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A Route-based Method for Turning Ratio Estimation: Application to the Grenoble Downtown Traffic Flow and Density Reconstruction

Martin Rodriguez-Vega*, Carlos Canudas-de-Wit* and Hassen Fourati*

Abstract—In this paper, we propose a route-based approach to improve the flow and density estimation methods for the case of urban traffic networks. For this proposal, a traffic assignment problem is solved at first, whose outputs are used to estimate the turning ratios for all intersections. Such information is not usually available at each node of the network. Second, the estimated turning ratios are used to better reconstruct the dynamic state of the network: the flow and density. To validate the proposed methods, we use real traffic data collected in the city of Grenoble in France, which include the measurement of Origin-Destination matrices, some turning ratios for validation, mean speeds of road sections and traffic flows at the boundaries of the network.

I. INTRODUCTION

Actually, the development of vehicle mobility in smart cities is one of the main problems in transportation research. Methods to estimate the flow and density of roads in urban networks is of great importance to promote such progress in this area. The widespread use of novel heterogeneous data sources such as Floating Car Data (FCD), Bluetooth (BT) and Wifi vehicle identifiers, among others, has helped in the development of data-based estimation approaches that provide more accurate estimates of traffic states, i.e. speed, flow, and density, in each road of a traffic network [1]. Then, the estimates are used by city operators to implement strategies for congestion reduction or by users to decide about the fastest route to take to reach the destination.

Currently, many of the state-of-the-art methods for traffic states estimation require the knowledge of the Turning Ratios (TRs), which model how flow split at intersection from the inbound to outbound roads. For instance, in [2] the authors use the TRs with data from connected vehicles to estimate the flow propagation in an urban network. Similarly, in [3], the authors propose a data-based density and flow estimation method, using heterogeneous data sources (flow and TR sensors with FCD) which are suitable for applications in large-scale networks.

In practice, as the number of intersections in a network can become very large, the TRs are quite hard to know everywhere, then to circumvent this problem, the interest is focused on their estimation based on some observed variables. For instance, node-based approaches use the local characteristics of the road adjacent to an intersection to calculate the TRs. The authors in [4] use geometrical properties of roads to identify the most likely turns, [5] uses

the connectivity of each outgoing road (dead-ends are less likely to be taken than an artery), and [3] uses the Functional Road Classification (FRC) [6] which assigns importance to each road according to its function [7]. Although node-based approaches provide an inexpensive way to estimate the TRs, they can introduce biases as they do not take into account the behavior of drivers that travel from Origins to Destinations.

To solve this problem, the Traffic Assignment (TA) can be used as an alternative approach for the estimation of TRs. Such method considers the flow of vehicles in an Origin-Destination (OD) pair, and distributes it into the possible routes that make this connection according to some criteria (e.g. minimize the total traveling time of each driver) [8]. In the literature, TA receives a lot of attention, in the case of traffic and infrastructure planning [9], and the prediction of congestion patterns [10]. Although TA is a powerful tool to understand path selection and congestion formation, it is computationally intensive, specially for large-scale traffic networks as the number of paths to consider grows exponentially. Some works in the literature propose more efficient algorithms to solve TA problem [11], [12], but they are restricted to specific networks and may not be applicable for general networks. Thus, to the best knowledge of the authors, TA is typically unsuited for real-time flow and density estimation applications in large-scale traffic networks.

In this paper, we propose a new method to estimate the TRs using a route-based TA approach, and use this information later to estimate, in real-time, the flow and density of traffic in a large-scale network of city. The proposed TA solution takes as inputs measured OD matrices and boundary flows of the network. As an optional input, few measured TRs can be included to provide more accurate results, if available. At this step, the calculation is done offline which better reduce the cost computation. Once all TRs are obtained, the real-time flow and density estimator proposed in [3] is used, which consists in using speed measurements from FCD in conjunction with the current density estimates to calculate road outflows. We validate the proposed methods using real traffic data collected in the downtown of Grenoble city, in France. The results show that the proposed method produces more accurate estimates when compared to node-based approaches.

The paper is organized as follows: Section II presents the main notions and definitions used throughout the paper. Section III describes the proposed methods and the sub-processes needed for its implementation. Section IV presents the study case and real traffic data used for validation of the method, and Section V discusses the obtained results. Finally,

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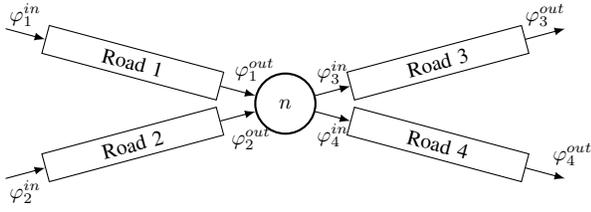


Fig. 1: Flow exchange at an intersection.

Section VI presents some conclusions.

II. DEFINITIONS, PRELIMINARIES AND MODELS

We define a traffic network as a set $\{\mathcal{N}, \mathcal{E}, \mathcal{T}\}$ where $\mathcal{N} = \{1, 2, \dots, N\}$ are the nodes corresponding to intersections (or nodes), $\mathcal{E} = \{1, 2, \dots, E\}$ are edges corresponding to the roads (or links), and $\mathcal{T} \subset \mathcal{E} \times \mathcal{E}$ is a set of road pairs (i, j) that correspond to turns from i to j that can legally be made at an intersection. Each road $i \in \mathcal{E}$ is characterized by physical properties such as length ℓ_i , free-flow speed v_i^{max} , number of lanes, etc. The state of road i refers to the values of the inflow $\varphi_i^{in}(t)$, outflow $\varphi_i^{out}(t)$, and density $\rho_i(t)$. The entire traffic state is the collection of these quantities in vector form $\varphi^{in}(t), \varphi^{out}(t), \rho(t) \in \mathbb{R}^E$, respectively, which are linked by the conservation equation

$$\dot{\rho}(t) = L^{-1}(\varphi^{in}(t) - \varphi^{out}(t)) \quad (1)$$

where L is the matrix of road lengths, $L_{i,i} = \ell_i$, and the i -th element of $\rho(t)$ is $\rho_i(t)$ (analogously for $\varphi^{in}(t)$ and $\varphi^{out}(t)$). Denote by $\mathcal{E}^{in} \subset \mathcal{E}$ the sets of roads at the boundaries from which cars enter the network (flow origins), and by $\mathcal{E}^{out} \subset \mathcal{E}$ the set of roads at the boundaries from which cars exit the network (flow destinations), respectively.

To describe how flows split at intersections, we assume the existence of the TR parameters, which apply locally at each intersection. A TR $r_{i,j} \in [0, 1]$ is the proportion of vehicles that turn from road i to road j , such that $(i, j) \in \mathcal{T}$. A simple intersection is shown in Fig. 1. Thus, the incoming flow for each road can be expressed in terms of the outflows of the upstream roads

$$\varphi_j^{in}(t) = \sum_{i|(i,j) \in \mathcal{T}} r_{i,j} \varphi_i^{out}(t), \quad \forall j \in \mathcal{E} \setminus \mathcal{E}^{in}. \quad (2)$$

Note: The super-indexes in and out for the flows correspond to roads and not intersections. Thus, φ_i^{out} is the outflow of road i (that then enters an intersection), and φ_i^{in} is the inflow to road i (that comes from an intersection).

To estimate the TRs, we use a route-based approach which is based on a traffic assignment problem using the OD matrix of the network. A route (or path) for a given OD pair $(o, d) \in \mathcal{E}^{in} \times \mathcal{E}^{out}$, is an ordered set \mathcal{P} of adjacent roads that begins

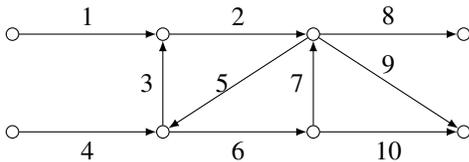


Fig. 2: Example network. Boundary roads are $\mathcal{E}^{in} = \{1, 4\}$ and $\mathcal{E}^{out} = \{8, 9, 10\}$.

with o and ends with d , i.e., $\mathcal{P} = (o, i_1, i_2, \dots, d)$, where $i_k \in \mathcal{E}$. Multiple paths may exist for each OD pair. Consider the example shown in Fig. 2, and the OD pair $(4, 8)$, for which two possible paths are $\mathcal{P}_1 = (4, 3, 2, 8)$ and $\mathcal{P}_2 = (4, 6, 7, 8)$. We denote by \mathbb{P} the set of all considered paths for all OD pairs.

Assume that there is an indexing $p = 1, 2, \dots, |\mathbb{P}|$ of the paths. Each path $\mathcal{P}_p \in \mathbb{P}$ has an associated steady-state flow f_p , and let $\mathbf{f} \in \mathbb{R}^{|\mathbb{P}|}$ be the concatenated vector of path flows. These flows are constrained by the given OD flows: define $\psi_{(o,d)}$ as the steady-state OD flow between the pair (o, d) , then, the sum of all path-flows connecting o to d must be equal to $\psi_{(o,d)}$,

$$\psi_{(o,d)} = \sum_{p=1}^{|\mathbb{P}|} \Omega_p^{(o,d)} f_p \quad (3)$$

where $\Omega_p^{(o,d)} = 1$ if path p connects the OD pair (o, d) , and is 0 else. Written in matrix form, this becomes

$$\boldsymbol{\psi} = \Omega \mathbf{f} \quad (4)$$

where $\boldsymbol{\psi}$ is the concatenated vector of all OD flows, and Ω is called the path-OD incidence matrix.

Similarly, each road $i \in \mathcal{E}$ has a unique steady-state flow $\varphi_i^{in} = \varphi_i^{out} = \varphi_i$, which are given by the path flows: the flow in road i is the sum of all the path-flows that contain road i ,

$$\varphi_i = \sum_{p=1}^{|\mathbb{P}|} L_{i,p} f_p \quad (5)$$

where

$$L_{i,p} = \begin{cases} 1 & \text{if } i \in \mathcal{P}_p \\ 0 & \text{else} \end{cases} \quad (6)$$

In matrix form, this is written as

$$\boldsymbol{\varphi} = L \mathbf{f} \quad (7)$$

where L is the link-path incidence matrix and maps the path-flows \mathbf{f} to link flows $\boldsymbol{\varphi}$.

III. ROUTE-BASED ESTIMATION METHOD

We propose to use a route-based method to estimate the TRs, which can be done offline using average OD flows during a long aggregation period (e.g. daily). As an

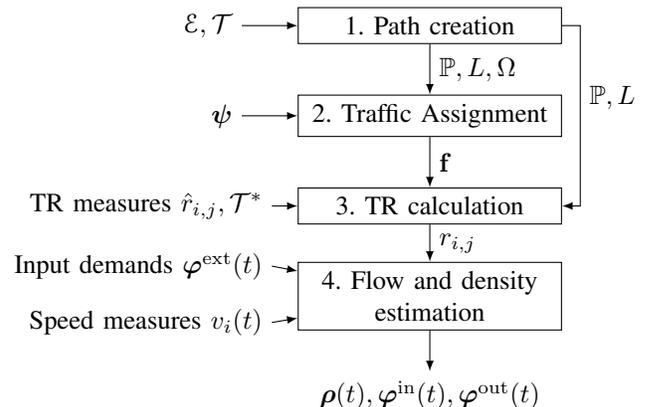


Fig. 3: Overview of the proposed methodology.

application, the TR estimates are used as an input to a flow and density reconstruction method. The overall diagram of the proposed methodology is shown in Fig. 3 where the main steps are

- 1) Create a list of paths that connect each origin to each destination.
- 2) Taking as input the OD flows, solve the TA problem to determine the flow in each path. If available, TR data from measured intersections can be included.
- 3) Use the computed path flows to estimate the TRs for each intersection.
- 4) Use the TR values with boundary inflow and speed measurements to estimate the road flows and densities.

The details of each step are described below.

A. Path creation

Route-based methods require first the creation of a set of considered paths \mathbb{P} between each OD pair. To construct this set, we select the paths with the smallest traveling time in free-flow,

$$c(\mathcal{P}) = \sum_{i \in \mathcal{P}} \ell_i / v_i^{max}. \quad (8)$$

This problem is solved by the well-known Dijkstra's algorithm [13], which gives a set of K shortest (or in this case fastest) paths for a given (o, d) pair. In practice, this calculations can be time consuming, so we introduce a slightly modified version of the algorithm to speed-up computations. A new condition is added such that if the new found paths are too slow in comparison to the fastest path, the algorithm stops.

Our version will find at least K_{min} paths between o and d , and after K_{min} paths have been found, the algorithm will continue to look for paths until the new path is ϵ times slower than the fastest path. If K_{max} paths are found, the algorithm stops regardless of the length-tolerance condition. This modification allows to have a variable number of paths for each pair, so the total number of iterations is reduced while keeping relevant (not too slow) paths.

Algorithm 1. K -fastest paths

Inputs:

- Road graph $\{\mathcal{E}, \mathcal{T}\}$
- Origin link $o \in \mathcal{E}^{in}$.
- Destination link $d \in \mathcal{E}^{out}$.
- Minimum number of paths K_{min}
- Maximum number of paths K_{max}
- Length tolerance $\epsilon > 1$

Initialize:

- Set of optimal paths $\mathbb{P} = \emptyset$.
- Number of paths found to each link $n_i = 0, \forall i \in \mathcal{E}$.
- Stack of paths $\mathcal{B} = \{\{o\}\}$

Steps:

- While $\mathcal{B} \neq \emptyset$ and $n_t < K_{max}$
 - Find $\mathcal{P} \in \mathcal{B}$ such that $c(\mathcal{P}) \leq c(\mathcal{P}') \quad \forall \mathcal{P}' \in \mathcal{B}$
 - Let i be the final link in \mathcal{P} .
 - $n_i \leftarrow n_i + 1$

- $\mathcal{B} \leftarrow \mathcal{B} \setminus \{\mathcal{P}\}$.
- if $i = d$
 - * $\mathbb{P} \leftarrow \mathbb{P} \cup \{\mathcal{P}\}$
 - * if $c_d \geq K_{min}$ and $c(\mathcal{P}) / \min(c(\mathbb{P})) > \epsilon$
 - **Return** \mathbb{P} .
- if $c_i \leq K_{max}$
 - * For each j such that $(i, j) \in \mathcal{T}$
 - $\mathcal{B} \leftarrow \mathcal{B} \cup \{\mathcal{P} \cup \{j\}\}$
- **Return** \mathbb{P} .

B. Traffic assignment

The TA problem consists in determining the flows in each path of a considered set \mathbb{P} , given a vector of OD flows ψ . We assume that drivers choose their route such that it minimizes total travel time. This is referred to as User Equilibrium (UE), whose solutions are constrained by the Wardrop criteria [14]. In the literature, TA problems under UE are commonly written as the following non-linear convex optimization problem [15],

$$\begin{aligned} \min_{\mathbf{f}} \quad & \sum_{i \in \mathcal{E}} \int_0^{\varphi_i} t_i(x) dx \\ \text{s.t.} \quad & \varphi = L\mathbf{f}, \quad \Omega\mathbf{f} = \psi \\ & \varphi \leq \varphi^{max}, \quad \mathbf{f} \geq 0 \end{aligned} \quad (9)$$

where φ^{max} is a vector of road capacities, and $t_i(\varphi)$ is the travel time in road i for a given flow. In this work, we use the commonly used BPR function [16] defined as

$$t_i(\varphi) = \frac{\ell_i}{v_i^{max}} \left(1 + \beta \left(\frac{\varphi}{\varphi_i^{max}} \right)^\alpha \right) \quad (10)$$

where α and β are tuning parameters. This function was first proposed by the Bureau of Public Roads, now the Federal Highway Administration (FHA) in the United States [16]. Common choices for the parameters are $\alpha = 4, \beta = 0.15$.

This TA problem formulation can be modified to include additional TR information. Suppose some intersections are equipped with sensors such that measured TRs $\hat{r}_{i,j}$ are available for turns (i, j) in a set $\mathcal{T}^* \subset \mathcal{T}$ (see Section IV-B). We can modify (9) by including this data to obtain a new optimization problem. From the TR definition, for each measured ratio there is an equation of the form

$$\varphi_{i,j} - \hat{r}_{i,j} \varphi_i = 0 \quad (11)$$

where $\varphi_{i,j}$ is the flow on turn $(i, j) \in \mathcal{T}^*$. These flows can be written in terms of the path flows,

$$\sum_{p=1}^{|\mathbb{P}|} L_{i,p} L_{j,p} f_p - \hat{r}_{i,j} \sum_{p=1}^{|\mathbb{P}|} L_{i,p} f_p = 0, \quad (12)$$

where the first term is the sum of flows in all paths that contain both roads i and j , and the second term is the sum of flow in paths that contain road i . This set of equations can be written in matrix form as $Q(\mathcal{T}^*)\mathbf{f} = \mathbf{0}$, with

$$Q_{k,p} = L_{i_k,p} (L_{j_k,p} - \hat{r}_{i_k,j_k}) \quad (13)$$

where (i_k, j_k) is the k -th element of \mathcal{T}^* . We introduce this condition as a soft constraint in the objective function of (9),

$$\min_{\mathbf{f}} \quad \sum_{i \in \mathcal{E}} \int_0^{\varphi_i} t_i(x) dx + \gamma \|Q(\mathcal{T}^*)\mathbf{f}\|_2 \quad (14)$$

under the same constraints, where γ is a tuning parameter to control how relative important the satisfaction of the TR measures are in comparison to the regular TA cost.

C. Turning ratio calculation

The resulting vector of path flows \mathbf{f} can be used to estimate the TR values for each intersection. Consider an arbitrary turn $(i, j) \in \mathcal{T}$, whose corresponding TR is calculated by

$$r_{i,j} = \left(\sum_{p=1}^{|\mathbb{P}|} L_{i,p} L_{j,p} f_p \right) / \sum_{p=1}^{|\mathbb{P}|} L_{i,p} f_p. \quad (15)$$

This is the ratio of the total flow of paths containing both roads i, j , and those containing road i . As the former set of paths is always a subset of the latter, this ratio is in $[0, 1]$, unless all the incoming flows are zero.

D. Flow and density estimation

In a previous work [3], we proposed a data-based method to estimate flows and densities for all roads, using speed, turning ratio and external input demand data from sensors. In the previous section, (2) allows to write road inflows in terms of the turning ratios, except for the roads at the boundaries of the network \mathcal{E}^{in} . For these roads, we have

$$\varphi_i^{\text{in}}(t) = \varphi_i^{\text{ext}}(t) \quad \forall i \in \mathcal{E}^{\text{in}} \quad (16)$$

where $\varphi_i^{\text{ext}}(t)$ is the external input demand for road i , and are assumed to be measured. Define the TR matrix $R \in \mathbb{R}^{E \times E}$ whose elements $R_{i,j}$ are the TRs if $(i, j) \in \mathcal{T}$ or are 0 else. Then (2) and (16) can be written in vector form as

$$\varphi^{\text{in}}(t) = R^{\top} \varphi^{\text{out}}(t) + \varphi^{\text{ext}}(t) \quad (17)$$

where $\varphi^{\text{ext}}(t)$ is a vector whose i -th entry is $\varphi_i^{\text{ext}}(t)$ if $i \in \mathcal{E}^{\text{in}}$, or is 0 else.

We assume that the speed of each road $v_i(t)$ is known, and let $V(t)$ be a diagonal matrix with $V_{i,i}(t) = v_i(t)$. Consider the hydrodynamic relation

$$\varphi^{\text{out}}(t) = V(t)\rho(t). \quad (18)$$

Substitution of (17) and (18) into (1) yields the following open-loop density estimator,

$$\dot{\hat{\rho}}(t) = L^{-1}(R^{\top} - \mathbb{I})V(t)\hat{\rho}(t) + L^{-1}\varphi^{\text{ext}}(t). \quad (19)$$

In [3], we proved that under error-free measurements and if the rate of change of the mean speed $V(t)$ is slow enough, this estimator converges asymptotically.

IV. EXPERIMENTAL SETUP

In this paper, we use data provided by the GTL-Ville¹, which is an experimental platform developed by the ERC Scale-Freeback project [17] for the real-time collection of traffic data coming from a network of installed sensors in the city of Grenoble in France. It covers an area of 1.4 Km by 1 Km in the city center, which is composed of 463 intersections, 804 road sections, and 1234 turns. The information collected using heterogeneous data sources are:

¹<http://gtlville.inrialpes.fr/>

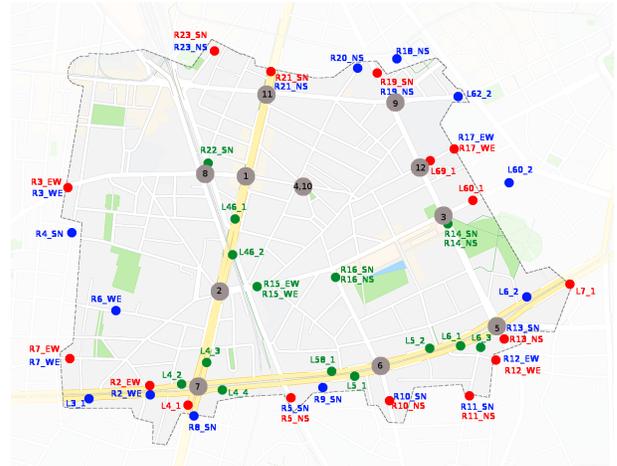


Fig. 4: Location of flow and turning ratio sensors. Flow sensors in color with text corresponding to the identifier in the database. TR sensor in gray with labeling numbers from 1 to 12.

A. Flow sensors

Two sensor technologies are currently being used to measure the vehicular flow (number of vehicle passages during a period of time) in a selection of roads: magnetic induction loops, and microwave radars. There are 40 flow sensors inside the region of interest as shown in blue, red, and green in Fig. 4. These colors correspond to the classification of the data provided by the sensors:

- In blue, sensors that provide boundary inflow data, located in roads in \mathcal{E}^{in} .
- In red, sensors that provide boundary outflow data, located in roads in \mathcal{E}^{out} .
- In green, sensors that provide data for cross-validation, located in roads on the network's interior.

Flow data is available in real-time with sampling rate of 1 min.

B. Turning Ratio sensors

To measure the TR parameters for a selection of 12 intersections, Bluetooth (BT) reader devices were located at the adjacent roads. These sensors detect the unique identifiers (IDs) of vehicles equipped with other BT devices and match these IDs with the other sensors to obtain the inbound and outbound road for each detected vehicle. To protect user privacy, only the total number of detected vehicles making each turn during a period of 1 hour is available. The selected intersections where TR sensors were installed are shown as gray circles in Fig. 4 with corresponding label numbers from 1 to 12.

TR data was collected during measuring campaigns, where data for each intersection was collected during a week, between September 21 and October 28, 2020. The TR values are computed as

$$\hat{r}_{i,j} = \text{Counts}(i, j) / \sum_k \text{Counts}(i, k) \quad (20)$$

where $\text{Counts}(i, j)$ is the number of detected vehicles that

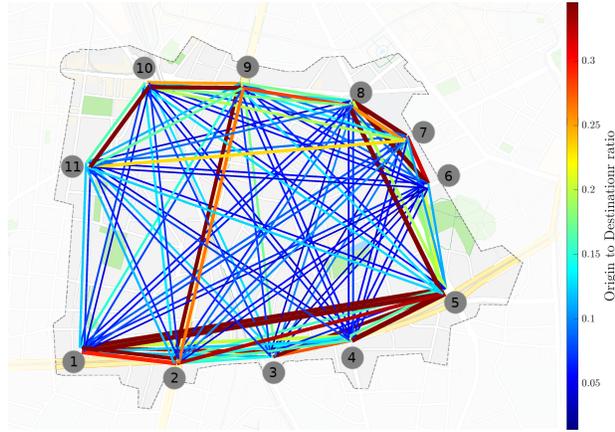


Fig. 5: Location of sensors for measurement of oD matrices, and the total measured OD ratios, normalized with respect to each origin. The two lines connecting each OD pair follow the right-hand rule to specify direction.

enter the intersection through road i and left via road j , during the total duration of the campaign.

C. Origin-Destination matrix sensors

BT sensors placed in specific locations were used to measure the OD matrices. In total, 11 sensors were located around the boundaries of the network, at the most important roads. OD data was collected during a two-weeks measuring campaign from February 24 to March 11, 2021. As BT devices only detect a sample of the population, only the OD rates can be estimated, using an analogous equation to (20). The location of sensors are shown as gray circles labeled from 1 to 11 in Fig. 5.

In the figure, the colored lines show the computed OD rates between each OD pair. The line color and thickness represents the percentage of vehicles going from each origin to all destinations. The ratios are normalized with respect to the origins, i.e., the sum of rates from one origin to all destinations is 1. As OD pairs are bidirectional (roads can have both directions), the right-hand driving rule is used to separate direction, i.e., for a vertical pair, the north-to-south direction is to the left, and for an horizontal pair, the east-to-west direction is on top. The data agrees with the expected behavior, as roads with high importance receive a much higher rate than least important ones. For instance, the OD pairs (1,5) and (5,1) are connected by the most important artery in this zone which sees very high flows, and have the highest OD rates in the figure. A similar behavior is seen for pairs (2,9) and (5,8).

D. Floating car data

Floating Car Data (FCD) refers to vehicle traces that are collected for a subset of drivers, usually obtained from GPS devices. For the region of interest, data is obtained through a third party, Tomtom², a well-known company for assisted navigation. Due to privacy protection regulations, we do not have access to direct GPS traces. Instead, this information

²<https://www.tomtom.com/>



Fig. 6: Speed data in the GTL-Ville. **Top:** Colored lines correspond to the roads with speed data in real time. **Bottom:** Time plot of a selected road for one day.

is aggregated and only the average speed of all reporting vehicles in a road section during a time period is available, for a subset of roads in the network. This data is obtained in real-time with a frequency of 1 min.

Figures 6 shows the available speed in the GTL-Ville platform. In the map representation, the colored lines show the speed data for each road in real time (green are high speed values and red are low speed values). The bottom of the figure shows a time plot of the speed of one road for one day.

V. RESULTS

For validation purposes, we consider the flow and speed data collected during February 8th, 2021. First, we use Algorithm 1 to calculate the paths between each OD pair, using $K_{min} = 2$, $K_{max} = 10$, and $\epsilon = 1.2$. This resulted in a total of 3437 paths.

The OD flow vector ψ was computed using the OD rates as described in Section IV-C, in conjunction with the boundary flows from Section IV-A, averaged during the entire day. As there are more roads at the boundaries than OD rate measurement sites, each road was mapped to the closest OD measure.

We consider three methods for calculating the TR values:

a) *Method 1: Using regular TA:* With the set of paths \mathbb{P} and the OD flow vector ψ as input, (9) is solved to calculate the path flow vector \mathbf{f} , and then (15) to estimate the TRs. For this, we used the well-known open-source convex optimization solver CVX in Matlab [18], [19].

b) *Method 2: Using TA with TR measures:* This follows an analogous procedure as the Method 1 but uses the modified cost function (14) with the TR data described in Section IV-B. The heuristic value $\gamma = 1/|\mathcal{T}^*| = 1/142$ was used.

c) Method 3: Using FRC: For comparison purposes, we also used the TR values produced by the node-based approach we proposed in [3], which makes use of the FRC. In this case, we consider 7 different classes, going from 1 (most important, e.g. Highways) to 7 (least important, e.g. local residential roads). To each class c , a weight θ_c is given, and each TR $r_{i,j}^{\text{FRC}}$ is computed as

$$r_{i,j}^{\text{FRC}} = \theta_{c_j} / \sum_{k|(i,k) \in \mathcal{T}} \theta_{c_k}, \quad (21)$$

where c_i is the FRC of road i , and the superindex is to specify the TR estimation method. The weights θ are computed through a flow-matching optimization problem using the measured boundary flows and imposing the measured TRs as constraints (see [3] for details). This method assumes that vehicles tend to turn to the most important road in each intersection, independent from destination.

A. Turning ratio estimation

TABLE I: Error distribution of TRs using route-based approaches

Method	Mean	Std. Deviation
Regular TA	0.7%	24.9%
TA with TR measures	0.7%	23.8%

The measured TR values were compared with the estimates provided by the TA based methods. The resulting errors followed a Gaussian distribution with parameters shown in Table I. The route-based TR estimates are seen to be close to the measured values. As expected, the inclusion of TR data in the TA problem decreases the standard deviation of the errors by 4.4%. The errors can be further decreased by increasing the value of γ , although too high values could produce unrealistic results by artificially increasing the total travel time of routes chosen by vehicles.

B. Flow and density estimation

The TR estimates using the three methods previously described were used to estimate the dynamic flow and density in each road of the network using (18) and (19). For validation, we used the flow measurements for the roads equipped with validation sensors. Figure 7 shows the measured flow values as ground truth, and the dynamic flow estimates using the TR values estimated using the three methods.

In general, the three approaches provide close estimates of the ground truth flow values. Note that for sensors R15_EW and R16_NS, the use of the TA approach (both with and without TR measures) significantly improves the estimation accuracy, especially when TR measures are included. As error metrics, we use the Relative Mean Error (RME)

$$\text{RME}_i = \left| \int_0^T (\varphi_i^{\text{out}}(t) - \hat{\varphi}_i^{\text{out}}(t)) dt \right| / \int_0^T \varphi_i^{\text{out}}(t) dt, \quad (22)$$

and the Relative Absolute Error (RAE)

$$\text{RAE}_i = \int_0^T |\varphi_i^{\text{out}}(t) - \hat{\varphi}_i^{\text{out}}(t)| dt / \int_0^T \varphi_i^{\text{out}}(t) dt \quad (23)$$

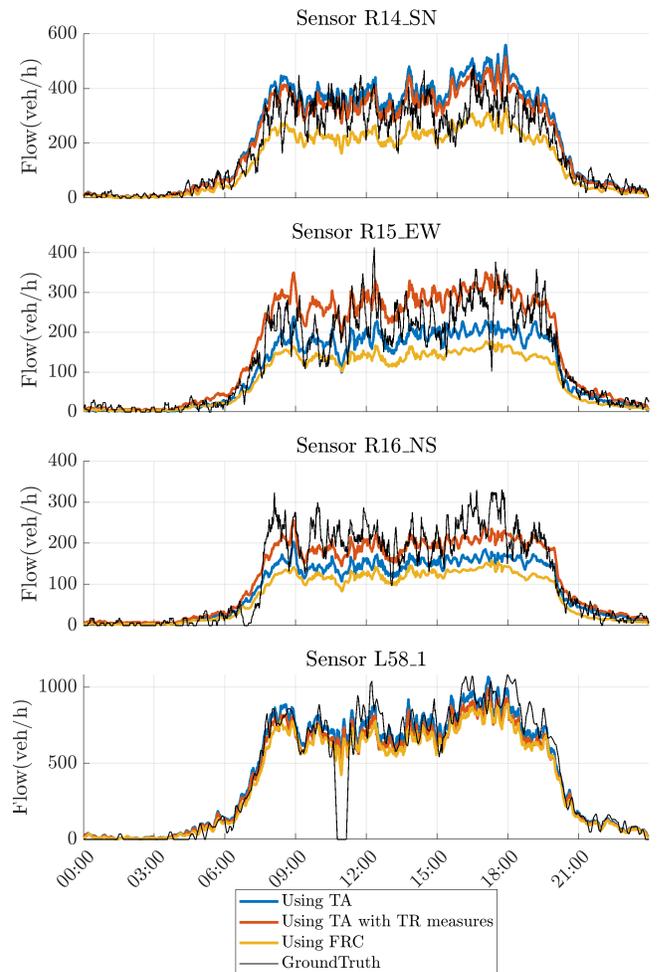


Fig. 7: Time series of the real and estimated flows using different TR values.

where $\varphi_i^{\text{out}}(t)$ and $\hat{\varphi}_i^{\text{out}}(t)$ are the real and estimated flows in road i , respectively. To take into account the data from all cross-validation sensors, we use the Cumulative Distribution Function (CDF) for each metric: $y = \text{CDF}(x)$ is the fraction of roads y which have an error value less than or equal to x . Figure 8 shows the resulting CDF for both error metrics for all the validation sensor locations.

In general, the TR values obtained using route-based TA approaches outperform the node-based approach, and the inclusion of TR measures in the TA approach improves the accuracy even more: 80% of the roads have an RME below 38%, 29%, and 21%, for Methods 3, 1 and 2, respectively. Comparing the TR estimates from Methods 1 and 3, both have very similar behavior up to an RME of 28% and RAE of 35%. From this point on, the estimates from Method 3 start producing higher errors, except for one location, where Method 1 produces a high jump in RME from 33% to 72%. However, this outlier is not present in the results of Method 2. This is because the high-error location in Method 1 corresponds to a sensor in the interior of the network R16_NS, whose real behavior is not correctly captured by only minimizing traveling time. By inclusion of the local TR information in the optimization problem, more detailed

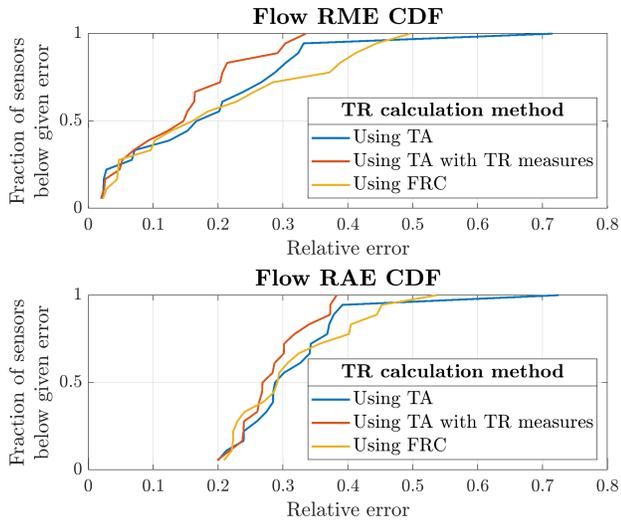


Fig. 8: Relative error CDF for flow estimation using different methods for TR computation.

user route selection can be recovered increasing the accuracy, as seen in the Method 2 results.

VI. CONCLUSIONS

In this paper, we have explored the use of route-based traffic assignment methods under user equilibrium, to estimate turning ratios at intersections. These estimates are then used in a computationally efficient and scalable dynamic flow and density estimator. We also consider a slight modification of the Assignment problem such that direct measurements of a set of Turning Ratios can be used when available. To validate these methods, we use real traffic data collected in the city of Grenoble and then compared it against a local node-based approach to estimate Turning Ratios, which we proposed in a previous work.

The results show that in general, the route-based approaches outperform the accuracy of the flow estimates compared to the considered node-based method. Furthermore, the case where Turning Ratio measurements are added to the classical methods provided the best reconstruction, greatly surpassing the other methods. The price to pay for the increased accuracy of route-based methods is the increase in computational cost, as the listing of paths between all Origin-Destination pairs and the TA optimization problems can incur a lot of computations. However, these computations can be performed offline with low-frequency average inputs.

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