XY cutting plane of the temperature from the anode

183 chamber to the nozzle throat. The highest temperature (around 8000 K) is  
 184 close to the centerline axis because of the low temperature imposed at the water-  
 185 cooled walls. Arc instabilities are visible in the constrictor. A three-dimensional  
 186 unsteady modeling tool is needed to capture these instabilities.

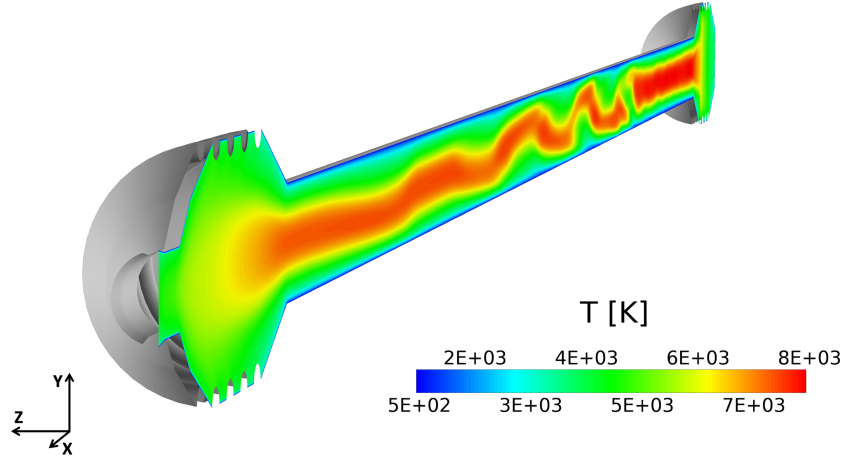


Figure 6: Temperature XY cutting plane from the anode chamber to the nozzle throat,  $t = 0.5738$  s.

187 Figure 7 shows the YZ cutting plane of the temperature for different times  
 188 at  $X = 1.5$  m. The shape of the high-temperature electric arc core shows  
 189 an unsteady behavior. The dynamic of the arc indicates that the instability  
 190 saturates next to the wall. The core cannot get close to the constrictor wall due  
 to the injection of low-temperature air and due to the water cooling system.

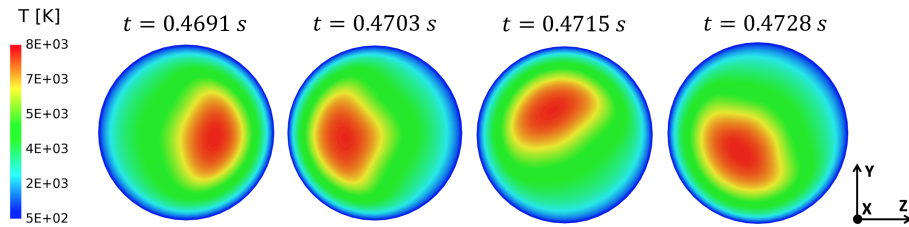


Figure 7: Temperature YZ cutting plane at  $X = 1.5$  m for different times.

191

192 Figures 8 and 9 show the XY cutting plane of the velocity at the cathode



193 and anode chambers. The inlet mass flow is  $0.05 \text{ kg} \cdot \text{s}^{-1}$  in the anode chamber  
 194 and  $0.4 \text{ kg} \cdot \text{s}^{-1}$  in the constrictor. The gas velocity is higher next to the  
 195 centerline axis. The highest velocity of  $1.9 \text{ km} \cdot \text{s}^{-1}$  is located at the end of the  
 196 computational domain, just after the nozzle throat. Figure 9 shows that the  
 197 flow is impacted by the arc instabilities in the constrictor.

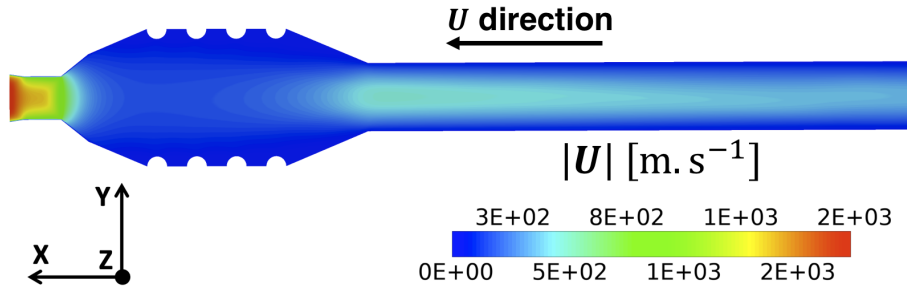


Figure 8: Velocity XY cutting plane in the constrictor and the cathode chamber,  $t = 0.5738$  s.

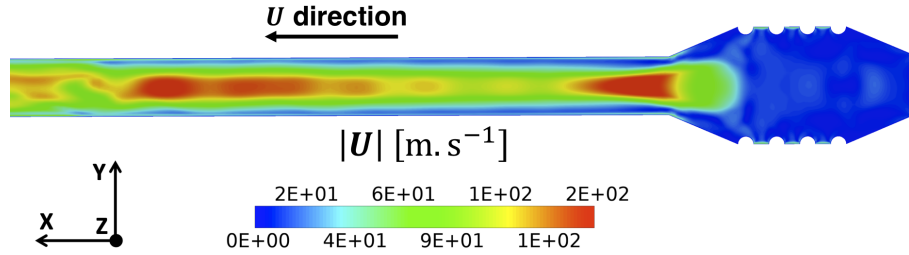


Figure 9: Velocity XY cutting plane in the constrictor and the anode chamber,  $t = 0.5738$  s.

Figure 10 shows cutting planes of the vorticity  $\omega$  computed as follows

$$\omega = \partial_x \times \mathbf{u} \quad (42)$$

198 As described in section 2.4, the flow is injected at the inter-anodes and con-  
 199 strictor. The XY cutting plane shows the vorticity magnitude for the entire  
 200 geometry. The highest vorticity magnitude is located close to the wall. The  
 201 YZ cutting planes show the vorticity magnitude in color, the YZ velocity  $\mathbf{u}_{yz}$   
 202 using streamlines at  $X = [1, 1.5] \text{ m}$ , and the axial velocity  $\mathbf{u}_x$  at  $X = 2 \text{ m}$ . The

203 locations of the vortex cores change along the centerline axis, and the velocity  
 204 is mainly axial at  $X = 2$  m. Future work will study the strong interactions  
 205 between wall shear and arc instabilities that impact the vorticity and play an  
 206 essential role in flow mixing.

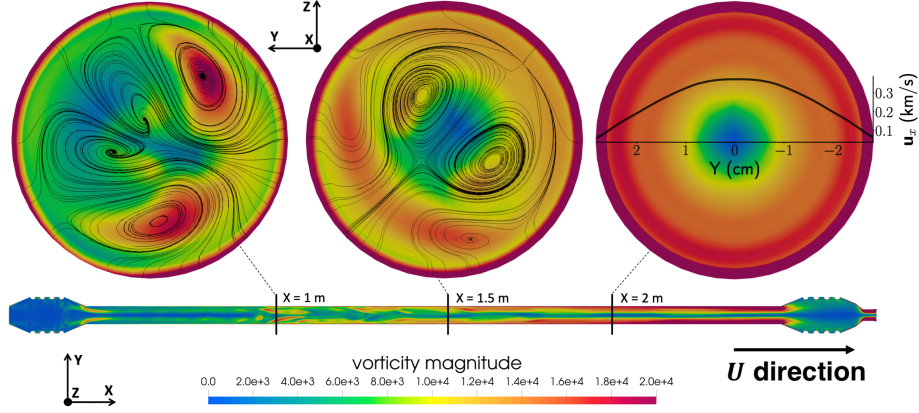


Figure 10: XY and YZ cutting planes of the vorticity  $\omega$ ,  $t = 0.5738$  s. The color shows the vorticity magnitude, the streamlines represent the velocity in the YZ plane  $u_{yz}$  at  $X = [1, 1.5]$  m, and the plot shows the axial velocity  $u_x$  at  $X = 2$  m.

#### 207 4.2. Electromagnetism and radiation

208 Figure 11 shows the comparison between the Joule heating and radiative  
 209 heating/cooling profiles along the centerline axis. The radiative heat flux is  
 210 the predominant cooling process of the gas medium. In the same figure, a  
 211 snapshot of the radiative heating/cooling and Joule heating profiles presents  
 212 fluctuations along the axis resulting from the electric arc instabilities. The  
 213 radiative heating/cooling follows the Joule heating curve trend and is mostly  
 214 negligible along the constrictor centerline. Joule heating is almost null at the  
 215 start of the anode chamber and at the end of the cathode chamber.

216 Figure 12 shows the Joule heating and the radiative heating/cooling profiles  
 217 along the radial axis at half-length of the constrictor. The radiative cooling  
 218 ( $\partial_x \cdot q^{rad} > 0$ ) is due to the re-radiation of the heat from the electric arc core,  
 219 where the temperature is the highest. The radiative cooling is almost negligible  
 220 compared to Joule heating. The radiative heating ( $\partial_x \cdot q^{rad} < 0$ ) is strong

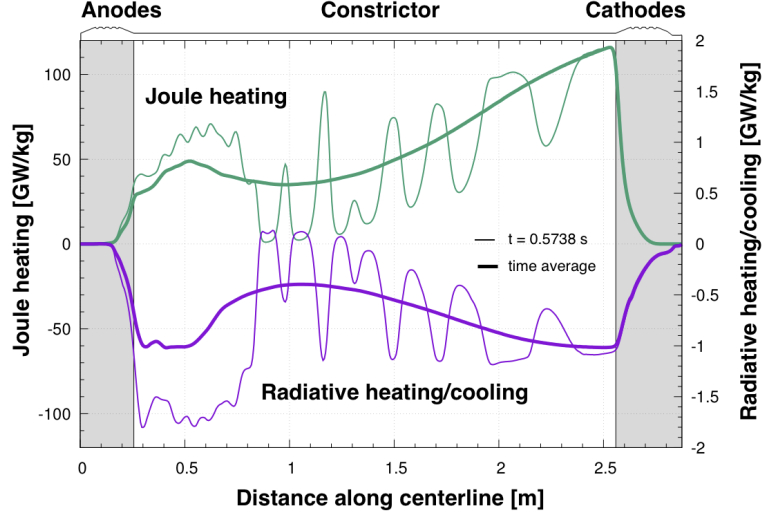


Figure 11: Joule heating ( $|J_{imp}|$ ) and radiative heating/cooling ( $-\partial_x \cdot \mathbf{q}^{rad}$ ) profiles along the centerline axis.

221 outside the electric arc core and is higher than the Joule heating next to the  
 222 wall ( $2.30 \text{ cm} < \text{radius} < 2.85 \text{ cm}$ ).

223 Figure 13 shows the cutting plane of the radiative heating/cooling from the  
 224 anode chamber to the nozzle throat. The blue color represents the radiative  
 225 cooling ( $\partial_x \cdot \mathbf{q}^{rad} > 0$ ) which occurs at the center of the electric arc. The  
 226 red color shows the radiative heating ( $\partial_x \cdot \mathbf{q}^{rad} < 0$ ) at the surroundings of the  
 227 electric arc core. The transverse cutting planes show the electric arc instabilities  
 228 and the importance of the three-dimensional calculations of the radiation. In  
 229 this work, the discretization of the radiative heat flux is done using 24 discrete  
 230 directions and 100 spectral bands.

231 Figure 14 shows the cutting plane of the imposed current density from the  
 232 anode chamber to the nozzle throat. The current density is mainly constrained  
 233 close to the centerline of the constrictor due to the cold injection of air at the  
 234 wall. The total current is equally distributed among the anodes due to the BC  
 235 explained in Eq. (34). The continuity of current from Eq. (13) imposes the  
 236 same total current at the anodes and the cathodes. The electric ballast system,

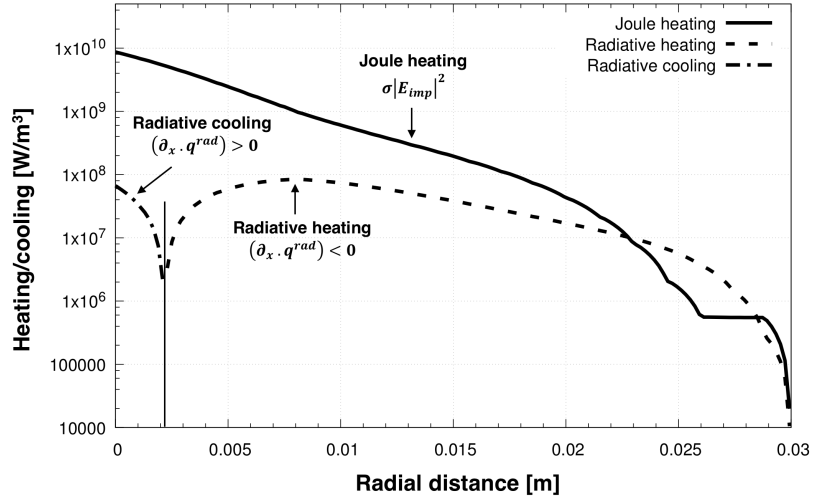


Figure 12: Joule heating and radiative heating/cooling profile in the radial direction halfway along the constrictor,  $t = 0.5738$  s.

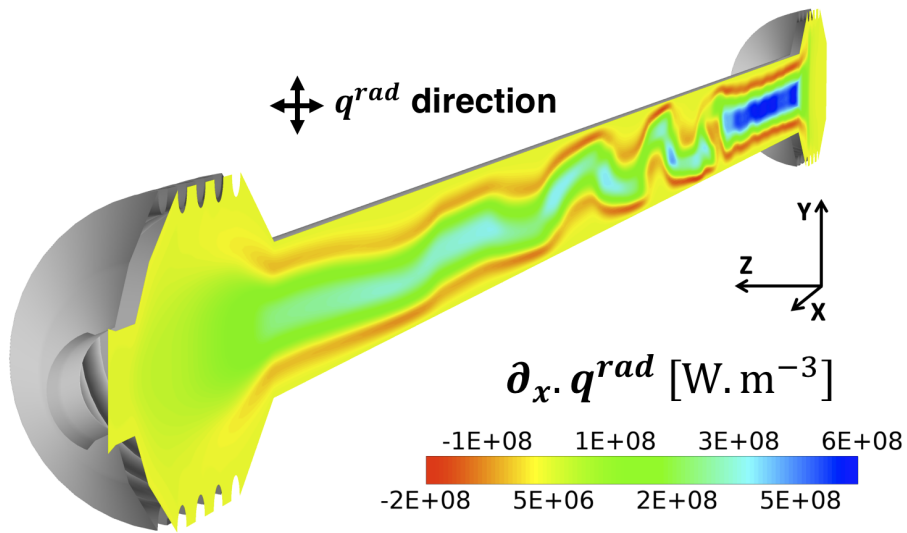


Figure 13: Divergence of the radiative heat flux from the anode chamber to the nozzle throat,  $t = 0.5738$  s. The blue and red colors correspond to radiative cooling and heating, respectively.

237 implemented using Eq. (37), balances the current at each cathode to force an  
 238 even distribution. The peak of the imposed current density is about  $3 \text{ MA} \cdot$   
 239  $\text{m}^{-2}$ .

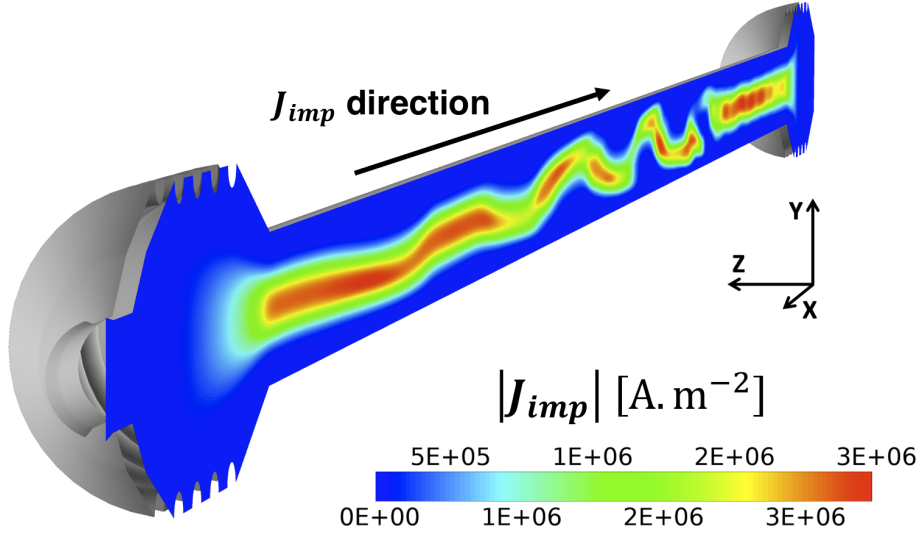


Figure 14: Imposed current density from the anode chamber to the nozzle throat,  $t = 0.5738$  s.

240 Figure 15 shows the cutting plane of the imposed magnetic field from the  
 241 anode chamber to the nozzle throat. The imposed magnetic field is generated  
 242 around the imposed current density according to Ampere's law (Eq. (3)). The  
 243 maximum magnitude of the imposed magnetic field is about 0.02 T. The mag-  
 244 netic field direction is mainly perpendicular to the imposed current density. The  
 245 magnetic field is null at the center of the electric arc and increases in the radial  
 246 direction. Towards the wall, it decreases again. The curvature of the electric  
 247 arc creates a stronger magnetic field at the concave part of the electric arc,  
 248 which results in the kink instability that is shown in Fig. 15. The intensity of  
 249 the imposed magnetic field decreases in the electrode chambers due to the more  
 250 diffuse imposed current density.

251 Figure 16 shows the cutting plane of the external magnetic field created by  
 252 the electrodes from the anode chamber to the nozzle throat. The magnitude

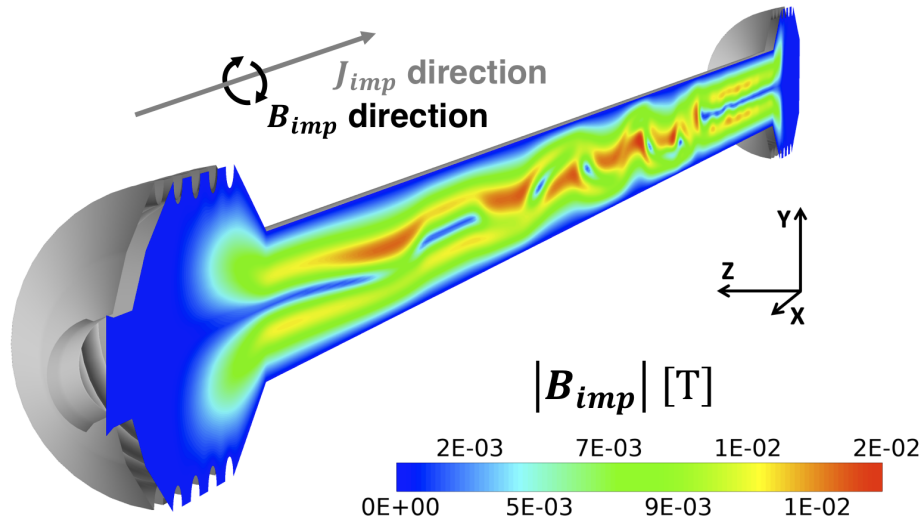


Figure 15: Imposed magnetic field from the anode chamber to the nozzle throat,  $t = 0.5738$  s.

253 of the external magnetic field is almost null in the constrictor. The maximum  
 254 magnitude is about 0.01 T and is found around the electrodes. The direction is  
 255 primarily longitudinal to the imposed current density direction.

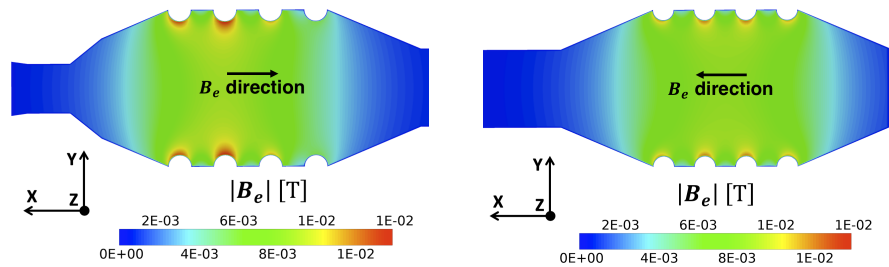


Figure 16: External magnetic field in the cathode (left) and anode (right) chambers,  $t = 0.5738$  s.

256 Figure 17 shows the current as a function of time at the different cathodes.  
 257 The sum of the 4 cathode currents gives a total current of 1170 A. Cathode 1  
 258 is the closest electrode to the constrictor. Cathode 4 is the nearest electrode to  
 259 the nozzle throat. A peak of current appears on all the electrodes at 0.487 s.  
 260 Then the 4 currents stabilize to a quasi constant value. The simulation seems

261 to achieve a statistically steady state.

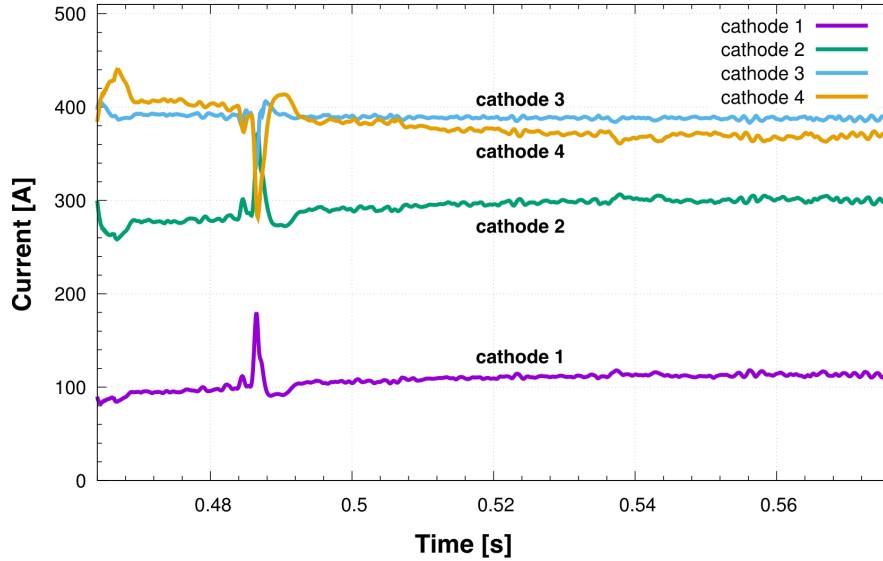


Figure 17: Current at the cathodes as a function of time.

262 *4.3. Arc instabilities*

263 Trelles [38] describes different types of instabilities that can play a dominant  
264 role in the dynamic geometry of a constrained electric arc. The fluid dynamic  
265 instabilities are produced by the large gradient of velocity, temperature, density,  
266 and viscosity between the cold gas and the plasma created by Joule heating.  
267 Shear flow instabilities are often considered one of the most prominent, leading  
268 to the growth of a small perturbation downstream that may drive the flow  
269 from laminar to turbulent. The thermal instabilities are caused by the non-  
270 uniform cooling of the plasma that produces a deflection of the electric arc. The  
271 injection of cold gas forces the arc to deviate away from the cold source. The  
272 magnetic field, both the external and that self-induced by the electric current,  
273 is responsible for the sausage and kink instabilities. The non-uniform plasma  
274 distribution and the curvature of the magnetic field impact these instabilities.  
275 Sausage instabilities occur when there is a change of cross section in the electric

276 arc. In this regard, a smaller cross section translates to a stronger magnetic field.  
277 Kink instabilities occur when the electric arc curves. Figure 18 shows the iso-  
278 surface of the imposed current density at  $1 \text{ MA} \cdot \text{m}^{-2}$ , a value representative  
279 of the main part of the electric arc. The color represents the magnitude of the  
280 total magnetic field. For high current density, the total magnetic field is  
281 primarily due to the imposed magnetic field in the constrictor. Even though  
282 the arc instabilities in Fig. 18 are a combination of all the instabilities, the kink  
283 instability is visible where the magnetic field is stronger on the concave side  
284 and weaker on the convex side of the plasma column. The electric arc seems to  
285 stabilize at the entrance of the electrode chambers. More advanced analysis of  
286 the arc behavior is required to fully understand and characterize the complex  
287 dynamic interaction of the cold gas as well as the inherent properties of thermal  
288 plasma. Future work will see the implementation of finite-rate chemistry in the  
289 existing model and may lead to new insights in low-pressure regions where the  
290 assumption of Local Thermal Equilibrium (LTE) breaks down.

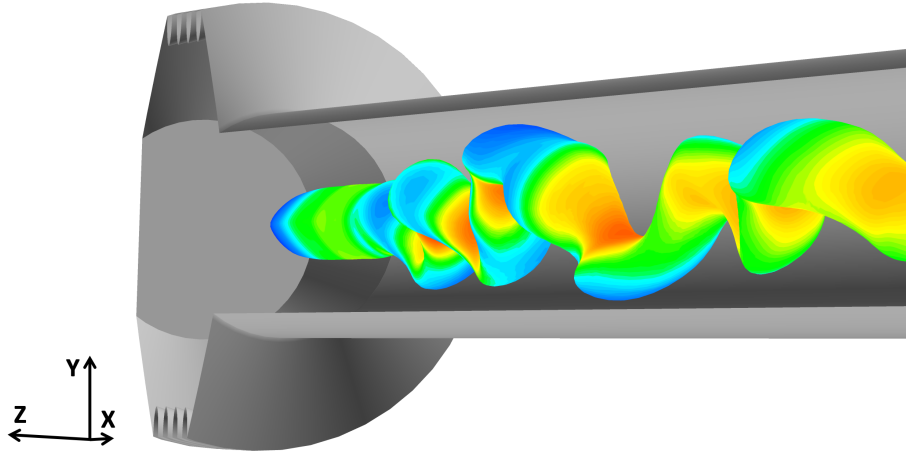


Figure 18: Arc instabilities in the constrictor,  $t = 0.5738 \text{ s}$ . The iso-surface shows the current density ( $1 \text{ MA} \cdot \text{m}^{-2}$ ). The color represents the magnitude of the total magnetic field.



#### 291 4.4. Arc attachment

292 Figures 19 and 20 show the iso-surface of the current density ( $30 \text{ kA} \cdot \text{m}^{-2}$ )  
293 at two different times ( $t = 0.4785 \text{ s}$  and  $t = 0.4880 \text{ s}$ ). The color represents the  
294 magnitude of the total magnetic field. The external magnetic field, predomi-  
295 nantly present around the electrode chambers, has a more significant influence  
296 on the arc attachment to the electrodes than the imposed magnetic field, almost  
297 negligible in these regions. The Neumann boundary condition of the imposed  
298 electric potential forces a constant and uniform current density among the an-  
299 odes. On the other hand, the Dirichlet boundary condition at the cathodes  
300 sets a constant imposed electric potential, which translates to a non-uniform  
301 and non-constant electric current among the cathodes. The location of the arc  
302 attachment changes over time at the cathodes. The high temperature in the  
303 cathode chamber makes the air conductive and allows a diffuse arc attachment.  
304 The ballast configuration balances the electric potential at each electrode to  
305 force an even distribution of the total current. It is shown that the attachment  
306 at the cathodes is mainly driven by the upstream arc instabilities. The kink in-  
307 stabilities generated in the constrictor force the arc to move closer to a specific  
308 region of the electrodes, which, in turn, provokes the attachment of the arc.  
309 Unlike the studies from Trelles [38], this LTE model does not need an artifi-  
310 cially high electric conductivity near the cathodes by choosing an interpolation  
311 scheme that conserves the electric flux.

## 312 5. Conclusion

313 Further development of arc jet technologies has been limited by an incom-  
314 plete understanding of unstable plasma flow occurring in the constricted arc  
315 heater. The direct observation of the complete arc dynamics is extremely dif-  
316 ficult inside an arc jet, and its numerical modeling is highly challenging. The  
317 complexity of plasma flow simulations comes from the partial knowledge of  
318 constrained arc instabilities. The interactions of cold gas and plasma flow are  
319 strongly influenced by the radiation and the magnetic field that lead to com-

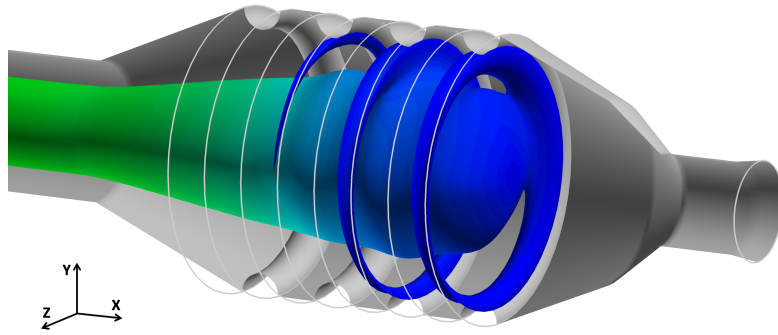


Figure 19: Arc attachment at the cathodes,  $t = 0.4785$  s. The iso-surface shows the current density ( $30 \text{ kA} \cdot \text{m}^{-2}$ ). The color represents the magnitude of the total magnetic field.

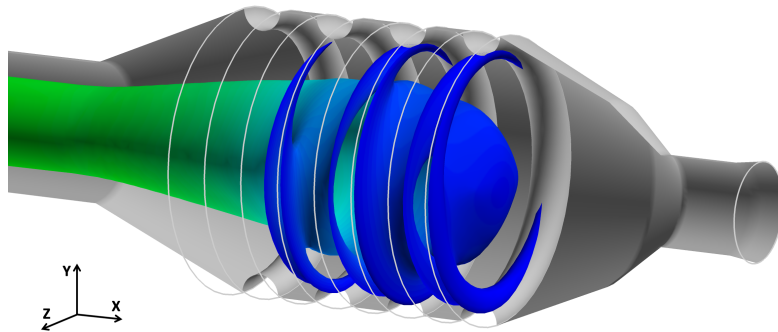


Figure 20: Arc attachment at the cathodes,  $t = 0.4880$  s. The iso-surface shows the current density ( $30 \text{ kA} \cdot \text{m}^{-2}$ ). The color represents the magnitude of the total magnetic field.

320 plicated coupled instabilities. A separation of the different physics involved is  
321 necessary to understand these instabilities. Arc attachment at the electrodes is  
322 a central part of the dynamics of the arc. The development of the analysis tool  
323 ARC Heater Simulator (ARCHEs) software provides the capability to perform  
324 three-dimensional time-dependent plasma flow simulations, which are necessary  
325 to capture the complex behavior of the arc. The massively parallel simulation  
326 support inherited in ARCHEs from its modular OpenFOAM architecture makes  
327 this work possible. This tool provides estimates of the flow state at the inlet of  
328 the nozzle, which are used as inlet conditions for well-established aerothermal  
329 techniques for estimating the aerothermal environment at the test articles. The  
330 formulation used in ARCHEs has been demonstrated to be efficient to attain  
331 convergence on very stiff non-linear systems encountered in plasma flow simu-  
332 lations. The unsteady flow is simulated from the anode chamber to the nozzle  
333 throat. Local Thermodynamic Equilibrium is assumed in the high-pressure arc  
334 heater. The imposed electric potential, the imposed magnetic field, and the  
335 external magnetic field are computed through Maxwell equations using a gener-  
336 alized geometric-algebraic multi-grid solver and are coupled to the flow through  
337 the Navier-Stokes equations. The three-dimensional radiation is solved using  
338 an accurate multi-band spectral discretization and Gaussian quadrature direc-  
339 tional discretization. The inlet mass flux and the total current were chosen to  
340 be close to the operating conditions of large arc jet complexes used for TPS  
341 evaluation. These numerical simulations performed on NASA's Pleiades super-  
342 computer have demonstrated their suitability for describing electric arc/cold  
343 flow interactions and flow instabilities encountered in arc heaters. This model  
344 is capable of describing the arc behavior and attachment at the cathodes with-  
345 out adding a reattachment model. The magnetic field plays an essential role  
346 in the flow characterization from the anode chamber to the nozzle throat. The  
347 location and frequency of the arc attachment at the cathodes are mainly driven  
348 by the upstream arc instabilities. A more complex analysis of the instabili-  
349 ties is needed to explain certain arc behaviors, such as the stabilization at the  
350 electrode chambers. The high computational cost of these simulations limits

351 the spatial resolution for wall turbulence analysis. These numerical simulations  
352 were obtained using 1200 cores of the NASA’s Pleiades supercomputer during  
353 three weeks, i.e., approximately  $6 \times 10^5$  core-hours. Comparison to measure-  
354 ments within arc heaters, such as those of Winter et al. [47], is still needed to  
355 validate this study. These experimental data are difficult to obtain due to the  
356 extremely harsh environment within the heaters. Future work should include  
357 a non-adiabatic boundary condition of the temperature at the electrodes. This  
358 requires a two-temperature model as well as a model of the energy transfer  
359 between the electrodes and the gas. The assumption of equilibrium chemistry  
360 in the entire arc heater from the anode chamber to the nozzle throat must be  
361 verified by adding a finite-rate chemistry model coupled with the fluid. Includ-  
362 ing heavy species in the thermodynamic and transport properties in addition to  
363 this model may pave the way to new areas of research, including the simulations  
364 of other complex plasma applications such as space propulsion. Even though  
365 large research efforts are still needed, this work has proven the effectiveness and  
366 efficiency of a finite volume approach for obtaining high-fidelity modeling of the  
367 governing equations involved in thermal plasma flows.

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379 **Appendix A. Thermodynamic and transport properties**

380 The equilibrium chemistry compositions and the thermodynamic and trans-  
 381 port properties (Fig. A.21, A.22, and A.23) are computed using the Muta-  
 382 tion++ library [35]. LTE is assumed in the entire domain. Most of the domain  
 383 where the arc instabilities occur has a high pressure (Fig. 4) which justifies the  
 384 use of LTE. Future work will take into account the low-pressure regions where  
 385 LTE breaks down. The air mixture used in this work is composed of the fol-  
 386 lowing 13 species:  $e^-$ ,  $N$ ,  $N^+$ ,  $O$ ,  $O^+$ ,  $NO$ ,  $NO^+$ ,  $N_2$ ,  $N_2^+$ ,  $O_2$ ,  $O_2^+$ ,  $Ar$  and  
 387  $Ar^+$ .

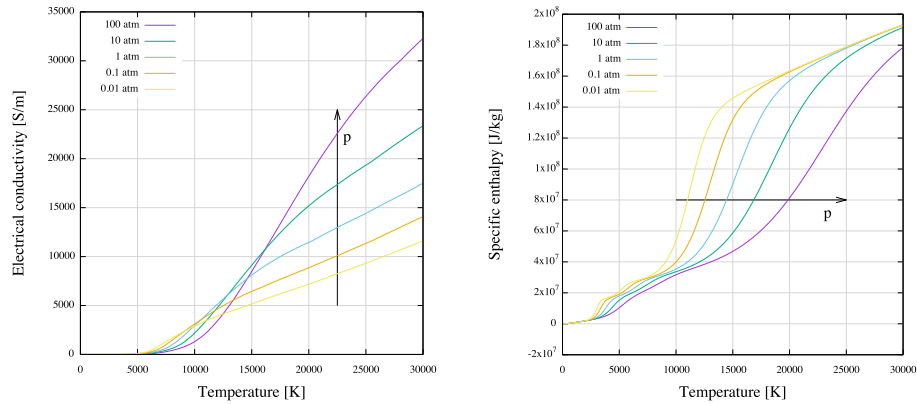


Figure A.21: Electrical conductivity (left) and enthalpy (right) as a function of temperature.

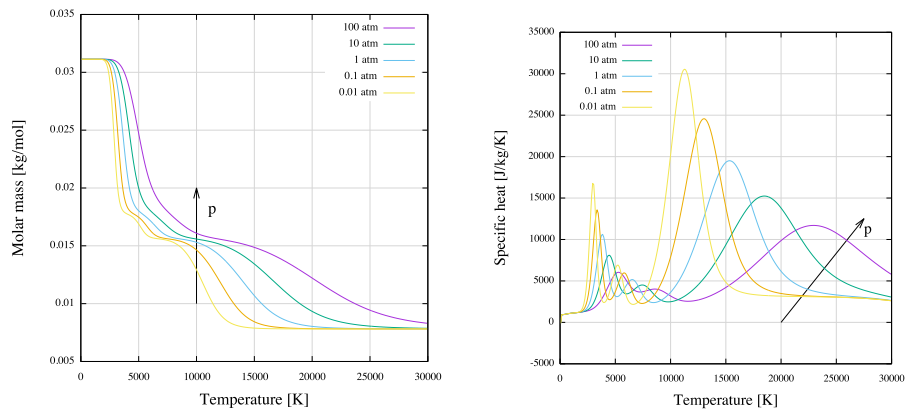


Figure A.22: Molar mass (left) and specific heat (right) as a function of temperature.

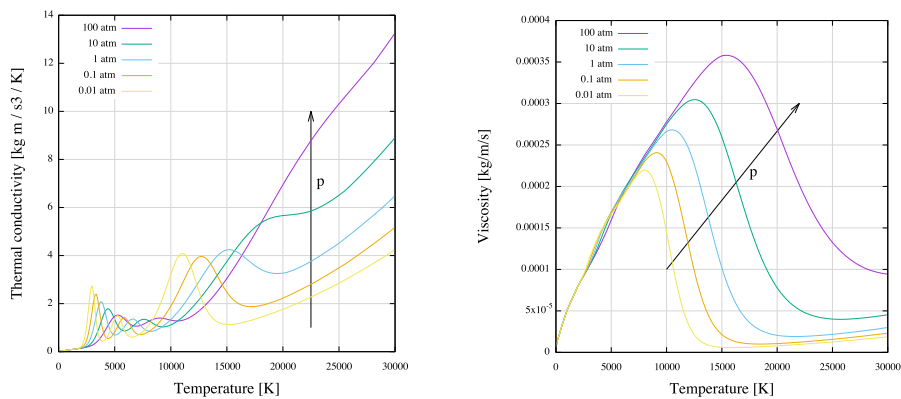


Figure A.23: Thermal conductivity (left) and viscosity (right) as a function of temperature.

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