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Three-dimensional unsteady model of arc heater plasma flow

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Abstract

Arc jets are unique facilities used to evaluate the performance of Thermal Protection Systems (TPS) for hypersonic vehicles. They produce high pressure and high enthalpy plasma flow to simulate the extreme heat encountered during atmospheric entry. The constricted arc heater part of an arc jet increases the test gas temperature by Joule heating. This study details the development of the three-dimensional unsteady plasma flow analysis tool, ARCHeS (ARC Heater Simulator). Coupled Navier-Stokes, radiative transfer and Maxwell equations yield current density, magnetism, radiation, and flow field solutions. The present work constitutes the first demonstration of an unsteady three-dimensional plasma flow simulation of high pressure and high enthalpy arc heater that captures kink instabilities of the electric arc. It is found that the arc attachment is mainly driven by upstream arc instabilities.

Keywords: Arc jet, Arc heater, Radiative transfer, Joule heating, Plasma, Arc instability, Arc attachment.

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Nomenclature

\mathcal{I}_ν^d	Discrete radiative intensity in direction Ω^d , $\text{W} \cdot \text{m}^{-2}$
\mathcal{I}_ν	Radiative intensity, $\text{W} \cdot \text{m}^{-2} \cdot \text{sr}^{-1}$
\mathbf{A}	Magnetic vector potential, $\text{T} \cdot \text{m}$
\mathbf{B}	Magnetic field, T
\mathbf{E}	Electric field, $\text{V} \cdot \text{m}^{-1}$
\mathbf{J}	Current density, $\text{A} \cdot \text{m}^{-2}$
\mathbf{n}	Normal direction
\mathbf{q}	Heat flux, $\text{W} \cdot \text{m}^{-2}$
\mathbf{t}	Tangential direction
\mathbf{u}	Velocity, $\text{m} \cdot \text{s}^{-1}$
\mathcal{H}	Planck's constant, $\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-1}$
\dot{m}	Mass flux, $\text{kg} \cdot \text{s}^{-1}$
\mathbf{S}	Outgoing surface normal, m^2
a_d	Gaussian quadrature weight
B_ν	Banded black body spectral radiative intensity, $\text{W} \cdot \text{m}^{-2}$
c	Speed of light in the medium., $\text{m} \cdot \text{s}^{-1}$
e	Internal energy, $\text{J} \cdot \text{kg}^{-1}$
E_0	Stagnation energy, $\text{J} \cdot \text{kg}^{-1}$
h	Enthalpy, $\text{J} \cdot \text{kg}^{-1}$
H_0	Stagnation enthalpy, $\text{J} \cdot \text{kg}^{-1}$
I	Electric current, A
k_B	Boltzmann's constant, $\text{J} \cdot \text{K}^{-1}$
k_i	Incoming part of the radiative directional split
k_o	Outgoing part of the radiative directional split
L_0	Characteristic length, m
M	Mach number
N_a	Number of anodes
N_b	Number of spectral bands
N_d	Number of directions

p	Pressure, Pa
Q_J	Joule heating, $\text{W} \cdot \text{m}^{-3}$
R_b	Resistance of the ballast configuration, Ω
Re_m	Magnetic Reynolds number
S	Surface area, m^2
T	Temperature, K
U_0	Characteristic velocity, $\text{m} \cdot \text{s}^{-1}$
V	Volume, m^3

Greek

Ω^d	Discrete direction
α	Diffusivity, $\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$
ω	Vorticity, s^{-1}
$\underline{\tau}$	Viscous stress tensor, $\text{kg} \cdot \text{m} \cdot \text{s}^{-2}$
ϵ_ν	Emissivity
η	Magnetic diffusivity, $\text{m}^2 \cdot \text{s}^{-1}$
γ	Heat capacity ratio
κ_ν	Linear absorption coefficient, m^{-1}
μ	Mixture viscosity, $\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$
μ_0	Vacuum magnetic permeability, $\text{H} \cdot \text{m}^{-1}$
ϕ	Electric potential, V
ψ	Compressibility, $\text{s}^2 \cdot \text{m}^{-2}$
ρ	Mass density, $\text{kg} \cdot \text{m}^{-3}$
σ	Electrical conductivity, $\text{S} \cdot \text{m}^{-1}$

Subscripts

w	Wall
e	External
imp	Imposed
ind	Induced
a	Anodes
b	Spectral band

c Cathodes
nc Neighbor cell
tot Total

Superscripts

cond Conductive
d Direction
rad Radiative

¹ **1. Introduction**

² High enthalpy arc jet wind tunnels are the established ground-test technology used to evaluate the performance of Thermal Protection Systems (TPS) materials for human and robotic exploration missions. By providing a highly energetic flow over prolonged test times, arc jet facilities enable testing materials under extreme aerothermal heating experienced by spacecraft during planetary entry.

⁸ Arc jets are usually composed of an anode chamber, a long constrictor, a cathode chamber, a converging/diverging nozzle, and a test chamber. The number of electrodes, commonly made of copper, around the anode and cathode chambers is different among arc jet facilities. The large difference in electric potential between the anodes and the cathodes generates the extreme current necessary to heat the test gas via Joule heating. The electrodes and the walls have an internal water-cooling system to remove the heat created by the electric arc. The constrictor, where most of the heating occurs, is usually built as a succession of gas-injecting disks that are insulated from each other by a ceramic material. The mixture of the cold gas injected through the disks is representative of the planetary atmosphere of interest to a test campaign. This study focuses on the arc heater part of an arc jet from the anode chamber to the nozzle throat.

²⁰ Although the supersonic aerothermal modeling technology from the nozzle throat to the test chamber is well established and routinely used, the inlet conditions from the arc heater section have always been highly approximated in the literature [28, 6, 15]. An accurate model of the plasma physics that occur in the arc heater is needed to understand the gas conditions at the nozzle throat. High fidelity heat and mass transfer analysis tools of the sample material response in the test chamber are an active area of research [26, 3, 23, 8].

²⁷ The importance of understanding the arc attachment and arc instabilities in arc jets was studied by [2, 36]. The interaction between the electric arc and the surrounding medium results in strong gradients in both the fluid and the electromagnetic fields, which can lead to various types of instabilities [5].

31 Numerical methods to compute the radiative transfer are available in the literature, such as ray tracing [48, 20, 22] and spatial and spectral discretization
32 [4, 25, 24, 1]. Research by Trelles et al. [40, 37, 38, 39] investigated finite element
33 modeling of industrial arc plasma torches using Argon gas. The collection of
34 work by Trelles et al. includes detailed models of the plasma. However, given
35 the empirical nature of the radiation model used, this work is more suitable
36 for smaller geometries, and lower total enthalpy flows. Park et al. [27], Sakai
37 et al. [32, 31, 33, 30], and Lee et al. [19, 18] have studied numerical models of
38 plasma flow in arc heaters. Wang et al. [44, 42, 43] have worked on modeling
39 3D vacuum arcs used in industrial applications, such as circuit breakers. Some
40 of their results have been compared to anode jet experimental data. A three-
41 dimensional unsteady simulation of NASA’s Aerodynamic Heating Facility was
42 performed by Sahai et al. [29] using the Fully-Implicit Navier-Stokes (FIN-S)
43 solver [16, 17]. Their finite element solver included Joule heating, 3D radiative
44 transfer, and non-equilibrium model for the plasma. A simplified geometry was
45 used for the anode and cathode chambers in their study. The computational
46 cost of the solver limited the integration in time.
47

48 The objective of this study is to develop a better understanding of the multi-
49 physics of the plasma flow inside arc heaters which will ultimately lead to a
50 more accurate characterization of the use of an arc jet in testing TPS materials.
51 The state-of-the-art ARC Heater Simulator (ARCHeS) analysis tool provides a
52 parallel, unstructured, finite volume solver based on the OpenFOAM library.
53 ARCHeS includes the solution of three-dimensional radiative transfer, imposed
54 current density, imposed magnetic field, external magnetic field, and turbulence.
55 Realistic boundary conditions are added to the model. The open-source third
56 party library Mutation++, developed at the von Karman Institute for Fluid
57 Dynamics, is used to compute the equilibrium gas mixture composition as well
58 as thermodynamic and transport properties [35]. Three-dimensional and time-
59 dependent simulations presented in this work show plasma instabilities in the
60 constrictor of the arc heater. Analysis of the electric arc dynamics will provide a
61 better intuitive understanding of the complex behavior of plasma flow observed

62 in arc jets. Massively parallel and efficient simulation capability in ARCHeS
 63 allows long-time integration, necessary for the development of the plasma insta-
 64 bilities.

65 The paper is organized as follows. In section 2, the governing equations and
 66 the boundary conditions used in the plasma flow model are presented. Section
 67 3 details the numerical model applied to solve the governing equations. In sec-
 68 tion 4, overall results are presented, including arc instabilities and attachment.
 69 Finally, section 5 summarizes the conclusions drawn from this study.

70 **2. Governing Equations**

71 *2.1. Mass, momentum, and energy equations*

The governing equations, described in Eq. (1), are the mass, momentum,
 and energy conservation laws.

$$\partial_t \rho + \boldsymbol{\partial}_x \cdot (\rho \mathbf{u}) = 0 \quad (1a)$$

$$\partial_t (\rho \mathbf{u}) + \boldsymbol{\partial}_x \cdot (\rho \mathbf{u} \mathbf{u}) = -\boldsymbol{\partial}_x p + \boldsymbol{\partial}_x \cdot \underline{\boldsymbol{\tau}} + \mathbf{J} \times \mathbf{B} \quad (1b)$$

$$\begin{aligned} \partial_t (\rho E_0) + \boldsymbol{\partial}_x \cdot (\rho H_0 \mathbf{u}) &= \boldsymbol{\partial}_x \cdot (\underline{\boldsymbol{\tau}} \cdot \mathbf{u} - \mathbf{q}^{cond}) + \sigma |\mathbf{E}|^2 + \mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) - \boldsymbol{\partial}_x \cdot \mathbf{q}^{rad} \\ &\quad (1c) \end{aligned}$$

72 H_0 , E_0 and e are the stagnation enthalpy, stagnation energy, and internal
 energy.

$$H_0 = E_0 + \frac{p}{\rho} = e + \frac{|\mathbf{u}|^2}{2} + \frac{p}{\rho} \quad (2)$$

73 The Lorentz force and work, the Joule heating, and the radiative heat-
 74 ing/cooling are detailed in the following subsections.

75 *2.2. Electromagnetic equations*

Chapter 3.4 of Goedbloed & Poedts [12] and section 5 of Giordano [10] give
 a general review of the coupling between plasma and Maxwell's equations. The
 governing equations, detailed in Eq. (1), remain unclosed. Maxwell's equations

result in the following dynamic equations for the current and magnetic fields. Ampere's law is given by

$$-\frac{1}{c^2} \partial_t \mathbf{E} + \boldsymbol{\partial}_x \times \mathbf{B} = \mu_0 \mathbf{J} \quad (3)$$

Given the non-relativistic system in the arc heater ($U_0 \ll c^2$), Ampere's law, Eq. (3), reduces to

$$\boldsymbol{\partial}_x \times \mathbf{B} = \mu_0 \mathbf{J} \quad (4)$$

The generalized Ohm's law, as shown in Eq. (40) of Ref. [10], is given by

$$\mathbf{J} = \sigma (\mathbf{E} + \mathbf{u} \times \mathbf{B}) \quad (5)$$

The magnetic Reynolds number, detailed in Eq. (6), compares magnetic induction to magnetic diffusion.

$$Re_m = \frac{U_0 L_0}{\eta} \quad (6a)$$

$$\eta = \frac{1}{\mu_0 \sigma} \quad (6b)$$

In the considered arc heaters, the magnetic Reynolds number is about 10^{-3} , which corresponds to a highly resistive plasma. This system with low Re_m is dominated by magnetic diffusion, and inductive terms are neglected. Using the quasi-static approximation, the Ohm's law, from Eq. (5), reduces to

$$\mathbf{J} = \sigma \mathbf{E} \quad (7)$$

In this work, the electromagnetic four-potential formulation [14] is used to write

$$\mathbf{E} = -\boldsymbol{\partial}_x \phi - \partial_t \mathbf{A} \quad (8a)$$

$$\mathbf{B} = \boldsymbol{\partial}_x \times \mathbf{A} \quad (8b)$$

In this case, the current is driven by a differential of electric potential between the electrodes, and an external magnetic field in the electrode region is added to prevent the arc attachment at one point. The total electric field is split into two parts in order to simplify the boundary conditions. The first part is an imposed electric field, \mathbf{E}_{imp} , where the fluid is assumed stationary compared to the imposed electric potential. Therefore, the fluid is treated as an electric conductor. The second part is an induced electric field, \mathbf{E}_{ind} , generated by fluctuations of the total magnetic vector potential and the gradient of the induced electric potential. The split of the total electric field follows

$$\mathbf{E}_{imp} = -\partial_{\mathbf{x}}\phi_{imp} \quad (9a)$$

$$\mathbf{E}_{ind} = -\partial_t(\mathbf{A}_{imp} + \mathbf{A}_e + \mathbf{A}_{ind}) - \partial_{\mathbf{x}}\phi_{ind} \quad (9b)$$

$$\mathbf{E} = \mathbf{E}_{imp} + \mathbf{E}_{ind} \quad (9c)$$

Ohm's law, from Eq. (7), yields

$$\mathbf{J}_{imp} = -\sigma\partial_{\mathbf{x}}\phi_{imp} \quad (10a)$$

$$\mathbf{J}_{ind} = -\sigma[\partial_t(\mathbf{A}_{imp} + \mathbf{A}_e + \mathbf{A}_{ind}) + \partial_{\mathbf{x}}\phi_{ind}] \quad (10b)$$

$$\mathbf{J} = \mathbf{J}_{imp} + \mathbf{J}_{ind} \quad (10c)$$

The induced fields generated by the movement of the charges in the plasma flow were not computed in this work. As shown in Eq. (6), the low magnetic Reynolds number indicates that advection is relatively unimportant, which means that the electric field (\mathbf{E}) is determined by the boundary conditions (\mathbf{E}_{imp}) rather than the flow (\mathbf{E}_{ind}). Therefore, the induced electric field and the magnetic vector potential were neglected. Future work will compare the

induced and imposed fields.

$$\mathbf{E}_{ind} = \mathbf{A}_{ind} = \mathbf{0} \quad (11)$$

⁷⁹ *2.2.1. Imposed electric field*

We define the imposed electric field as the electric field that is created by the potential difference between the electrodes. The electric field is sustained inside the plasma due to the finite conductivity, which generates a conductive current that heats the gas. Consequently, the imposed current continuity comes from the divergence of Eq. (4) that yields the current conservation relation,

$$\partial_{\mathbf{x}} \cdot \mathbf{J}_{imp} = 0. \quad (12)$$

⁸⁰ The plasma is considered to be quasi-neutral in the whole domain, hence ne-
⁸¹ glecting the sheaths at the walls and electrodes.

Substituting Eq. (10a) in Eq. (12) gives

$$\partial_{\mathbf{x}} \cdot (-\sigma \partial_{\mathbf{x}} \phi_{imp}) = 0 \quad (13)$$

By solving for the electric potential using Eq. (13), the electric field is computed as

$$\mathbf{E}_{imp} = -\partial_{\mathbf{x}} \phi_{imp} \quad (14)$$

The Joule heating is expressed as follows

$$Q_J = \sigma |\mathbf{E}_{imp}|^2 = \sigma |-\partial_{\mathbf{x}} \phi_{imp}|^2 \quad (15)$$

⁸²

⁸³ *2.2.2. Imposed magnetic field*

The imposed magnetic field is defined as the magnetic field induced by the imposed current. Using charge neutrality of the plasma, the imposed vector

magnetic field must satisfy,

$$-\partial_x^2 \mathbf{A}_{imp} = -\mu_0 \sigma \partial_x \phi_{imp} \quad (16)$$

After solving for the imposed magnetic vector potential using Eq. (16), the imposed magnetic field is computed as follows,

$$\mathbf{B}_{imp} = \partial_x \times \mathbf{A}_{imp} \quad (17)$$

This ensures that $\partial_x \cdot \mathbf{B}_{imp} = 0$. Note that in regions where the flow is not conductive, i.e., $\sigma \approx 0$, the imposed magnetic vector potential satisfies the Laplace equation.

$$\partial_x^2 \mathbf{A}_{imp} \approx 0 \quad (18)$$

84 2.2.3. External magnetic field

85 The external magnetic field is the magnetic field that is generated outside
 86 the plasma by an external source. In the studied arc heaters, a magnetic field is
 87 created within the electrode to create a force that moves the attachment point
 88 of the current and protects the electrode from erosion [46], as represented in
 89 Fig. 1.

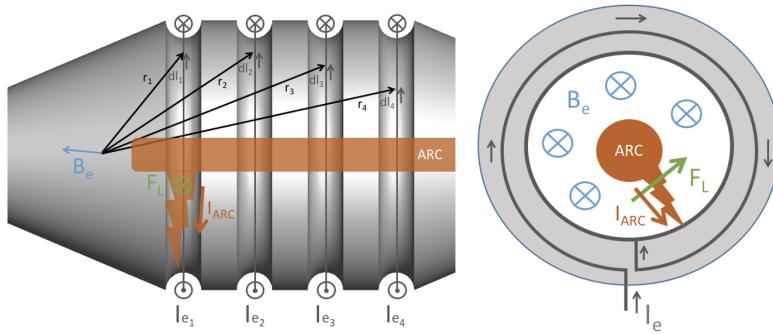


Figure 1: Representation of the magnetic spin coil in series with the arc current contained in each electrode. The external magnetic field creates a force that rotates the arc attachment point and reduces erosion.

The Biot-Savart law is used to express the external magnetic field \mathbf{B}_e , generated by the wire wrapped within the electrode assembly,

$$\partial_x^2 \mathbf{A}_e = -\mu_0 \mathbf{J}_e \quad (19)$$

The analytic solution of Eq. (19) is given by

$$\mathbf{A}_e(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}_e(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3 r' \quad (20)$$

For a thin wire of infinitesimal thickness, this results in

$$\mathbf{A}_e(\mathbf{r}) = \frac{\mu_0 I_e}{4\pi} \oint \frac{dl}{|\mathbf{r} - \mathbf{r}'|} \quad (21)$$

where \oint is the integral along the length of the wire. The external magnetic vector potential from Eq. (21) was numerically solved using 60 points for the contour integral. The external magnetic field was then computed as follows

$$\mathbf{B}_e = \partial_x \times \mathbf{A}_e \quad (22)$$

This formulation ensures that $\partial_x \cdot \mathbf{B}_e = 0$ in the computational domain.

2.3. Radiative heat flux

Radiative Transfer Equations (RTE) are solved in this work to characterize the radiative heat flux. In the literature on arc plasma torches modeling, simpler approaches are found to tackle radiation. The most common one considers the radiative flux as a function of the local properties through a Net Emission Coefficient (NEC) [21, 11]. Alternatively, the tangent slab approximation that reduces the RTE to one spatial dimension is often used for cylindrical geometries [34]. However, both approximations cannot be used in the presence of 3D instabilities with strong temperature gradients, as the radiation no longer depends on the local properties nor the physics can be reduced to cylindrical symmetry.

The total radiative heat flux is given by integrating the spectral radiative heat flux over the entire spectrum. The integral is approximated by a sum of

discrete spectral radiative heat fluxes with a number of bands N_b , as follows,

$$\mathbf{q}^{rad} \approx \sum_{\nu=1}^{N_b} \mathbf{q}_{\nu}^{rad} \quad (23)$$

Here, \mathbf{q}_{ν}^{rad} is given by integrating the radiative intensity in direction Ω over the complete solid angle. The Discrete Ordinates Method is used to discretize \mathbf{q}_{ν}^{rad} into a set of 1-D steady state RTE. The integral over the solid angle is replaced by a weighted sum of discrete radiative intensities. Gaussian quadrature is employed to determine discrete directions Ω_d and weights a_d for different degrees of approximation (N_d directions) as

$$\mathbf{q}_{\nu}^{rad} = \int_{4\pi} \Omega \mathcal{I}_{\nu}(\Omega) d\Omega \approx \sum_{d=1}^{N_d} a_d \mathcal{I}_{\nu}^d \Omega_d \quad (24)$$

104

For a non-scattering plasma, the 1-D steady state RTE of direction Ω_d is expressed as

$$\Omega_d \cdot \partial_x \mathcal{I}_{\nu}^d = \kappa_{\nu} B_{\nu} - \kappa_{\nu} \mathcal{I}_{\nu}^d \quad (25)$$

105 The linear absorption coefficient κ_{ν} was calculated following the Planck
 106 Mean Absorption Coefficient band-averaging model [9]. The Line-By-Line spec-
 107 tral properties in air plasma were computed by the Nonequilibrium Air Radi-
 108 ation (NEQAIR) program [45] using more than 500,000 wavelengths between
 109 0.04 μm and 20 μm .

The banded black body spectral radiance intensity is given by

$$B_{\nu}(T) = \int_{\nu}^{\nu+1} \frac{2\mathcal{H}\nu^3}{c^2} \frac{1}{\exp(\mathcal{H}\nu/k_B T) - 1} d\nu \quad (26)$$

110

The left-hand side of Eq. (25) integrated over the cell volume gives

$$\int_{V_c} \boldsymbol{\Omega}_d \cdot \partial_{\mathbf{x}} \mathcal{I}_{\nu}^d dV = \int_{V_c} \partial_{\mathbf{x}} \cdot (\boldsymbol{\Omega}_d \mathcal{I}_{\nu}^d) dV = \int_{S_c} \mathcal{I}_{\nu}^d \boldsymbol{\Omega}_d \cdot d\mathbf{S} \approx \sum_{c=1}^{N_{nc}} \mathcal{I}_{\nu,c}^d \boldsymbol{\Omega}_d \cdot \Delta \mathbf{S}_c \quad (27)$$

111 The surface integral in Eq. (27) is approximated by a summation over the
112 faces of face-centered values times the corresponding cell areas.

The right-hand side of Eq. (25) integrated over the cell volume gives

$$\int_{V_c} \kappa_{\nu} (B_{\nu} - \mathcal{I}_{\nu}^d) dV \approx \kappa_{\nu,c} (B_{\nu,c} - \mathcal{I}_{\nu,c}^d) V_c \quad (28)$$

113 The volume integral in Eq. (28) is approximated by the cell center value of
the integrand times the cell volume. Eq. (27) and (28) give

$$\sum_{c=1}^{N_{nc}} \mathcal{I}_{\nu,c}^d \boldsymbol{\Omega}_d \cdot \Delta \mathbf{S}_c = \kappa_{\nu,c} (B_{\nu,c} - \mathcal{I}_{\nu,c}^d) V_c \quad (29)$$

114 As represented in Fig. 2, Eq. (29) is split into an incoming part ($k_i \in \boldsymbol{\Omega}_d \cdot \Delta \mathbf{S}_c < 0$) and an outgoing part ($k_o \in \boldsymbol{\Omega}_d \cdot \Delta \mathbf{S}_c > 0$). This is expressed in mathematical terms as

$$\sum_{k_i=1}^{N_{k_i}} \mathcal{I}_{\nu,k_i}^d \boldsymbol{\Omega}_d \cdot \Delta \mathbf{S}_c + \mathcal{I}_{\nu,c}^d \sum_{k_o=1}^{N_{k_o}} \boldsymbol{\Omega}_d \cdot \Delta \mathbf{S}_c = \kappa_{\nu,c} (B_{\nu,c} - \mathcal{I}_{\nu,c}^d) V_c. \quad (30)$$

115 Consequently, an explicit expression of the radiative intensity at the cell c is obtained as follows

$$\mathcal{I}_{\nu,c}^d = \frac{\sum_{k_i=1}^{N_{k_i}} \mathcal{I}_{\nu,k_i}^d \boldsymbol{\Omega}_d \cdot \Delta \mathbf{S}_c + \kappa_{\nu,c} B_{\nu,c} V_c}{\sum_{k_o=1}^{N_{k_o}} \boldsymbol{\Omega}_d \cdot \Delta \mathbf{S}_c + \kappa_{\nu,c} V_c} \quad (31)$$

116 $\mathcal{I}_{\nu,c}^d$ is solved in Eq. (31) for each band and direction following a specific order,
117 different for each direction, represented in Fig. 2.

The divergence of the total radiative heat flux at each cell c is given by

$$\partial_x \cdot \mathbf{q}_c^{rad} = \sum_{\nu=1}^{N_b} \sum_{d=1}^{N_d} \frac{a_d \mathcal{I}_{\nu,c}^d \Omega_d \cdot \Delta S_c}{V_c} \quad (32)$$

118

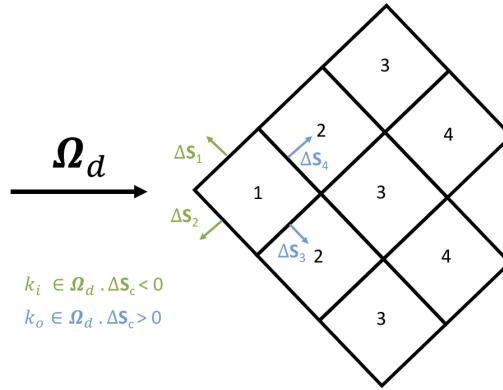


Figure 2: Representation of the directional split into an incoming and outgoing part and the specific advance order used in the computation of the radiative heat flux.

119 2.4. Boundary Conditions

120 The Boundary Conditions (BC) used are presented (see Fig. 3) in this section for the multiple physics phenomena involved in these arc heater simulations.

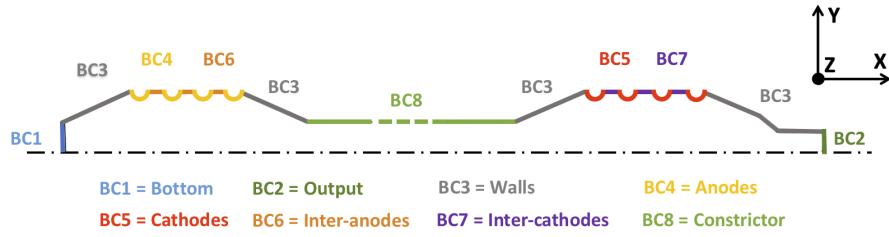


Figure 3: Representation of the boundary conditions in the studied arc heater. The flow is injected at the inter-anodes (BC6) and constrictor (BC8). BC2 is treated as an outflow boundary.

121

¹²² 2.4.1. Flow

The water-cooled arc heater walls (BC3), constrictor (BC8), and inter-electrodes (BC6 and BC7) are assumed to be at a constant temperature. The electrodes (BC4 and BC5) and the outlet (BC2) are assumed adiabatic. A constant air inlet mass flux is imposed at the inter-anodes (BC6) and constrictor (BC8). The swirling inlet velocity is computed as follows

$$\mathbf{u} = \frac{-\dot{m}}{\rho |\mathbf{S}|} (\mathbf{n} + \tan(\theta) \mathbf{t}) \quad \text{with} \quad \mathbf{t} = \mathbf{n}_{axis} \times \mathbf{n}. \quad (33)$$

¹²³ The tangential inclination angle is set at 45° to promote air mixing. Table
¹²⁴ 1 gives a summary of the BC that are used for the flow in this study.

	p [Pa]	T [K]	u [m/s]
BC1	$\partial_x p \cdot \mathbf{n} = 0$	$T_w = 500 \text{ K}$	$\mathbf{u} = \mathbf{0}$
BC2	$\partial_x p \cdot \mathbf{n} = 0$	$\partial_x T \cdot \mathbf{n} = 0$	$\partial_x \mathbf{u} \cdot \mathbf{n} = 0$
BC3	$\partial_x p \cdot \mathbf{n} = 0$	$T_w = 500 \text{ K}$	$\mathbf{u} = \mathbf{0}$
BC4	$\partial_x p \cdot \mathbf{n} = 0$	$\partial_x T \cdot \mathbf{n} = 0$	$\mathbf{u} = \mathbf{0}$
BC5	$\partial_x p \cdot \mathbf{n} = 0$	$\partial_x T \cdot \mathbf{n} = 0$	$\mathbf{u} = \mathbf{0}$
BC6	$\partial_x p \cdot \mathbf{n} = 0$	$T_w = 500 \text{ K}$	$\dot{m}_{BC6} = 0.05 \text{ kg/s}$
BC7	$\partial_x p \cdot \mathbf{n} = 0$	$T_w = 500 \text{ K}$	$\mathbf{u} = \mathbf{0}$
BC8	$\partial_x p \cdot \mathbf{n} = 0$	$T_w = 500 \text{ K}$	$\dot{m}_{BC8} = 0.4 \text{ kg/s}$

Table 1: Summary of the BC for the Navier-Stokes equations.

¹²⁵ 2.4.2. Electromagnetism

The power supply to the arc heater is a constant current supply, and therefore, the electric field will adjust to enable a constant current between the anode and cathode chambers. The anode current density is considered constant in this study, as follows

$$\sigma \partial_x \phi_{imp,a_i} \cdot \mathbf{n} = -\mathbf{J}_{imp,a_i} \quad \text{and} \quad I_{imp,a_i} = \int_{S_{a_i}} \mathbf{J}_{imp,a_i} \cdot d\mathbf{S} \quad (34)$$

¹²⁶

An unbalanced current at the electrodes will result in an uneven current attachment: the electrode with the highest current will experience the highest

heating near the surface, which results in the highest conductivity for the gas near the surface, thus reinforcing the location of this attachment. A stable concentrated current attachment will result in mechanical failure (erosion or melting) of the electrode. The use of a ballast configuration balances the electric potential at each electrode to force an even distribution of the total current. The same resistance is used for all electrodes. The configuration results in a mixed boundary condition at the cathodes. This reads in mathematical terms

$$-I_{imp,c_i} = \int_{S_{c_i}} \sigma \partial_x \phi_{imp,c_i} \cdot d\mathbf{S} = \frac{1}{R_b} (\phi_{imp,c_i} - \phi_G) \quad (35)$$

An explicit scheme is used to impose the above BC. This is done by setting the potential at the discretized time step n to

$$\phi_{imp,c_i}^n = \phi_G - R_b I_{imp,c_i}^{n-1} \quad (36)$$

with $I_{imp,c_i}^0 = 0$ initially, otherwise

$$I_{imp,c_i}^{n-1} = \int_{S_{c_i}} -\sigma \partial_x \phi_{imp,c_i}^{n-1} \cdot d\mathbf{S} \quad (37)$$

i.e., the potential at the cathodes is adjusted by using the electric current from the previous time step. Note that the continuity of the electric current ensures that

$$-\sum_{i=1}^{N_{c_i}} I_{imp,c_i} = \sum_{i=1}^{N_{a_i}} I_{imp,a_i} = I_{imp,tot} \quad (38)$$

The walls of segmented arc heaters are water-cooled copper rings that are isolated from each other by thin ceramic (non-conductive) spacers. The rings are electrically isolated from the ground, and the spacers prevent current along the walls. In this configuration, electrically floating rings ensure proper adjustment of the electric potential. The walls of the constrictor are made of floating rings except at the electrodes where voltage is applied to ensure constant current. As

such, charge will accumulate in each ring so that

$$\mathbf{J}_{imp,w} = \mathbf{0} \quad (39)$$

Thus, the electrical potential boundary condition at the wall is

$$\partial_{\mathbf{x}} \phi_{imp,w} \cdot \mathbf{n} = 0 \quad (40)$$

127 Table 2 summarizes the BC that are used for the electromagnetic equations
128 in this simulation.

	ϕ_{imp} [V]	\mathbf{A}_{imp} [T · m]	\mathbf{A}_e [T · m]
BC1	$\partial_{\mathbf{x}} \phi_{imp} \cdot \mathbf{n} = 0$	$\mathbf{A}_{imp} = \mathbf{0}$	$\mathbf{A}_e = \mathbf{0}$
BC2	$\partial_{\mathbf{x}} \phi_{imp} \cdot \mathbf{n} = 0$	$\mathbf{A}_{imp} = \mathbf{0}$	$\mathbf{A}_e = \mathbf{0}$
BC3	$\partial_{\mathbf{x}} \phi_{imp} \cdot \mathbf{n} = 0$	$\mathbf{A}_{imp} = \mathbf{0}$	$\mathbf{A}_e = \mathbf{0}$
BC4	$I_{imp,tot} = 1200$ A	$\mathbf{A}_{imp} = \mathbf{0}$	$\mathbf{A}_e = \mathbf{0}$
BC5	$\phi_{imp,c_i}^n = \phi_G - R_b I_{imp,c_i}^{n-1}$	$\mathbf{A}_{imp} = \mathbf{0}$	$\mathbf{A}_e = \mathbf{0}$
BC6	$\partial_{\mathbf{x}} \phi_{imp} \cdot \mathbf{n} = 0$	$\mathbf{A}_{imp} = \mathbf{0}$	$\mathbf{A}_e = \mathbf{0}$
BC7	$\partial_{\mathbf{x}} \phi_{imp} \cdot \mathbf{n} = 0$	$\mathbf{A}_{imp} = \mathbf{0}$	$\mathbf{A}_e = \mathbf{0}$
BC8	$\partial_{\mathbf{x}} \phi_{imp} \cdot \mathbf{n} = 0$	$\mathbf{A}_{imp} = \mathbf{0}$	$\mathbf{A}_e = \mathbf{0}$

Table 2: Summary of the BC for the Maxwell equations.

129 2.4.3. Radiation

The walls are made of unpolished copper and are assumed to behave as a gray body at the wall temperature. We set the emissivity of copper $\epsilon_\nu = 0.6$, constant for all wavelengths. The average radiative intensity from the wall surface, emitted by each band, is given by

$$\mathcal{I}_{\nu,w}^d = \epsilon_\nu B_\nu \quad (41)$$

130

131 **3. Numerical model**

132 The numerical framework is based on OpenFOAM, an open-source, modern,
133 and modular C++ finite volume library. This framework offers a wide range
134 of numerical schemes, interpolation schemes, integration methods, turbulence
135 models, thermodynamic models, and flow solvers. The governing equations are
136 sequentially solved as explained in Algorithm 1. The OpenFOAM semi-discrete
137 finite volume solver rhoCentralFoam has been chosen for modeling the flow.
138 The non-staggered central scheme that is used in this solver has proven to be
139 accurate in the simulations of high-speed compressible flows [13]. Additionally,
140 a $k - \epsilon$ model is applied to the flow turbulence [7]. Second order accuracy in
141 space is achieved with a total variation diminishing (TVD) scheme with van
142 Albada's limiter [41]. The Maxwell equations, described in Eq. (13) and (16),
143 that link the electric and magnetic fields are computed by using a generalized
144 Geometric-Algebraic Multi-Grid (GAMG) solver. The Mutation++ library [35],
145 dynamically linked to the OpenFOAM thermodynamic framework, is used to
146 compute equilibrium chemistry compositions and thermodynamic and trans-
147 port properties, as detailed in Appendix A. The discretization of radiative heat
148 flux uses a multi-band approximation of the non-gray spectral properties of
149 high-temperature air. The radiative directional integration is determined by
150 a Gaussian type quadrature. The contributions of the electric field, magnetic
151 field, and radiative heat flux are added as source terms to the momentum and
152 energy equations.

Algorithm 1 ARCHeS

```
1: function MAIN
2:   create time and mesh
3:   read and initialize user configurations
4:   read fields  $p^0, T^0, \mathbf{u}^0, \phi^0, \mathbf{A}^0, \partial_{\mathbf{x}} \cdot \mathbf{q}_{rad}^0$             $\triangleright$  flags for radiation and electromagnetism
5:   compute  $\alpha^0, \psi^0, \mu^0, \sigma^0, \gamma^0, e^0, \rho^0$ 
6:   initialize radiation and electromagnetism
7:   while RunTime do
8:     correct boundary conditions for  $\mathbf{u}^n, \sigma^n, \phi^n$ 
9:     compute  $\phi^{n+1}, \mathbf{A}^{n+1}$ 
10:    compute  $\partial_{\mathbf{x}} \cdot \mathbf{q}_{rad}^{n+1}$                                       $\triangleright$  solve electromagnetic equations
11:    compute fluxes and source terms
12:    compute  $\rho^{n+1}$                                           $\triangleright$  solve radiation
13:    compute  $\mathbf{u}^{n+1}$                                           $\triangleright$  solve mass equation
14:    compute  $e^{n+1}$                                           $\triangleright$  solve momentum equations
15:    compute  $T^{n+1}$                                           $\triangleright$  solve energy equation
16:    compute  $\alpha^{n+1}, \psi^{n+1}, \mu^{n+1}, \sigma^{n+1}, \gamma^{n+1}$ 
17:    initialize radiation and electromagnetism
18:    compute  $p^{n+1}$                                           $\triangleright$  solve thermodynamics in function of  $p^n, T^{n+1}$ 
19:    correct  $p^{n+1}, \rho^{n+1}$  boundaries
20:    compute  $\mu_{eff}^{n+1}$ , and  $\alpha_{eff}^{n+1}$ 
21:    write fields
22:
```

153 **4. Results**

154 *4.1. Flow*

155 Figure 4 shows the pressure and temperature profiles along the centerline axis
156 of the arc heater. The pressure has a maximum value of 13.3 atm, almost steady
157 in the anode chamber. The pressure gently decreases along the constrictor from
158 13.2 to 9.2 atm. The pressure stays constant in the cathode chamber and then
159 drops again in the convergent part. The pressure is about 3.5 atm at the nozzle
160 throat. The time-averaged temperature computed from 0.4640 s to 0.5759 s
161 has a maximum value of \sim 8000 K. The temperature increases in the anode
162 chamber. The instantaneous time values of the temperature highly fluctuate in
163 the constrictor as a consequence of the arc instabilities. The temperature drops
164 from 7500 K at the cathode chamber to 4400 K at the nozzle throat.

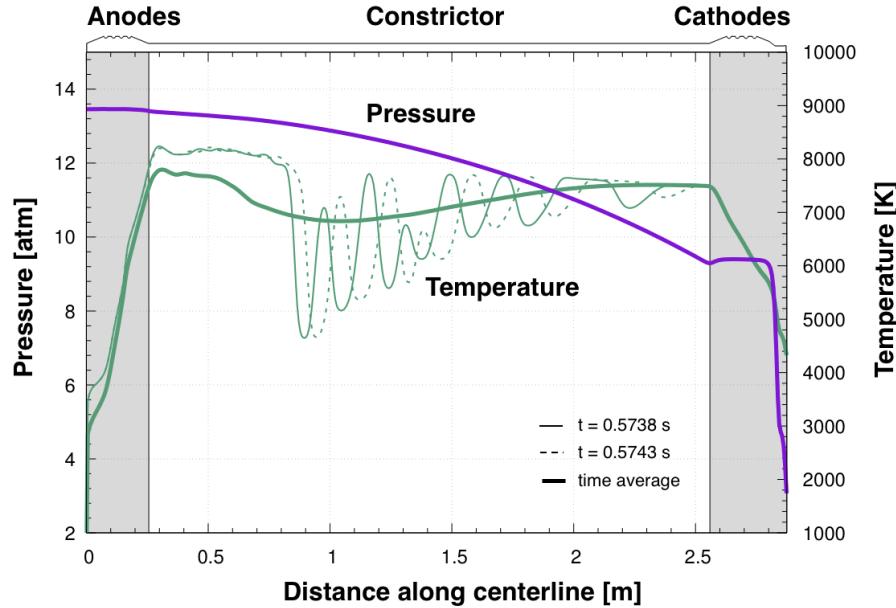


Figure 4: Pressure and temperature profiles along the centerline axis.

165 Figure 5 shows the enthalpy and velocity profiles along the centerline axis of
166 the arc heater. The time-averaged enthalpy and velocity were computed from
167 0.4640 s to 0.5759 s. The enthalpy follows the trend of the temperature (Fig.

168 4). The maximum of the time-averaged enthalpy is about $24 \text{ MJ} \cdot \text{kg}^{-1}$. For
 169 both instantaneous and time-averaged, the gas enthalpy increases in the anode
 170 chamber and decreases in the cathode chamber. In the constrictor region, the
 171 averaged value drops only to increase again so that the value at the exit of the
 172 constrictor is similar to the one at the entrance. Alternatively, instantaneous
 173 values show high fluctuation that is indicative of arc instabilities. Instantaneous
 174 and time-averaged velocities along the centerline follow a similar trend. The
 175 velocity is low in the anode chamber due to the absence of air injection, which
 176 is introduced along the inter-anodes and the constrictor (Fig. 3). The velocity
 177 increases up to $550 \text{ m} \cdot \text{s}^{-1}$ in the constrictor, drops in the divergent part, and
 178 increases again in the convergent part of the cathode chamber. The flow is
 179 subsonic in the anode chamber, the constrictor, and the cathode chamber. The
 180 Mach number equals 1 at the nozzle throat. The flow becomes supersonic after
 181 the nozzle throat.

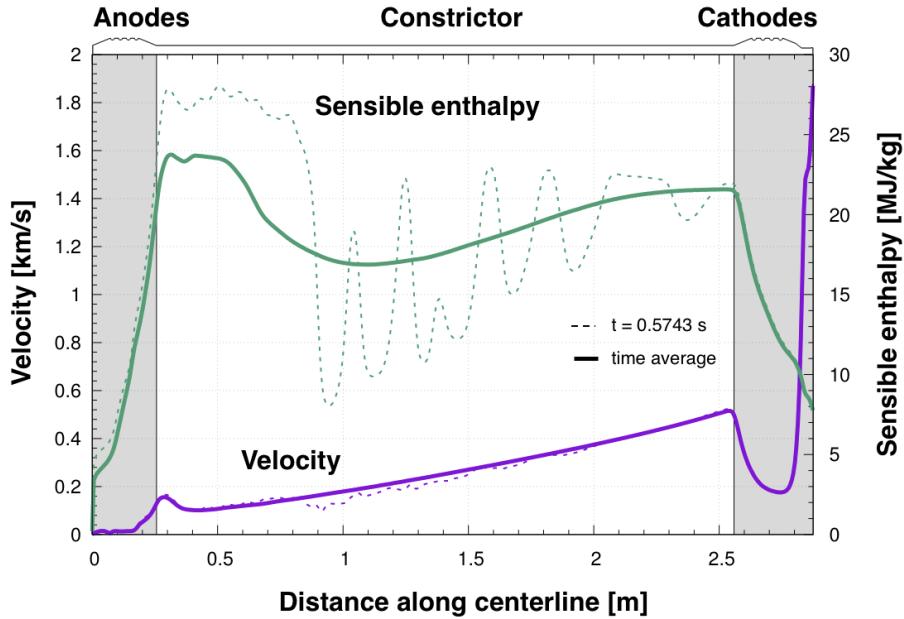


Figure 5: Velocity and enthalpy profiles along the centerline axis.

182 Figure 6 shows the XY cutting plane of the temperature from the anode

183 chamber to the nozzle throat. The highest temperature (around 8000 K) is
 184 close to the centerline axis because of the low temperature imposed at the water-
 185 cooled walls. Arc instabilities are visible in the constrictor. A three-dimensional
 186 unsteady modeling tool is needed to capture these instabilities.

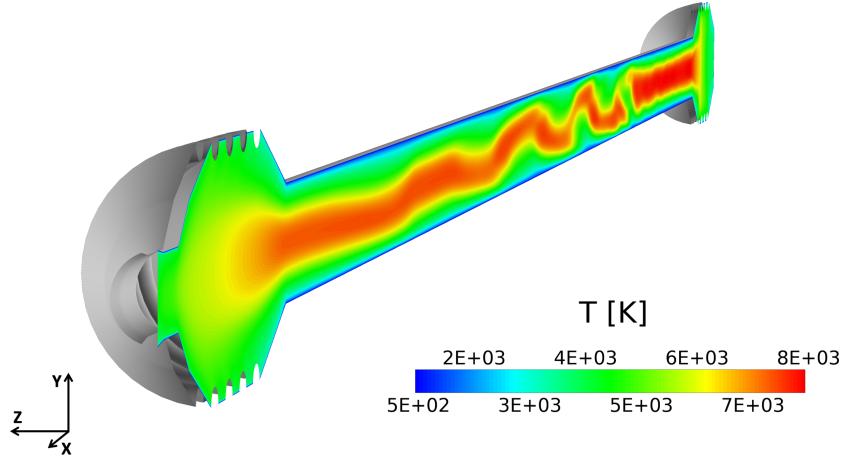


Figure 6: Temperature XY cutting plane from the anode chamber to the nozzle throat, $t = 0.5738$ s.

187 Figure 7 shows the YZ cutting plane of the temperature for different times
 188 at $X = 1.5$ m. The shape of the high-temperature electric arc core shows
 189 an unsteady behavior. The dynamic of the arc indicates that the instability
 190 saturates next to the wall. The core cannot get close to the constrictor wall due
 to the injection of low-temperature air and due to the water cooling system.

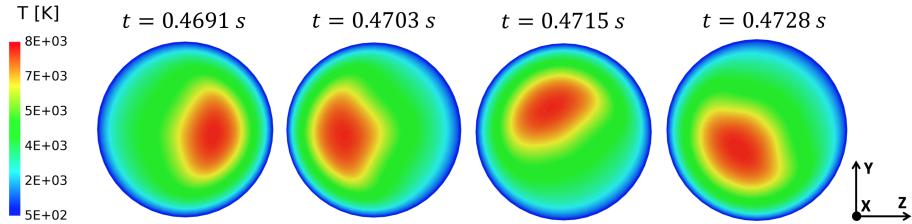


Figure 7: Temperature YZ cutting plane at $X = 1.5$ m for different times.

191
 192 Figures 8 and 9 show the XY cutting plane of the velocity at the cathode

193 and anode chambers. The inlet mass flow is $0.05 \text{ kg} \cdot \text{s}^{-1}$ in the anode chamber
 194 and $0.4 \text{ kg} \cdot \text{s}^{-1}$ in the constrictor. The gas velocity is higher next to the
 195 centerline axis. The highest velocity of $1.9 \text{ km} \cdot \text{s}^{-1}$ is located at the end of the
 196 computational domain, just after the nozzle throat. Figure 9 shows that the
 197 flow is impacted by the arc instabilities in the constrictor.

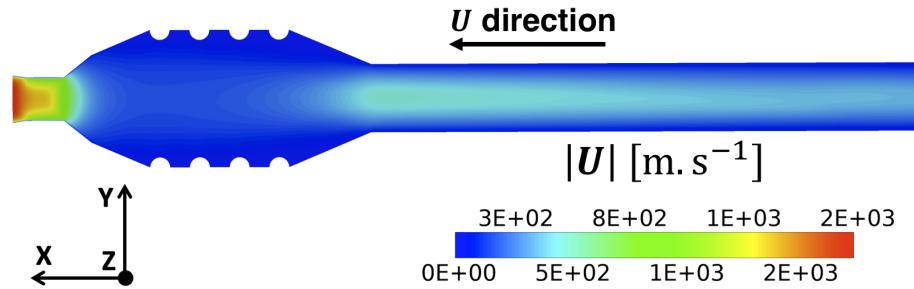


Figure 8: Velocity XY cutting plane in the constrictor and the cathode chamber, $t = 0.5738 \text{ s}$.

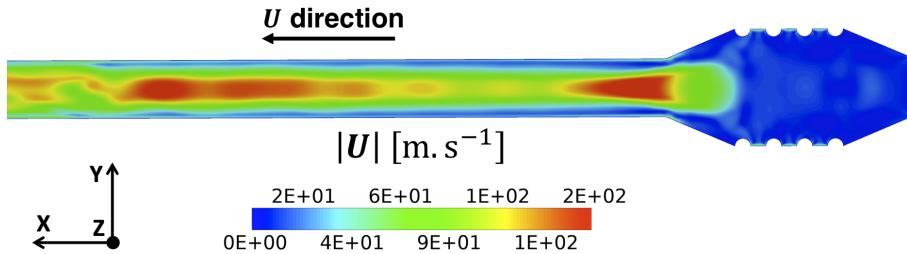


Figure 9: Velocity XY cutting plane in the constrictor and the anode chamber, $t = 0.5738 \text{ s}$.

Figure 10 shows cutting planes of the vorticity ω computed as follows

$$\omega = \partial_x \times u \quad (42)$$

198 As described in section 2.4, the flow is injected at the inter-anodes and con-
 199 strictor. The XY cutting plane shows the vorticity magnitude for the entire
 200 geometry. The highest vorticity magnitude is located close to the wall. The
 201 YZ cutting planes show the vorticity magnitude in color, the YZ velocity u_{yz}
 202 using streamlines at $X = [1, 1.5] \text{ m}$, and the axial velocity u_x at $X = 2 \text{ m}$. The

locations of the vortex cores change along the centerline axis, and the velocity is mainly axial at $X = 2$ m. Future work will study the strong interactions between wall shear and arc instabilities that impact the vorticity and play an essential role in flow mixing.

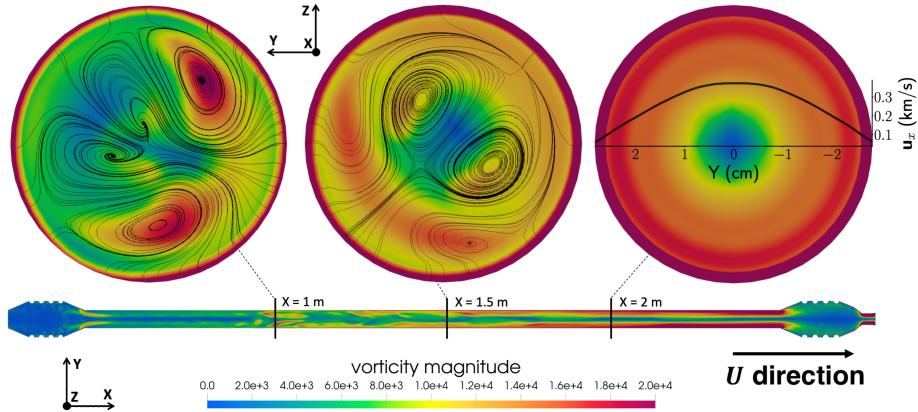


Figure 10: XY and YZ cutting planes of the vorticity ω , $t = 0.5738$ s. The color shows the vorticity magnitude, the streamlines represent the velocity in the YZ plane u_{yz} at $X = [1, 1.5]$ m, and the plot shows the axial velocity u_x at $X = 2$ m.

4.2. Electromagnetism and radiation

Figure 11 shows the comparison between the Joule heating and radiative heating/cooling profiles along the centerline axis. The radiative heat flux is the predominant cooling process of the gas medium. In the same figure, a snapshot of the radiative heating/cooling and Joule heating profiles presents fluctuations along the axis resulting from the electric arc instabilities. The radiative heating/cooling follows the Joule heating curve trend and is mostly negligible along the constrictor centerline. Joule heating is almost null at the start of the anode chamber and at the end of the cathode chamber.

Figure 12 shows the Joule heating and the radiative heating/cooling profiles along the radial axis at half-length of the constrictor. The radiative cooling ($\partial_x \cdot q^{rad} > 0$) is due to the re-radiation of the heat from the electric arc core, where the temperature is the highest. The radiative cooling is almost negligible compared to Joule heating. The radiative heating ($\partial_x \cdot q^{rad} < 0$) is strong

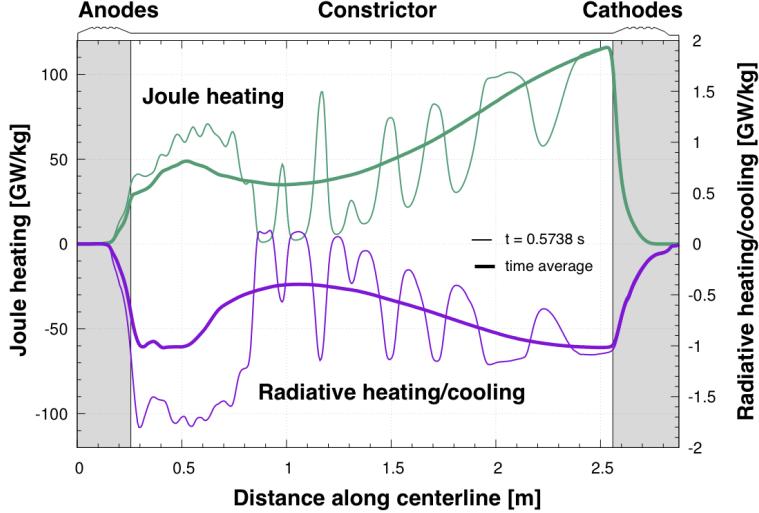


Figure 11: Joule heating ($|J_{imp}|$) and radiative heating/cooling ($-\partial_x \cdot q^{rad}$) profiles along the centerline axis.

221 outside the electric arc core and is higher than the Joule heating next to the
 222 wall ($2.30 \text{ cm} < \text{radius} < 2.85 \text{ cm}$).

223 Figure 13 shows the cutting plane of the radiative heating/cooling from the
 224 anode chamber to the nozzle throat. The blue color represents the radiative
 225 cooling ($\partial_x \cdot q^{rad} > 0$) which occurs at the center of the electric arc. The
 226 red color shows the radiative heating ($\partial_x \cdot q^{rad} < 0$) at the surroundings of the
 227 electric arc core. The transverse cutting planes show the electric arc instabilities
 228 and the importance of the three-dimensional calculations of the radiation. In
 229 this work, the discretization of the radiative heat flux is done using 24 discrete
 230 directions and 100 spectral bands.

231 Figure 14 shows the cutting plane of the imposed current density from the
 232 anode chamber to the nozzle throat. The current density is mainly constrained
 233 close to the centerline of the constrictor due to the cold injection of air at the
 234 wall. The total current is equally distributed among the anodes due to the BC
 235 explained in Eq. (34). The continuity of current from Eq. (13) imposes the
 236 same total current at the anodes and the cathodes. The electric ballast system,

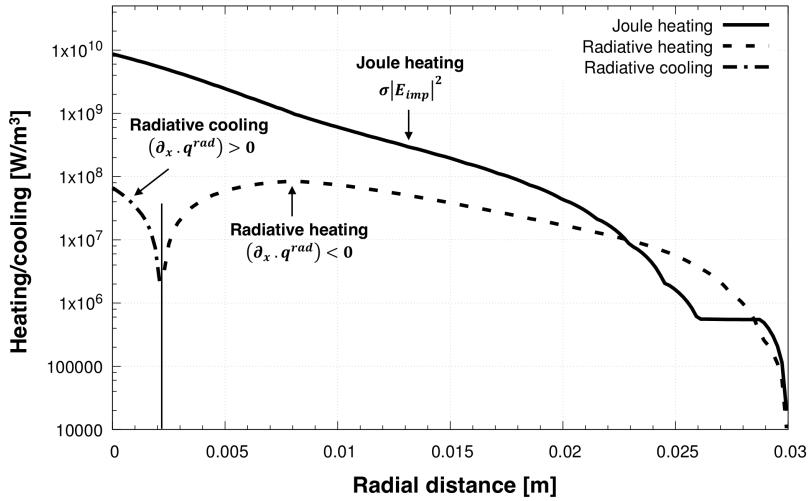


Figure 12: Joule heating and radiative heating/cooling profile in the radial direction halfway along the constrictor, $t = 0.5738$ s.

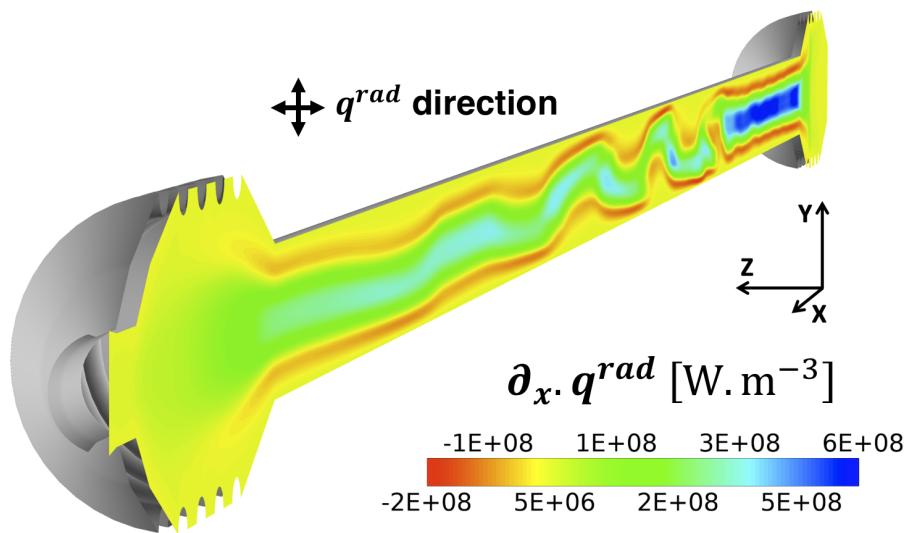


Figure 13: Divergence of the radiative heat flux from the anode chamber to the nozzle throat, $t = 0.5738$ s. The blue and red colors correspond to radiative cooling and heating, respectively.

²³⁷ implemented using Eq. (37), balances the current at each cathode to force an
²³⁸ even distribution. The peak of the imposed current density is about $3 \text{ MA} \cdot \text{m}^{-2}$.
²³⁹

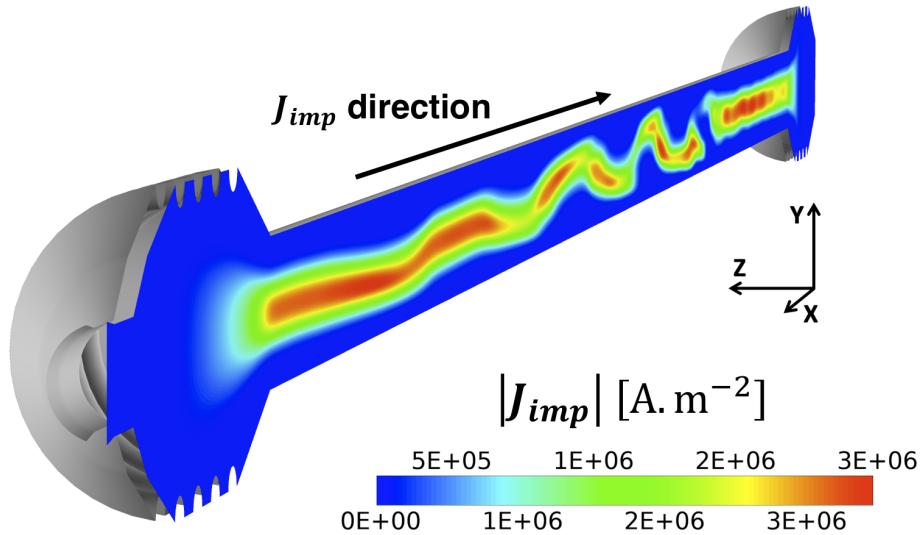


Figure 14: Imposed current density from the anode chamber to the nozzle throat, $t = 0.5738 \text{ s}$.

²⁴⁰ Figure 15 shows the cutting plane of the imposed magnetic field from the
²⁴¹ anode chamber to the nozzle throat. The imposed magnetic field is generated
²⁴² around the imposed current density according to Ampere's law (Eq. (3)). The
²⁴³ maximum magnitude of the imposed magnetic field is about 0.02 T. The mag-
²⁴⁴ netic field direction is mainly perpendicular to the imposed current density. The
²⁴⁵ magnetic field is null at the center of the electric arc and increases in the radial
²⁴⁶ direction. Towards the wall, it decreases again. The curvature of the electric
²⁴⁷ arc creates a stronger magnetic field at the concave part of the electric arc,
²⁴⁸ which results in the kink instability that is shown in Fig. 15. The intensity of
²⁴⁹ the imposed magnetic field decreases in the electrode chambers due to the more
²⁵⁰ diffuse imposed current density.

²⁵¹ Figure 16 shows the cutting plane of the external magnetic field created by
²⁵² the electrodes from the anode chamber to the nozzle throat. The magnitude

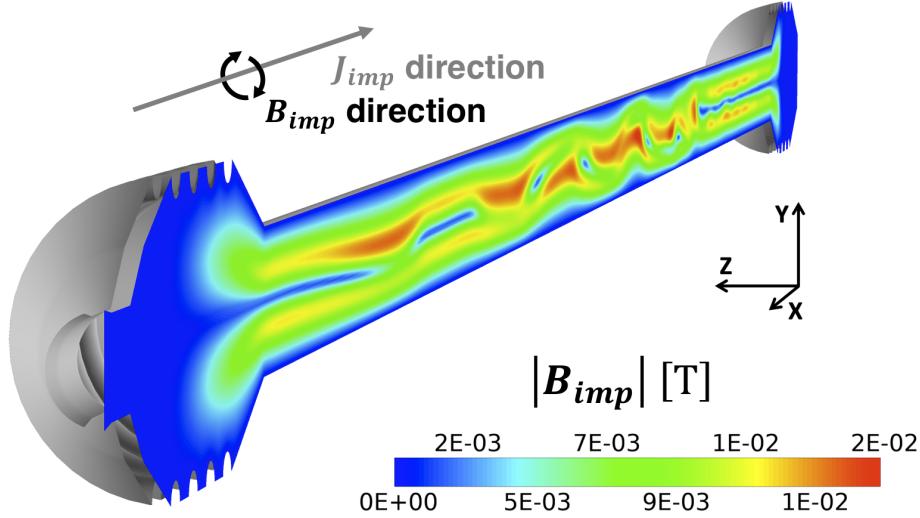


Figure 15: Imposed magnetic field from the anode chamber to the nozzle throat, $t = 0.5738$ s.

253 of the external magnetic field is almost null in the constrictor. The maximum
 254 magnitude is about 0.01 T and is found around the electrodes. The direction is
 255 primarily longitudinal to the imposed current density direction.

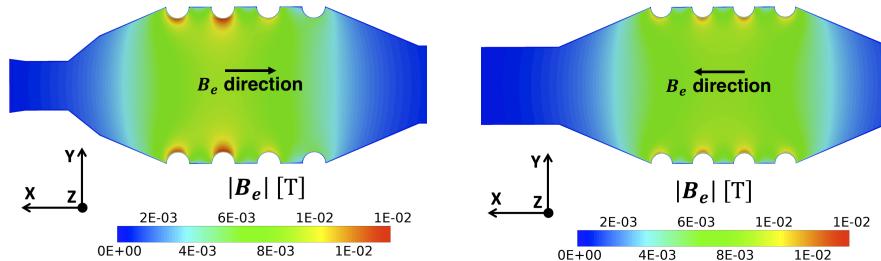


Figure 16: External magnetic field in the cathode (left) and anode (right) chambers, $t = 0.5738$ s.

256 Figure 17 shows the current as a function of time at the different cathodes.
 257 The sum of the 4 cathode currents gives a total current of 1170 A. Cathode 1
 258 is the closest electrode to the constrictor. Cathode 4 is the nearest electrode to
 259 the nozzle throat. A peak of current appears on all the electrodes at 0.487 s.
 260 Then the 4 currents stabilize to a quasi constant value. The simulation seems

261 to achieve a statistically steady state.

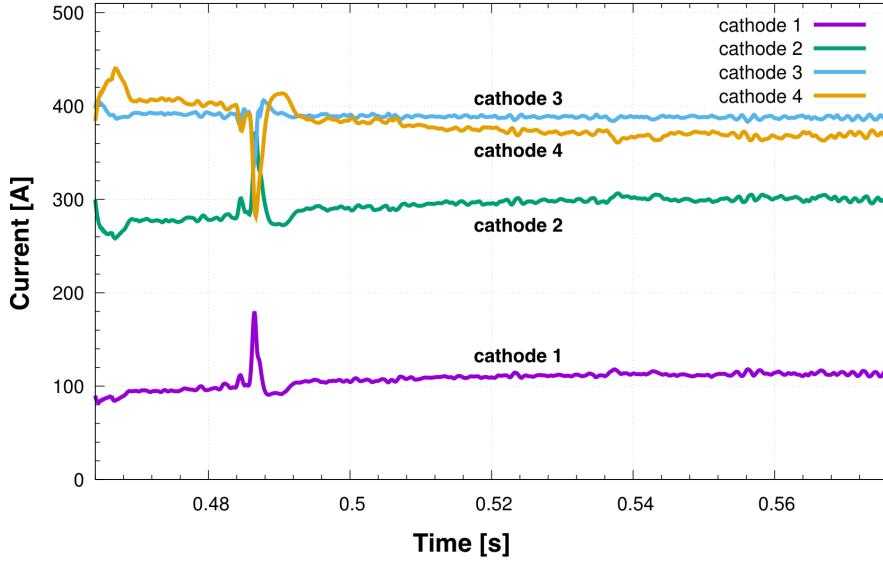


Figure 17: Current at the cathodes as a function of time.

262 4.3. Arc instabilities

263 Trelles [38] describes different types of instabilities that can play a dominant
264 role in the dynamic geometry of a constrained electric arc. The fluid dynamic
265 instabilities are produced by the large gradient of velocity, temperature, density,
266 and viscosity between the cold gas and the plasma created by Joule heating.
267 Shear flow instabilities are often considered one of the most prominent, leading
268 to the growth of a small perturbation downstream that may drive the flow
269 from laminar to turbulent. The thermal instabilities are caused by the non-
270 uniform cooling of the plasma that produces a deflection of the electric arc. The
271 injection of cold gas forces the arc to deviate away from the cold source. The
272 magnetic field, both the external and that self-induced by the electric current,
273 is responsible for the sausage and kink instabilities. The non-uniform plasma
274 distribution and the curvature of the magnetic field impact these instabilities.
275 Sausage instabilities occur when there is a change of cross section in the electric

276 arc. In this regard, a smaller cross section translates to a stronger magnetic field.
 277 Kink instabilities occur when the electric arc curves. Figure 18 shows the iso-
 278 surface of the imposed current density at $1 \text{ MA} \cdot \text{m}^{-2}$, a value representative
 279 of the main part of the electric arc. The color represents the magnitude of
 280 the total magnetic field. For high current density, the total magnetic field is
 281 primarily due to the imposed magnetic field in the constrictor. Even though
 282 the arc instabilities in Fig. 18 are a combination of all the instabilities, the kink
 283 instability is visible where the magnetic field is stronger on the concave side
 284 and weaker on the convex side of the plasma column. The electric arc seems to
 285 stabilize at the entrance of the electrode chambers. More advanced analysis of
 286 the arc behavior is required to fully understand and characterize the complex
 287 dynamic interaction of the cold gas as well as the inherent properties of thermal
 288 plasma. Future work will see the implementation of finite-rate chemistry in the
 289 existing model and may lead to new insights in low-pressure regions where the
 290 assumption of Local Thermal Equilibrium (LTE) breaks down.

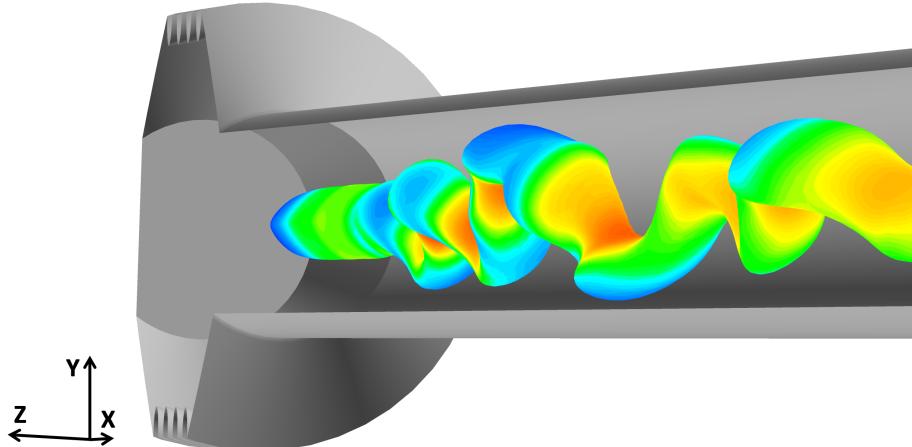


Figure 18: Arc instabilities in the constrictor, $t = 0.5738 \text{ s}$. The iso-surface shows the current density ($1 \text{ MA} \cdot \text{m}^{-2}$). The color represents the magnitude of the total magnetic field.

291 *4.4. Arc attachment*

292 Figures 19 and 20 show the iso-surface of the current density ($30 \text{ kA} \cdot \text{m}^{-2}$)
293 at two different times ($t = 0.4785 \text{ s}$ and $t = 0.4880 \text{ s}$). The color represents the
294 magnitude of the total magnetic field. The external magnetic field, predomi-
295 nantly present around the electrode chambers, has a more significant influence
296 on the arc attachment to the electrodes than the imposed magnetic field, almost
297 negligible in these regions. The Neumann boundary condition of the imposed
298 electric potential forces a constant and uniform current density among the an-
299 odes. On the other hand, the Dirichlet boundary condition at the cathodes
300 sets a constant imposed electric potential, which translates to a non-uniform
301 and non-constant electric current among the cathodes. The location of the arc
302 attachment changes over time at the cathodes. The high temperature in the
303 cathode chamber makes the air conductive and allows a diffuse arc attachment.
304 The ballast configuration balances the electric potential at each electrode to
305 force an even distribution of the total current. It is shown that the attachment
306 at the cathodes is mainly driven by the upstream arc instabilities. The kink in-
307 stabilities generated in the constrictor force the arc to move closer to a specific
308 region of the electrodes, which, in turn, provokes the attachment of the arc.
309 Unlike the studies from Trelles [38], this LTE model does not need an artifi-
310 cially high electric conductivity near the cathodes by choosing an interpolation
311 scheme that conserves the electric flux.

312 **5. Conclusion**

313 Further development of arc jet technologies has been limited by an incom-
314 plete understanding of unstable plasma flow occurring in the constricted arc
315 heater. The direct observation of the complete arc dynamics is extremely dif-
316 ficult inside an arc jet, and its numerical modeling is highly challenging. The
317 complexity of plasma flow simulations comes from the partial knowledge of
318 constrained arc instabilities. The interactions of cold gas and plasma flow are
319 strongly influenced by the radiation and the magnetic field that lead to com-

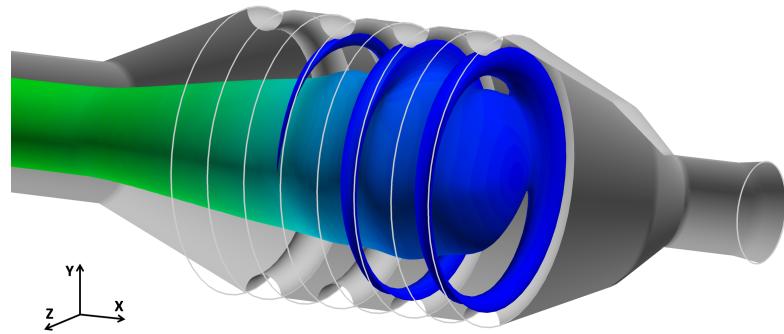


Figure 19: Arc attachment at the cathodes, $t = 0.4785$ s. The iso-surface shows the current density ($30 \text{ kA} \cdot \text{m}^{-2}$). The color represents the magnitude of the total magnetic field.

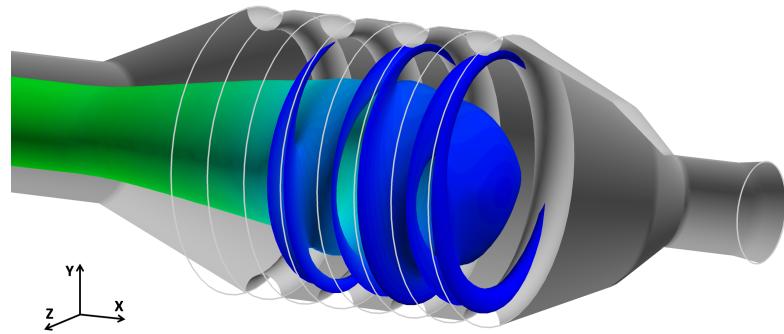


Figure 20: Arc attachment at the cathodes, $t = 0.4880$ s. The iso-surface shows the current density ($30 \text{ kA} \cdot \text{m}^{-2}$). The color represents the magnitude of the total magnetic field.

320 plicated coupled instabilities. A separation of the different physics involved is
321 necessary to understand these instabilities. Arc attachment at the electrodes is
322 a central part of the dynamics of the arc. The development of the analysis tool
323 ARC Heater Simulator (ARCHeS) software provides the capability to perform
324 three-dimensional time-dependent plasma flow simulations, which are necessary
325 to capture the complex behavior of the arc. The massively parallel simulation
326 support inherited in ARCHeS from its modular OpenFOAM architecture makes
327 this work possible. This tool provides estimates of the flow state at the inlet of
328 the nozzle, which are used as inlet conditions for well-established aerothermal
329 techniques for estimating the aerothermal environment at the test articles. The
330 formulation used in ARCHeS has been demonstrated to be efficient to attain
331 convergence on very stiff non-linear systems encountered in plasma flow simu-
332 lations. The unsteady flow is simulated from the anode chamber to the nozzle
333 throat. Local Thermodynamic Equilibrium is assumed in the high-pressure arc
334 heater. The imposed electric potential, the imposed magnetic field, and the
335 external magnetic field are computed through Maxwell equations using a gener-
336 alized geometric-algebraic multi-grid solver and are coupled to the flow through
337 the Navier-Stokes equations. The three-dimensional radiation is solved using
338 an accurate multi-band spectral discretization and Gaussian quadrature direc-
339 tional discretization. The inlet mass flux and the total current were chosen to
340 be close to the operating conditions of large arc jet complexes used for TPS
341 evaluation. These numerical simulations performed on NASA's Pleiades super-
342 computer have demonstrated their suitability for describing electric arc/cold
343 flow interactions and flow instabilities encountered in arc heaters. This model
344 is capable of describing the arc behavior and attachment at the cathodes with-
345 out adding a reattachment model. The magnetic field plays an essential role
346 in the flow characterization from the anode chamber to the nozzle throat. The
347 location and frequency of the arc attachment at the cathodes are mainly driven
348 by the upstream arc instabilities. A more complex analysis of the instabili-
349 ties is needed to explain certain arc behaviors, such as the stabilization at the
350 electrode chambers. The high computational cost of these simulations limits

351 the spatial resolution for wall turbulence analysis. These numerical simulations
352 were obtained using 1200 cores of the NASA's Pleiades supercomputer during
353 three weeks, i.e., approximately 6×10^5 core-hours. Comparison to measure-
354 ments within arc heaters, such as those of Winter et al. [47], is still needed to
355 validate this study. These experimental data are difficult to obtain due to the
356 extremely harsh environment within the heaters. Future work should include
357 a non-adiabatic boundary condition of the temperature at the electrodes. This
358 requires a two-temperature model as well as a model of the energy transfer
359 between the electrodes and the gas. The assumption of equilibrium chemistry
360 in the entire arc heater from the anode chamber to the nozzle throat must be
361 verified by adding a finite-rate chemistry model coupled with the fluid. Includ-
362 ing heavy species in the thermodynamic and transport properties in addition to
363 this model may pave the way to new areas of research, including the simulations
364 of other complex plasma applications such as space propulsion. Even though
365 large research efforts are still needed, this work has proven the effectiveness and
366 efficiency of a finite volume approach for obtaining high-fidelity modeling of the
367 governing equations involved in thermal plasma flows.

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379 **Appendix A. Thermodynamic and transport properties**

380 The equilibrium chemistry compositions and the thermodynamic and trans-
 381 port properties (Fig. A.21, A.22, and A.23) are computed using the Mutation++ library [35]. LTE is assumed in the entire domain. Most of the domain
 382 where the arc instabilities occur has a high pressure (Fig. 4) which justifies the
 383 use of LTE. Future work will take into account the low-pressure regions where
 384 LTE breaks down. The air mixture used in this work is composed of the fol-
 385 lowing 13 species: e^- , N , N^+ , O , O^+ , NO , NO^+ , N_2 , N_2^+ , O_2 , O_2^+ , Ar and
 386 Ar^+ .
 387

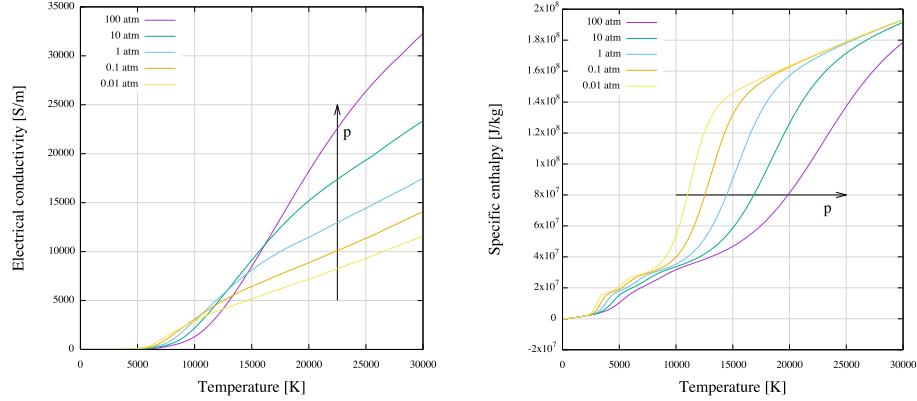


Figure A.21: Electrical conductivity (left) and enthalpy (right) as a function of temperature.

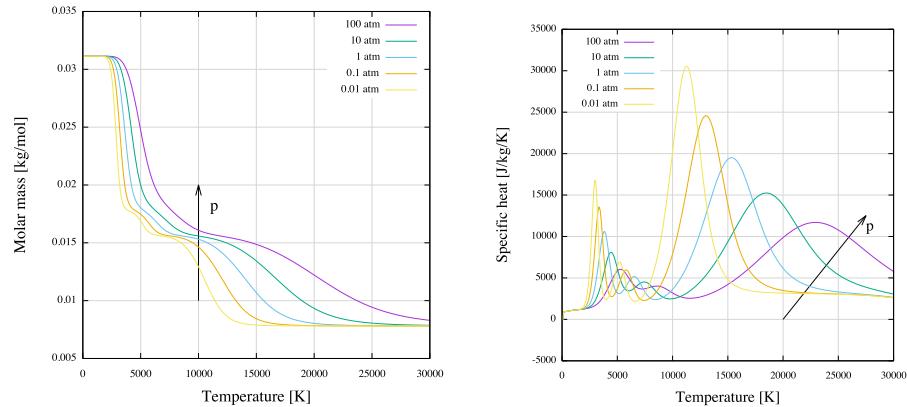


Figure A.22: Molar mass (left) and specific heat (right) as a function of temperature.

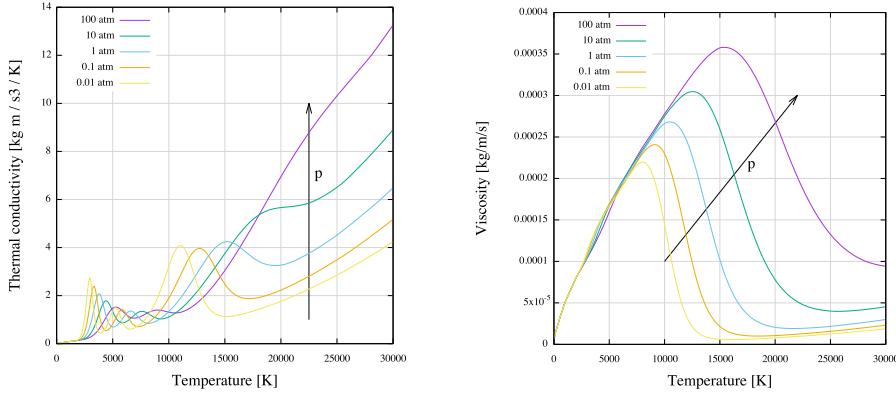


Figure A.23: Thermal conductivity (left) and viscosity (right) as a function of temperature.

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