



HAL
open science

Partial Paving Strategy: Application to optimize the Area Coverage Problem in Mobile Wireless Sensor Networks

Adda Boualem, Cyril de Runz, Marwane Ayaida

► **To cite this version:**

Adda Boualem, Cyril de Runz, Marwane Ayaida. Partial Paving Strategy: Application to optimize the Area Coverage Problem in Mobile Wireless Sensor Networks. *Journal of Wireless Mobile Networks, Ubiquitous Computing, and Dependable Applications*, 2022, 13 (2), pp.1-22. 10.22667/JOWUA.2022.06.30.001 . hal-03726364

HAL Id: hal-03726364

<https://hal.science/hal-03726364v1>

Submitted on 18 Jul 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Partial Paving Strategy: Application to optimize the Area Coverage Problem in Mobile Wireless Sensor Networks

Adda Boualem^{1*}, Cyril De Runz², Marwane Ayaida³

¹Ibn Khaldoun University, Computer Science Department, Tiaret, Algeria
adda.boualem@univ-tiaret.dz

²BdTLN, LIFAT, University of Tours, Blois, France
cyril.derunz@univ-tours.fr

³Université de Reims Champagne Ardenne, CReSTIC EA 3804, 51097 Reims, France
ayaida.marwane@univ-reims.fr

Abstract

Finding the minimum number of sensor nodes to ensure coverage of an area for a maximum of time duration is an NP-difficult problem, especially in large areas. The mathematical solutions proposed for this problem consist in answering to the question: "How to place the sensor nodes in n equal circular areas of a given radius to cover the area of interest as much as possible?" Discrete geometry models have been proposed in the literature to analyze and solve this mathematical problem, and we have seen that paving techniques are considered to be the most suitable methods for this kind of problems. Consequently, this paper provides a basic solution of discrete geometry, based on tiling a circle by a minimal number of circles to optimize the coverage problem in wireless sensor networks in this work entitled: "Partial Paving of a Circle by Equal Circles strategy (PPoCEC)". We have proven mathematically the effectiveness of our solution. The simulation which consists of a statistical evaluation step: Asymptotic Confidence Interval (ACI) calculation and another step of comparison with clustering strategies showed that the proposed strategy achieves coverage at 100% for the first 10 nodes deployed in the area of interest compared to the other coverage strategies in sensor networks proposed in the literature.

Keywords: Telling problem, Packing problem, Area Coverage, Power Consumption, Wireless Sensor Network

1 Introduction

The advancement in Micro Electro-Mechanical Systems (MEMS) technology has led to production of thin, cheaper and reliable sensors nodes. A network of sensors suffers from several technical weak points such as communication range, monitoring range, low battery, and network deployment circumstance problems such as the difficulty of building a sensor network in volcanoes, mountains, or in the oceans. The deployment with its two types considered a very effective solution to optimize the problem of coverage in wireless sensors networks (WSN), power consumption, routing, connectivity, and others. Deterministic deployment is effective in very limited areas where man can be involved to troubleshoot, configure, change batteries, and replace or move nodes. Random deployment is effective in unrestricted areas where human intervention is impossible or difficult. Advancement in MEMS technology has led to production of very thin sensor nodes that are less expensive and yet powerful in terms of storage and processing. Increasing network lifetime by keeping the entire coverage of the area of interest is one of the compromises desired by researchers in their work using deployment. Among the disadvantages of this solution for the random type are the non-equitable distribution of the sensor nodes and the difficulty

*Corresponding author: Ibn Khaldoun University, Computer Science Department, Tiaret, Algeria, Tel: +213-774186430

of applying the process of activating sensor nodes in the network. The necessity of knowing positions (best distribution) and remote-control in geographical areas is mandatory, where human intervention is difficult in certain danger zones. In this paper, we will propose a solution that can be used easily to search for an optimal solution to this problem as a powerful mathematical technique of geometry. The proposed approach is k-coverage (3-coverage), subdividing the area of interest (AoI) into 3 sub-areas. Each sub-area provided by a base station (BS). These base stations do not suffer from energy constraints. Each base station can communicate information gathered from sensor nodes of its sub-area to the same processing center.

The rest of this paper is organized as follows. Section 2 reviews the main works on the problem of coverage in WSN. In the first part (Section 2.1) some traditional strategies, and in the second part (Section 2.2) some others works for coverage with tiling strategies as packing and paving techniques (the cutting and wrapping techniques for planar, circular and spherical shapes tiling). The third part (Section 2.3) reviews some other coverage works by circles paving strategies. Section 3 defines formally the paving problem and the packing problem. The first part (section 3.1) give the formulation of the problem, the second part (section 3.2) give the definition of a closed multiply-connected set M , and finished by (section 3.3), that describes the basic assumptions used to define our coverage strategy in the third part. The part (section 3.4) describes the general assumptions in the fourth part. Sections 4 subdivided in four parts: the first part 4.1 presents the motivations behind the proposal of this strategy. The part 4.2 presents our strategy based on the discrete geometry for solve coverage problem in WSN. The part 4.3 justifies the choice of using 3 circles to pave a rectangular (square) area of interest. We finished by section 4.4 that describe the mathematical proof of our partial paving strategy.

The evaluation and simulation results are presented in Section 5. This section is subdivided in two parts. The first part (Section 5.1) validates the results found by a statistical analysis. In the second part (Section 5.2) we have presented the comparison strategies in (Section 5.2.1) and the evaluation step in (Section 5.2.2). The results analysis are discussed in Section 5.3. Finally, we have ended with a conclusion that summarizes the additions expected by the application of paving techniques, cutting and packaging in the coverage of geographical areas in Section 6.

2 RELATED WORK

This section is subdivided into two parts; the first part is to cite literature based on traditional methods often used to ensure coverage in wireless sensor networks. The second part consists of citing literature based on geometric paving methods (circles, squares, rectangles, ...).

2.1 Coverage with traditional methods

Currently, the use of wireless sensor networks is a major need in various surveillance applications, such as monitoring applications in: the environment, battlefields, borders, forests, intelligent spaces, and in the control of industrial and biological disasters.

The main objective of using of this technology is how to monitor with sufficient coverage an area of interest with a fewer sensor nodes during a longtime [9]. The scientific community has classified the solutions to the coverage problem in WSN by : (a) coverage strategies, (b) coverage algorithms and by (c) the fields involved in the solution.

- **Classification by coverage strategies:** Referring to [11], the coverage is classified according to the presence or absence of mobile sensors in the wireless sensor network. Those coverage types (Figure 1) uses strategies of the two main aspects: static (Table 1) and dynamic (Table 2) strategies.

Table 1: Some static coverage approaches

Reference	Application	Deployment	Node Type	1 st Objectif	2 nd Objectif
[1]	Generic	Deterministic	Data collector	Lifetime of WSN	Coverage
[18]	Monitoring	Deterministic	Sensor node	Coverage	Lifetime of WSN
[42]	Outdoor	Random	Sensor node	Coverage	Lifetime of WSN
[41]	Monitoring	Deterministic	Sensor node	Coverage	Connectivity

Table 2: Some Dynamic coverage approaches

Reference	Relocation trigger	1 st Objectif	2 nd Objectif	Contraintes
[54]	Weak coverage	Coverage	Travel distance	Time of convergence
[56]	High traffic	Network lifetime	Coverage	Time of convergence
[52]	Bad Image Coverage	Image Coverage	Travel distance	Time of convergence
[4]	High traffic	Coverage	Travel distance	Time of convergence

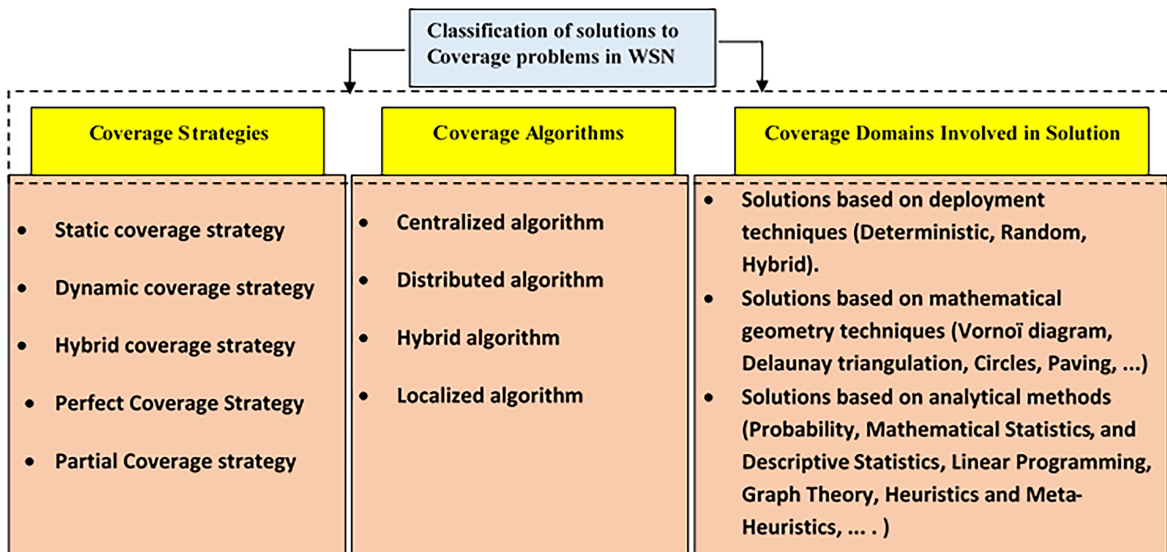


Figure 1: Classification of coverage problems solutions in WSN.

- **Classification by coverage algorithms:** In wireless sensor networks, the ability to transmit and receive data allows control and decision making through the sensors but also to perform tasks cooperatively, they are based on the classes of algorithms described as follows: centralized algorithm, distributed algorithm, hybrid algorithm, and localized algorithm. Figure 1 gives an overall schematic of the coverage hierarchy in WSNs, which is why several coverage concepts have emerged.

- **Classification by use fields:**

In the literature, some classify the solutions, algorithms and strategies dealing with the coverage problem by field of use, i.e. according to the technique of the field used (Figure 2), such as :

- Solutions based on deployment techniques (Deterministic, Random, and Hybrid). [16]
- Solutions based on the mathematical techniques of geometry (Vornoï diagram, Delaunay triangulation, Circles, Paving,...)
- Solutions based on analytical methods (Probability, Mathematical and Descriptive Statistics, Linear Programming, Graph Theory, Heuristics and Meta-Heuristics,...)

In this context, the coverage problem have been classified in our contributions into two main categories; (a) coverage in sensor networks based on deterministic models: Energetic Sleep-Scheduling via Probabilistic Interference K-Barrier Coverage with Truth-Table Technique in Sensor Network [15], Hybrid Model Approach for Wireless Sensor Networks Coverage Improvement [11], and (b) coverage in sensor networks based on uncertain models: An Evidential Approach for Area Coverage in Mobile Wireless Sensor Networks [12], Area Coverage Optimization in Wireless Sensor Network by Semi-random Deployment (SRDP) [16], Spiderweb strategy: application for area coverage with mobile sensor nodes in 3D wireless sensor network (Spiderweb strategy)[14], A New Dijkstra Front-Back Algorithm for Data Routing-Scheduling via energy-efficient Area Coverage in wireless Sensor Network [10], Fuzzy/Evidential Approach to Address the Area Coverage Problem in Mobile Wireless Sensor Networks [13].

Unsurprisingly, most of the proposed strategies, which focus on the coverage, the network connectivity, and lifetime and/or fault-tolerance, have addressed the cost factor of the deployed network in terms of the total number of used nodes. This is due to the high probability of node damage and loss in applications requiring these network features. In contrast, none of the proposed deployments have addressed all of the required properties. Moreover, lifetime and fault-tolerance properties in this review are restricted to node-based and node fault-tolerance. As far as we know, we have not found a deployment proposal which takes into account connectivity, coverage, heterogeneity, lifespan, node-based, and node-based network fault tolerance at the same time. For this reason, we propose a study which considers to optimize the three aspects in the same time: Coverage, Connectivity, Lifetime.

A main comparison between the different strategies proposed for (Coverage, Connectivity, and Energy-Efficient) is illustrated in Table 3 [3].

These tables (Table 3, Table 1 and Table 2) summarize a comparison in terms of static topology (static nodes), in terms of dynamic topology (mobile nodes), and in terms of techniques used. The objective of this comparison is to explain the strengths and weaknesses of coverage strategies in WSNs.

2.2 Coverage as packing and paving technique with tiling strategies

The coverage problem is inspired by mathematical puzzles in the seventies (70s), and eighties (80s) [30], [25], which consist of finding the minimum number of circles to cover a square or rectangle, and how is the minimum number of cameras needed to cover an entire gallery? The early researchers consider it a mathematical problem and they try to solve it by geometry methods. In recent decades, researchers

Table 3: A Comparison between Various Strategies Proposals in the Literature for (Coverage, Connectivity, Lifetime) [3]

Ref. No. Pub. Year	Paper Highlight	Algorithm Proposed/Used	Distributed/Centralized	Main Issues Handled	Main Drawbacks
[33] 2017	Potential Field Approach	Clustering, Virtual Force & Merging	Distributed & Centralized	Coverage, Sensor Movement	Obstacle Adaptability, Connectivity
[58] 2019	Virtual Force	Floor based Scheme	Distributed	Coverage, Sensor Movement, Connectivity	Computational Overhead
[49] 2019	Optimization Problem	-	Centralized	Power Consumption, Fault Tolerance	Coverage and connectivity
[45] 2019	Intelligent Mobile Sensor	Voronoi Diagram	Distributed	Energy Efficiency, Sensor Movement	Obstacle adaptability
[38] 2019	Incremental Deployment	-	Distributed	Coverage	NA
[20] 2018	Electrostatic Problem	-	NA	Connectivity	NA
[53] 2020	Mobility for Improving Coverage	-	Distributed	Coverage	NA
[59] 2019	Virtual Force	Hangerian, SMART, Extended SMART	Centralized	Coverage, Sensor Movement	Obstacle adaptability
[61] 2020	Non Uniform Sensor Distribution	MAND	Centralized	Energy Efficiency	Coverage
[34] 2016	Node Discovery	Machine Learning	Distributed	NA	coverage
[57] 2020	Dynamic Coverage	-	Distributed	Coverage	NA

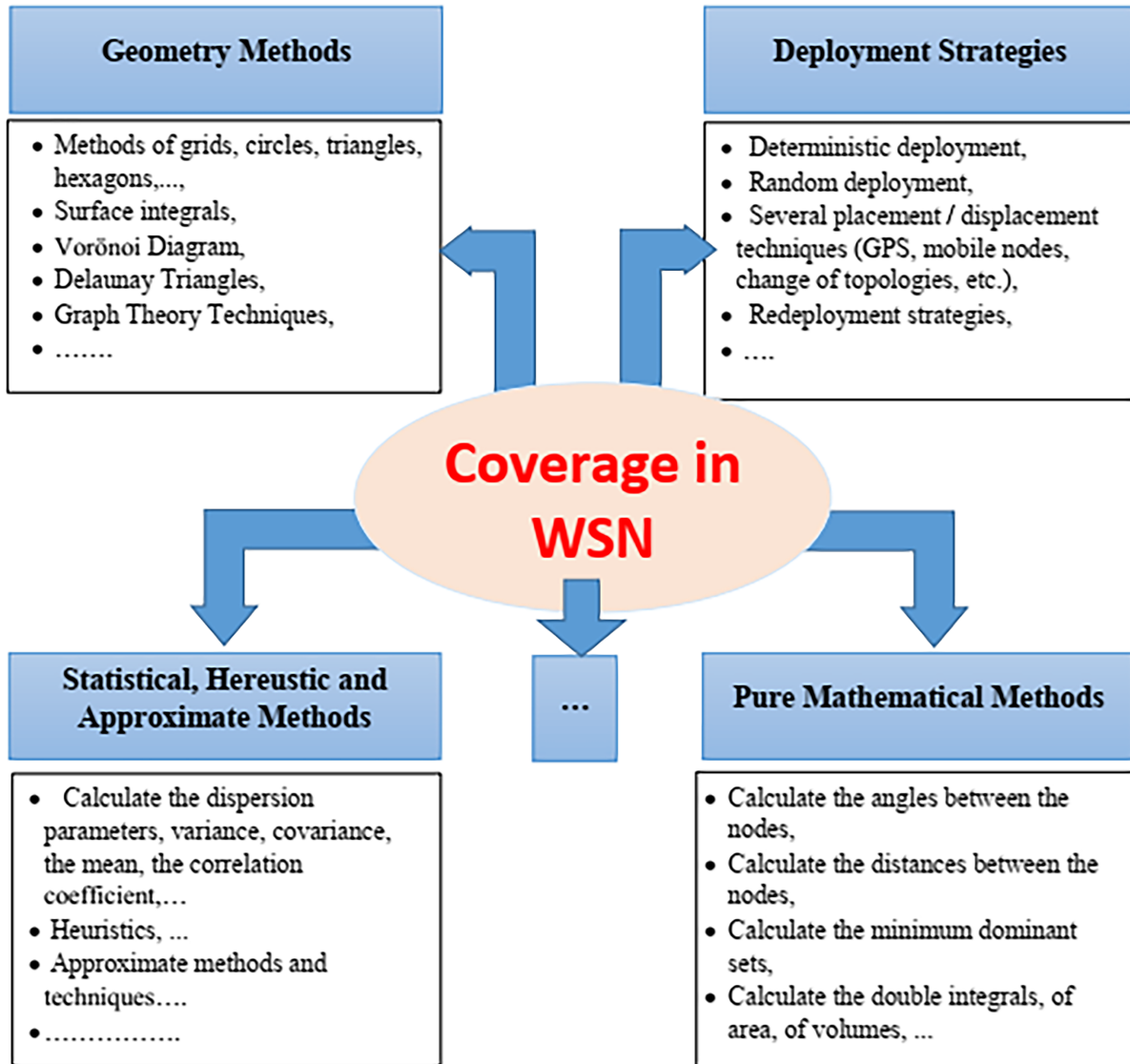


Figure 2: Pipeline of fields and strategies used in coverage in WSN

have used different techniques from different scientific fields for this problem, as described in Figure 2. One of the famous problems of discrete geometry in mathematics concerning the tiling of a plane by a number of finite circles is described in [17]: how to share a unit circle in n equal circles (of the same radius) without overlap so that the radius of the circles is as large as possible? The wrapping of circles of different shapes in a plane or sphere has been a fundamental subject of research in discrete geometry in recent decades. The packaging problem consists of trying to find a configuration of n equal circles not overlapping inside the zone, so as to maximize the radius of the circles [17]. This packaging problem has a double counterpart in tiling: how should the unit circle be covered with n equal circles without gaps so that the radius of the circles is as small as possible? Zahn [60] posed an intermediate problem between these two questions: how should we have n equal circles of given radius so that the area of the elementary circle part covered by the n circles is as large as possible? This

intermediate problem is the subject of this paper. In all the paper, after Fejes T'oth [22], we use the term circle also for a circular disk. The problem of coverage is a bit more difficult than the problem of packaging. Certainly, fewer results are known for the coverage problem. Although the solution is trivial up to $n = 4$ and for $n = 7$, other solutions are solved only up to $n = 10$ proven solutions were provided for $n = 5$ and $n = 6$ by Bezdek [8, 29] and for $n = 8, 9, 10$ by Fejes T'oth [21]. Conjectural solutions were given for $n = 11$ and 12 by Melissen [35] (improved by Nurmela [39]) and for $13 \leq n \leq 35$ by Nurmela Nurmela [39]. The intermediate problem is the least treatable of the three. To our knowledge, the only article dedicated to intermediaries the problem in the plan is that of Zahn [60] which provided a proven solution for $n = 2$ (for $0.5 < r < 1$) and hypothetical solutions for $3 \leq n \leq 10$ (for between three and six selected values of r). Apart from these results, there is not even any conjecture for other values of n . (It should be noted that a similar problem regarding the partial overlapping of the spherical surface by n equal circles has been raised and that proven solutions for $n = 2, 3, 4, 6, 12$ have been established by Fejes T'oth [22], [23], with putative solutions for $n = 5, 7$ to 11 and 13 produced later by Fowler and Tarnai [24], [43]. The main objective of this paper is to set up a model of discrete geometry (paving, cutting and packaging) to address the coverage problem, which also works for triple overlaps, and by which the solution to problem can be found, or that leads to an optimal arrangement at least locally. Cutting and packaging problems are complex scientific problems that cover a wide range of applications. These are very interesting *NP-hard* combinatorial optimization problems; that is, no procedure is able to solve them exactly in deterministic polynomial time. They are found in a variety of real-world applications, including production and packaging for the textile, apparel, marine, automotive, aerospace and food industries. These are bottleneck problems in computer-aided design, in which design plans have to be generated for industrial installations, electronic modules, nuclear and thermal installations, etc [47]. Cutting and packaging problems consist of wrapping a set of geometric objects / articles of fixed dimensions and shapes in a region of predetermined shape while taking into account technical considerations and problem design [47]. The package identifies the layout and position of the geometric objects that determine the dimensions of the containing shape and reach the extremum of a specific objective function [47]. When the geometric objects are objects ϕ , the compaction problem is reduced to a mathematical model whose constraints reflect the conditions of arrangement of the objects in the given region, the conditions of mutual non-intersection for the objects and any technological constraint which can not be reduced to purely geometric constraints [47], such as guillotine cuts for rectangular packaging. "One approach to solving this complex problem is to organize the ϕ objects according to a prescribed order in Ω , and to find the extremum local exact [47] However, the search for exact local extrema takes a lot of time without any guarantee of sufficient convergence towards the optimum [47]. The packaging of circular objects is a challenge for discrete and computer geometry [50]. This paper focuses on problems with two- and three-dimensional circular and / or spherical objects to address 2D and 3D coverage respectively in wireless sensor networks. To do this, we find that the problems of cutting and packaging are very useful. As a result, these problems have a wide range of applications ranging from natural sciences to technical design to everyday life. They include coverage of a geographic area with WSNs wireless sensor networks, cellular networks, storage of cylindrical drums in containers or their storage in open areas, bottles or cans of packaging in the smallest box, planting trees in a given area so as to maximize the density of the forest and the distance between trees, and so on [50]. With a large number of circular objects to pack, the optimal solution is very difficult to find. An optimal solution can be turned, reflected or reorganized circular objects; therefore, the number of equivalent optimal solutions increases as the number of circular objects increases from 2% to 4%. In addition, one or more circular objects can be moved slightly without affecting the optimal solution. In fact, there is a continuum of optimal solutions [50], [27], [2]. To do this, we want to use the cut by 3 circular or spherical objects. Finally, there is the question of the accuracy and precision of the calculations. Obviously, the packaging of circular objects poses optimization problems, but their classification into continuous

or discrete problems is unclear. Indeed, the positions of the circular objects are continued while the structure of an optimal pattern has a discrete nature [40]. The recent trend is to design technical solutions that deal "to some extent" with these two aspects simultaneously. Many packaging variants of planar circular objects have been formulated as non-convex optimization problems; with the hope that these formulations, once solved, use the available non-linear programming (NLP) solvers give approximate solutions of high quality [37]. However, most NLP solvers fail to identify the overall optimum. In fact, problems with circular objects are very difficult optimization problems, have a large number of variables and a local minimum. They must therefore be treated with algorithms combining local searches and heuristic procedures in order to explore the research space more widely [32],[19]. The recent trend has been to adopt approximation methods (heuristics) that combine global (heuristic) search methods and local exhaustive (exact) search methods for local minimums or their approximations [48]. This paper provides an in-depth study of the recent literature on circular object packing. The literature can be classified into two categories: work seeking optimal solutions or demonstrating the optimality of certain models, and work seeking to improve previously published results [31]. Regardless of their ranking, these documents look for the best solutions, whatever their margin of improvement or their cost of calculation [31]. In this state of the art, we will cite the works of literature, classifying the area of interest in two categories: the first category where the region of interest is a regular polygon ?, while the second category where the region of interest is circular ?. It is difficult to come up with a solution to find better coverage for a k-covered and k-connected sensor network deployment because coverage is an NP-hard problem. The space of all feasible solutions can be large enough.

2.3 Coverage by circles paving

The coverage of the circle is inspired and adopted to solve the problem of packing the circle. The coverage problem is to locate congruent geometric objects in a metric space so that its given surface lies entirely in their union. Usually, scientists treat circles covering the problem (CCP) in special cases, when the area covered is a square, circle, rectangle or regular triangle. In fact, there are two common directions for studying this problem. The first is to improve coverage without any constraint of optimality; the second is to check the optimality of a particular circle configuration. Some analytical results of the literature are detailed in [26]. The first computer results, using computer algorithms for plane coatings in the plane were obtained by Zahn [60]. He divided a large circle into small cells and applied local optimization methods to cover as many cells as possible while keeping the radius of small circles constant. The computational results for a square pavement appear to have been presented for the first time by Tarnai and Gaspar, who found coatings containing up to 10 circles [51]. Algorithms to cover sets simply connected by congruent circles using the quasi-differentiability of the objective function are presented in [28], heuristics and meta-heuristics can be found in [6], Integer and continuous optimization algorithms proposed by [36], geometric methods are suggested in [55]. A method for modifying the possible directions appears in [46] where the optimal overlays are given for different $n \leq 100$. Note that the approach proposed in [36] is based on a simulated annealing algorithm using the Voronoi tessellation of the square by relation to the centers of the circles. Numerous calculation results concerning the circle recovery problem are available on the Web, for example the Erich Packaging Center website. This theoretical problem is widely used to solve practical tasks in various fields of human activity. Examples of such tasks are the location of cell towers, life-saving stations, police stations, automated teller machines, hospitals, schools [7], the design of performance monitoring systems of objects distributed by wireless sensor networks [5] etc. Note that most known results are obtained in the case where covered areas or containers are subsets of the Euclidean (2D) or Euclidean (3D) space. Figure 3 summarizes the different paving techniques to ensure coverage in WSN.

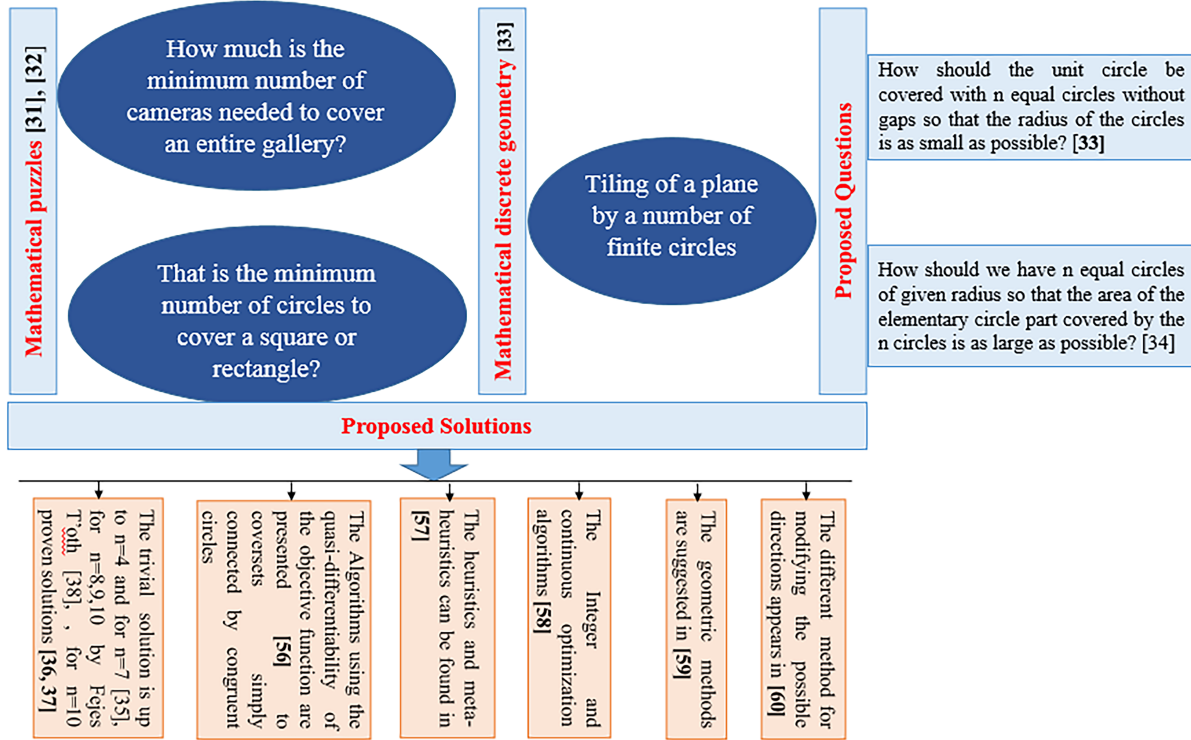


Figure 3: The Tiling strategies for coverage in WSN

3 Problem definition

The problem here is to try to lengthen the coverage area of the sensor network without disturbing the property as the area of interest is subdivided into 3-sub-areas. Each sub-area provided by a base station. Activating 3 sensor nodes for each sub-area in predetermined positions guarantees connectivity Figure 4. To do this, we will use the paving technique.

3.1 Formulation of the problem

Let X be a given metric space, a $D \subset X$ a bounded field, compact sets $B_k \in D/k = 1..m$, and n of overlapping circles $C_i(O_i)$ of centers $O_i = (x_i, y_i), i = 1..n$.

Let $0 < f(x, y) \leq \beta$ be a continuous function that gives a meaning to the instantaneous speed of motion at any point of D and $B = \bigcup_{k=1}^m B_k$ [62]

3.2 Definition of a closed multiply-connected set M

$$M = cl\left(\frac{D}{B}\right) \subset X \subseteq (R^2) \tag{1}$$

As cl is the closing operator (equation 1).

The distance in the space X is determined as follows (equation 2):

$$\rho(a, b) = \min_{\Gamma G(a, b)} \oint_{\Gamma} \frac{d\Gamma}{f(x, y)} \tag{2}$$

where $G(a, b)$ is the set of all the continuous curves which belong to X and connect the points a and b . In other words, the shortest route between two points is a curve that requires as little time as possible.

- **Definition 1**

$$P_n^*(r) = \bigcup_{i=1..n} C_i O_i \quad (3)$$

A covering of multiply-connected set M with the radius r if: $\forall i = 1..n, O_i \in M$ and $P_n(r) \cap D = D$ is represented by (Equation 3).

- **Definition 2** $P_n^*(r)$ is an optimal covering of M if r is minimal. Thus, it is necessary to find a partition of D on n segments $D_i, i = 1..n$, and the location of the circles centers $O_i^* \in M$, which provide minimum for equation (4)

$$R_* = \max_{i=1..n} \rho(O_i, dD_i) \quad (4)$$

where $\rho(O_i, dD_i)$ is the distance from the circle center O_i to the closed boundary of the corresponding segment D_i (equation 5).

$$\rho(O_i, dD_i) = \min_{x \in dD_i} \rho(O_i, x) \quad (5)$$

This protocol is realized under the following hypotheses:

3.3 Hypotheses

We assume that we have a 2D area of interest without obstacle A (width, height), N sensor nodes with a detection radius R_s and a communication radius $R_c = 2 * R_s$. All sensor nodes are mobile, and their mobility will be for one time until the total depletion of their batteries. We assume that the three proposed base stations are located at the centers. Their only main task is the communication of data collected by the appropriate sensor nodes.

We want to maximize the total covered area of the sensor network under the following constraints: All sensor nodes in a subarea can communicate with each other 3 base stations to the centers these circles covering the square area of interest (connectivity), such that the area of interest must be covered by three base stations for data communication (3-Subareas, 3-Base Stations), as shown in Figure 4. The detection and communication ranges of all sensor nodes are identical. The 3 base stations do not suffer from energy constraints or connectivity. For this study, assumptions are taken into consideration, such as:

3.4 Assumptions

To apply our strategy of area coverage in large areas, which consists of using semi-random deployment by mixing between the two types (random and deterministic), we took into consideration the following assumptions:

1. The subdivision of AoI under sub-areas, in order to cover the vast areas well, to achieve an equitable distribution of the nodes in the geographical area, to control connectivity, and to conserve energy consumption in the network.
2. The deployment of a large number of sensor nodes on each sub-area (sub-zone) by a device on a height a little lower and in calm circumstances around each of the sub-zones z_i , in order to guarantee a good distribution in the network.
3. The nodes used in our study are of two types; the first type is made to have a storage memory more than the ordinal nodes, the second type has the same characteristics of the first but a little heavy than the nodes of the first type.

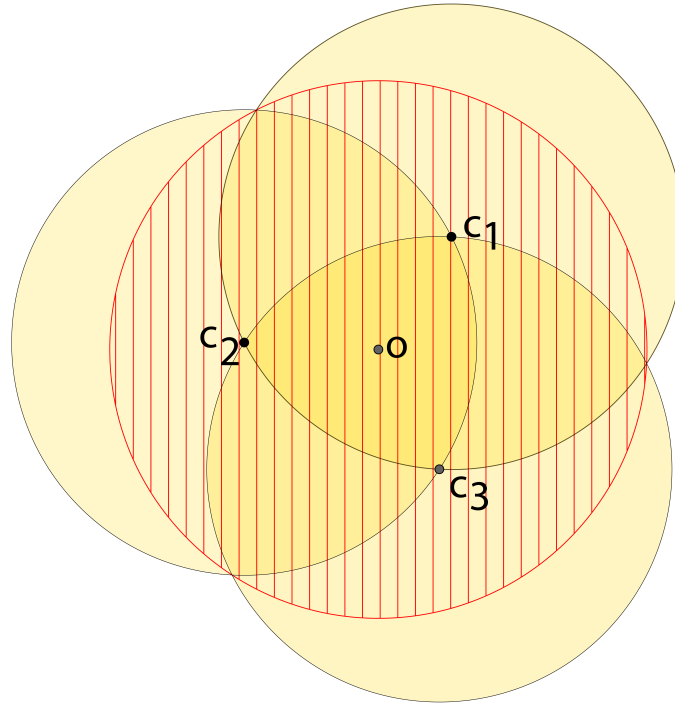


Figure 4: Example of Area of Interest (3-Subareas, 3-Base Stations)

4 The proposed approach

This part consists of showing the motivation and objectives behind the proposed strategy, the justification of tiling by only 3 circles, the mathematical formulation of the problem and ends with the demonstration of the strategy.

4.1 Motivation and Objectives

- The tiling of a circle by a minimal set of circles (3 circles) is proven by mathematical theories as a better tessellation. This paving helps to cover a square area (a circular area surrounded by a square) by up to 3 circles.
- The proposed strategy is to show that the minimal tiling of a square or circular area is 3-circles, and not 2-circles.
- To avoid using more base stations for each subarea, the 3 points build a triangle, and every 3 points in each subarea build 3 triangles.
- The proposed strategy in the first step is to subdivide the area of interest (considered a square zone) in 3 sub-circular areas. That is to say to pave a square area by 3 circles, like a first step.
- The second step is to mobilize the sensor nodes each 3 times maximum to ensure a fair connectivity and full coverage.
- The nodes which have moved 3 times (the number of displacements = 3), remain in last position until the total exhaustion of their energy reserves.

The tiling of a square area by three circular areas is shown in the Figure 5. (b), and in Figure 5.(c).

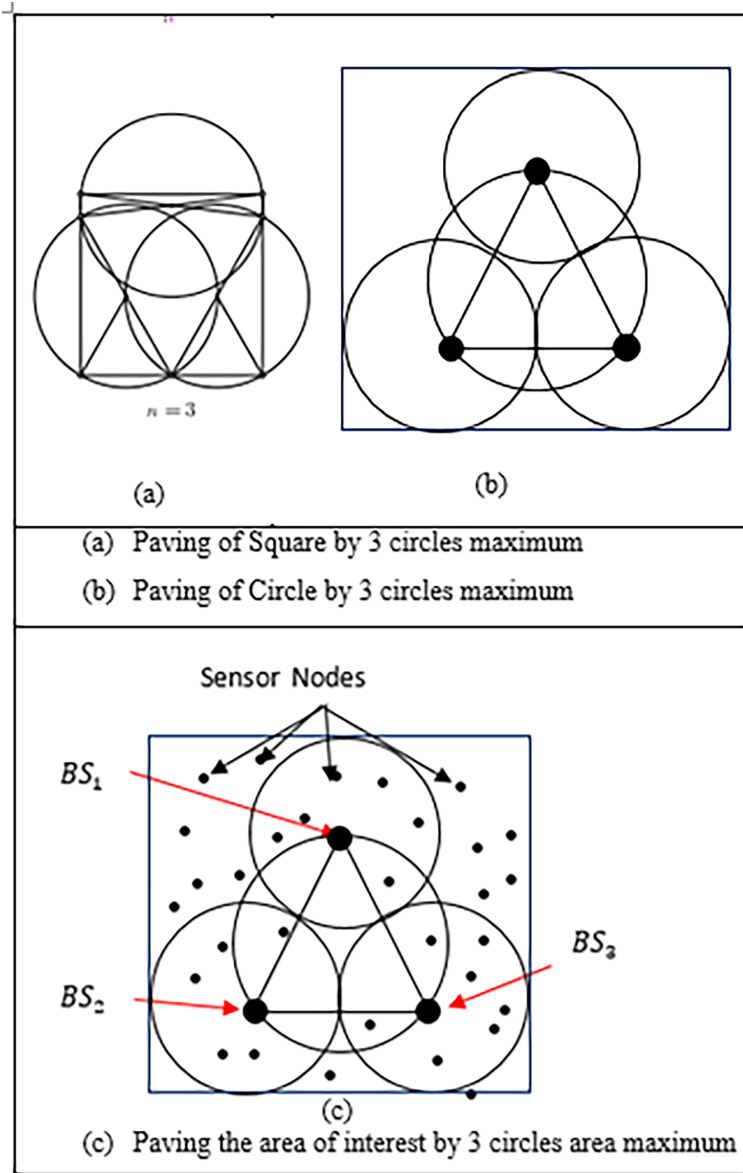


Figure 5: The Paving by circles elements

4.2 Description of the proposed approach to solve the problem

We will implement tiling by k-circles (3-circles) to find an optimal solution to the coverage problem in this part. The main lines of the proposed algorithm (Algorithm 1) are implemented as follows :

4.3 Choice of paving a square area with 3 circles

For tiling a circle by $n = 3$ isocircles, the centers of the three circles C_1 , C_2 and C_3 are located on the vertices of an equilateral triangle of side 1 unit. They cover the circle of center O and radius $r = 2/3 = 1.1547$ (Figure 5.(a)). If the radius r of the equal circles is in the interval (r_{maxn}, r_{minn}) , the equal ranks

Algorithm 1 Circles Paving based Area coverage pseudo algorithm

-
- 1: **Step 1: Deployment and clustering**
 - 2: → The sensor nodes are deployed randomly on the area of interest;
 - 3: → The three bases station are deployed in the three-centers of circular of subarea witch covering the AoI;
 - 4: → Each subarea considered in our strategy as a physical cluster;
 - 5: → The base substation (BS_i , $i = 1, 2, 3$) sends to each sensor node, the base station number corresponds;
 - 6: → Each BS_i , $i = 1, 2, 3$ send to each sensor node in the appropriate area the geographical coordinates of the three centers of the covering circles of the sub-area of interest: $C_i(x_{(c_{ik})}, y_{(c_{ik})}, z_{(c_{ik})}) / C_{i1}(x_{(c_{i1})}, y_{(c_{i1})}, z_{(c_{i1})}), C_{i2}(x_{(c_{i2})}, y_{(c_{i2})}, z_{(c_{i2})})$ et $C_{i3}(x_{(c_{i3})}, y_{(c_{i3})}, z_{(c_{i3})})$;
 - 7: **Loop:** Repeat until the network sensor nodes are exhausted;
 - 8: **Step 2: Positioning**
 - 9: → Each base station randomly activates three sensor nodes in the first time and sends them their three geographical positions respectively C_{i1} , C_{i2} , C_{i3} respectively;
 - 10: → The first node moves to the geographical point C_{i1} , the second node moves to the geographical point C_{i2} , and the third node to the geographical point C_{i3} ;
 - 11: **Step 3: Coverage**
 - 12: → Each sub-area is covered by 3 nodes, building the 3 circles that cover a circle (which surrounds a square);
 - 13: → The choice of three nodes is to guarantee connectivity and perfect coverage;
 - 14: → Data collected by any node is reported to BS_i . in turn, the secondary base station communicates this data to the main base station;
 - 15: → After a period, the three active sensor nodes simultaneously send their energy reserves;
 - 16: **Step 4: Renewal**
 - 17: → The exhausted nodes alert the corresponding base station of its situation;
 - 18: → In this case, BS_i randomly activates another sensor node, and do the same positioning and coverage procedure;
 - 19: **Return:**
 - 20: → Optimal coverage, guaranteed connectivity with minimal power consumption;
-

can then, at the optimum arrangement, intersect the unit circle and cross each other. Circle unit will have a part discovered by circles. The sum of the areas of the n equal circles is given, and therefore minimizing the uncovered area is the same problem as minimizing the sum of two areas: the area of the unit circle that is multiplied by the equal circles, the area of the parts of equal circles that lie outside the unit circle. If k of equal circles covers a given area, this area has a surplus of coverage $(k - 1)$. For the sake of simplicity, it is assumed that, in the arrangements studied, no more than three circles cover the same field, that is, the equal circles may have at most a triple overlap.

4.4 Mathematical proof

In the case where three equal circles intersect, the magnitude of the traction force Q_i occurring in the i^{th} edge of the triangle element is defined as the partial derivative of the common area A with respect to the length of the edge: (The force exerted on the edge is a pulling force because, with the compressive forces exerted on the strut elements along the three edges, an excess of coverage of the common area has already been subtracted three times, so that this common area has been discovered). Intersection of two equal circles, the strength of the strut is calculated as the length of rope. If the common area of these

two equal circles is intersected by a third circle, the force exerted on the strut will be equal to the length of the edge of the Voronoï cell, corresponding to this strut. Therefore, the respective edge force of the triangle element is equal to the difference between the string length and the edge length of the cell, as shown in Figure 6.

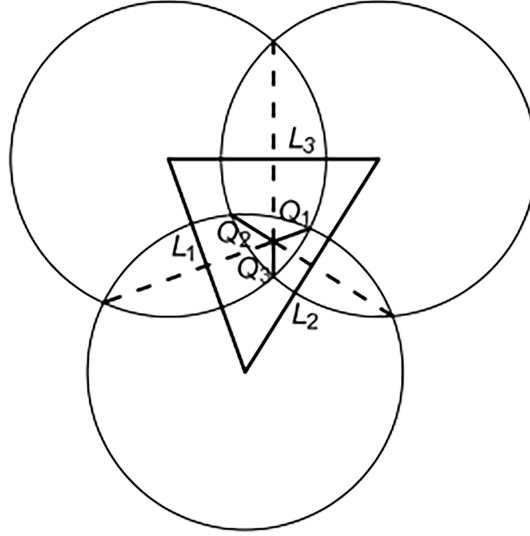


Figure 6: Edge force of the triangle element

As a result, if for the three edges ($i = 1, 2, 3$) the inequality $ai + bi < 0$, the triangle element is passive. We note that it may happen that only one of the forces on board is different from zero. This happens when the common part of two circles is contained by the third circle, that is, the common part of the three circles is the union of two equal circle segments. In this case, the force exerted on the edge is exactly the opposite of the force exerted on the strut by the strut element along this edge. In such a case, the system of internal forces and stiffness matrix of the structure remain unchanged if the triangular element and the strut element along the aforementioned edge are considered passive.

5 Evaluation and Simulation

5.1 Statistical evaluation: Asymptotic Confidence interval (ACI) calculation

Confidence interval estimation is a good alternative to significance testing in most cases from a statistical point of view. The confidence intervals provide a different interpretation of the results obtained. So, the confidence interval characterizes a statistical parameter. So, in our study, the confidence interval of the average percentage of measurements coverage (2D, 3D) allows to establish with a given level of confidence an interval around the observed average percentage in which the true average is likely to be found. It is expressed as $\text{Mean} \pm k_{ACI}$ (standard deviation of the mean), where k_{ACI} is the coverage factor of the confidence interval. Let f be the frequency of a character (in sample size n). In our case the character is "coverage". Asymptotic Confidence Interval proportion is defined by the relation (6) :

$$I_C = \left[f - u_\alpha \frac{\sqrt{f(1-f)}}{\sqrt{n}}, f + u_\alpha \frac{\sqrt{f(1-f)}}{\sqrt{n}} \right] \quad (6)$$

After the application of *ACI*, we can deduce that the proportion of sub-areas that are covered belong to this *ACI* interval with a confidence level equal to 95%. Where u_α is defined by the relation (7) :

$$u_\alpha = \Psi^{-1}(1 - \alpha/2) \quad (7)$$

In our case study:

- The used nodes number is 30.
- The confidence interval in this study is based on a sample, representing a series of 30 independent measures of coverage, in particular to estimate the average percentage of effective coverage.
- $95\% = 1 - \alpha$ then $\alpha = 0.05$ and $1 - \alpha/2 = 0.975$
In conclusion, $\Psi^{-1}(0.975) = 1.96$.
Frequently used values of Ψ^{-1} are stored in the following table (Table 4):

Table 4: Frequently values of Ψ^{-1}

	90%	95%	98%	99%
Ψ^{-1}	1.64	1.96	2.05	2.58

- The obtained results are recorded in the Table 5. Therefore, the average coverage reliability at 95% is between [0.8585, 0.8703].

5.2 Simulation

We compared the proposed strategy with some other well-known protocols that study deployment, clustering and the Active / Passive scheduling process. We compared our PPOCEC strategy with three strategies, namely, Semi-Random Deployment Protocol (SRDP) [16], hybrid strategy of three clustering models, namely, Classical Clustering, Connected Dominant Set and Voronoi Diagram (VD_C_CDS) [11] and Voronoi based Area Coverage (VAC) [44].

5.2.1 Strengths and weaknesses of simulation protocols

- **SRDP:** This is our proposed strategy, it is a clustering strategy based on deployment strategies. It aims to optimize the area coverage, connectivity and energy consumption in WSN. This strategy proposed a third type of hybrid deployment that combines the advantages and minimizes the disadvantage of the application of the random deployment and deterministic deployment in heterogeneous network. This type has two steps; the pre-configuration step and the scheduling process step. Performance evaluation of the proposed SRDP strategy is done using MATLAB tool. It has been run on a Dell LATITUDE E5410 Intel (R) Core (TM) i5 CPU M560, 2.67ghz, RAM=8Gb.
- The VD_C_CDS this strategy is proposed to make a fully distributed decision to select clusters and the minimum number of nodes to activate. The execution of this protocol starts with clustering such that the first n_i nodes that are randomly activated send a message to the neighboring nodes, the one that received the message first becomes a member of the cluster that contains n_i .

This strategy is proposed to make a fully distributed decision to select clusters and the minimum number of nodes to be activated. The execution of this protocol starts with a clustering phase. As a result, the first n_i nodes are randomly activated and broadcast a message to the neighboring nodes. The other ones that received the message become a member of the concerned clusters.

event network simulator.

5.2.2 Evaluation

The data used in this evaluation are recorded in Table 6. We compared our strategy PPOCEC by three (3) clustering strategies that use deployment, clustering and the Active / Passive scheduling process in this study: SRDP in terms of energy consumption. SRDP, VD_C_CDS and VAC in terms of coverage rate. We took the same simulation data cited in [16]. The deployment of a number of sensor nodes between 5 and 40; and an area of interest of dimensions $50m \times 50m$; to measure the coverage ratio, and thus to calculate the energy consumed during the 1000 registered primary time units.

To apply our protocol, we divide the area of interest into L-side of sub-areas such that: $L = R_c/1.42 = 15/1.42 = 10.56m$ ie: *25sub – areas*.

Table 6: Common data used in the simulations.

Configurations	Value
Area of Interest (AoI)	$50m \times 50m$
Communication Radius (R_c)	$15m$
Sensing Radius (R_s)	$15m$
Deployed Sensor Nodes Number	<i>5to40</i>
Time Units	<i>1000Seconds</i>

The results of simulation are shown in Figure 7 and Figure 8.

5.3 Results analysis

We simulated the total energy consumption data, the coverage ratio in the network for both protocols, the Partial Paving a Circle by Equal Circles (PPOCEC), the Voronoï based Area Coverage (VAC), The Clustering, Connected Dominating Set (C_CDS_D) and the Semi-Random Deployment Protocol (SRDP).

5.3.1 In terms of coverage

The PPOCEC protocol reaches a ratio of 1 if the number of deployed nodes is equal to 10. On the other hand, the coverage ratio when applying SRDP reaches 1 if the number of nodes arrives at 25 nodes as shown in Figure 7; which shows a gain on the minimum sensor nodes number that can be deployed to achieve a perfect coverage of the area of interest. In addition, the network keeps a perfect coverage lifetime better than when applying the protocol SRDP, VAC and C_CDS_VD. This is due to the fact that SRDP, C_CDS_VD and VAC use the clustering concept with the activation of a single Cluster-Head by a Cluster to act as monitoring and communication node of the data captured on the base station. This is explained by the fact that the PPOCEC strategy uses only three movements of each sensor node in the network and uses (3) three base stations to receive the collected data. This is explained by the fact that the PPOCEC strategy uses a perfect tiling strategy of the area of interest by only three (3) circles. This tiling is proven by the mathematical literature to be the best tiling solution by using a number of $n = 2$ or $n > 3$ circles. This shows the effectiveness of PPOCEC versus SRDP, C_CDS_VD and VAC strategies.

5.3.2 In terms of energy consumption

Figure 8 shows an equitable energy consumption in the network when applying the PPOCEC and SRDP strategies, modeled and proved respectively by the linear functions $Y = -0.3X + 500$ and $Y = 500 - 0.5X$.

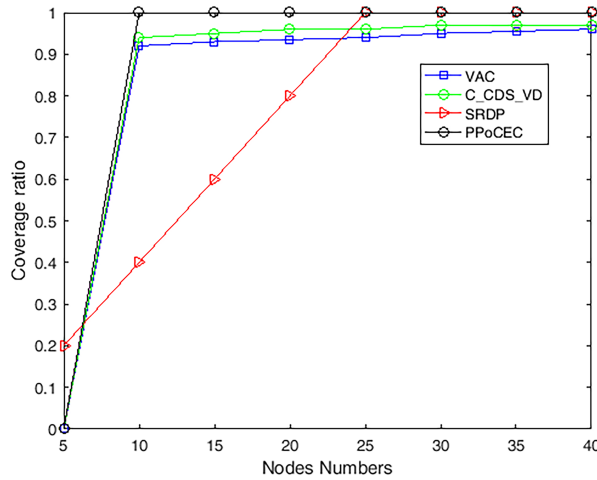


Figure 7: The Comparison of Coverage Ratio with PPOCEC and SRDP protocols

On the other hand, the energy of the network with SRDP depletes at 1000 unit time, but on the same time, PPOCEC arrived at 200 energy units. This means that a slight gain in energy consumption is achieved in the network. As a result, Figure 8 shows that the network is exhausted during the 1000 first time units if by applying the SRDP; On the other hand, the network consumes 300 units from their total energy in this time. This is explained by the fact that the PPOCEC strategy uses only three movements of each sensor node in the network and uses (3) three base stations to receive the collected data. This shows the effectiveness of PPOCEC versus SRDP.

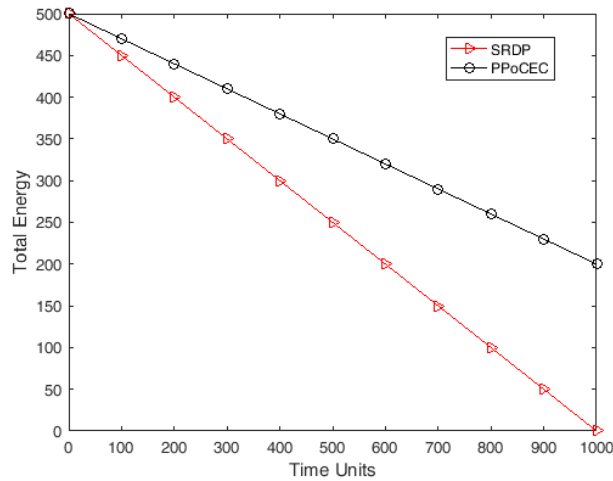


Figure 8: The Comparison of total energy consumption with PPOCEC and SRDP protocols

6 Conclusions

One of the best discrete geometric techniques is paving with a minimal number of circles. In this paper, with , we have shown a model based on 3-circles paving suitable for providing conjectural solutions

to the problem of coverage in wireless sensor networks. In this solution, we use the circles with equal radii, with a number equal to three to not increase the arithmetic complexity, the number of movements made by the sensor nodes, and the number of base stations. We do not produce the equilibrium paths to be followed by the sensor nodes to reach the appropriate point, but only the equilibrium position points, that is equilibrium positions for equal radius values. to ensure minimal coverage in the presence of connectivity. The application of our strategy marks the insufficiency in generalization for any type of coverage in WSN; area coverage, barriers coverage or target coverage, dynamic or static, periodic or permanent. For these reasons, in future work, we will apply this strategy for heterogeneous sensor networks, hybrid sensor networks, and we will address the trinomial problem (Coverage, Connectivity, Energy-efficiency) for either 2D and 3D Wireless Sensor Networks (WSN). Therefore, in large areas of interest.

References

- [1] Y. E. Ahmed, K. H. Adjallah, R. S. I. Kacem, and S. F. Babiker. Nds based methods for maximizing the lifespan of randomly deployed wireless sensor networks for infrastructures monitoring. *Computers & Industrial Engineering*, 115:17–25, 2018.
- [2] H. Akeb, M. Hifi, and R. M'Hallah. Beam search and non-linear programming tools for the circular packing problem. *International Journal of Mathematics in Operational Research*, 1(4), 2009.
- [3] F. Al-Turjman. Cognitive-node architecture and a deployment strategy for the future wsns. *Springer Mobile Networks and Applications*, 23(4):940–955, 2017.
- [4] S. Arjunan and P. Sujatha. A survey on unequal clustering protocols in wireless sensor networks. *Journal of King Saud University - Computer and Information Sciences*, 31(7):304–317, 2020.
- [5] S. Astrakov and A. Erzin. Efficient band monitoring with sensors outer positioning. *Optimizat. J. Math. Program and Operat. Res*, 62(10):1367–1378, 2013.
- [6] Z. Azimi, P. Toth, , and L. Galli. An electromagnetism metaheuristic for the unicast set covering problem. *European Journal of Operational Research*, 205(2):290–300, 2010.
- [7] B. Bànhelyi, E. Palatinus, and L. B.L. Optimal circle covering problems and their applications. *Central European Journal of Operations Research*, 23:815–832, 2015.
- [8] K. Bezdek. *Beiträge Algebra Geom*, chapter Uber einig Kreis überdeckungen, pages 7–13. Springer, 1983.
- [9] A. Boualem. *Stratégies d'amélioration de la couverture dans les réseaux de capteurs sans fil*. Phd thesis, High National School of Computer Science, 2021.
- [10] A. Boualem, M. Ayaida, Y. Dahmani, C. De Runz, and A. Maatoug. New dijkstra front-back algorithm for data routing-scheduling via efficient-energy area coverage in wireless sensor network. In *Proc of the 15th International Wireless Communications and Mobile Computing Conference, Tangier, Morocco*, pages 1971–1976. IEEE, 24-28 June 2019.
- [11] A. Boualem, M. Ayaida, and C. De Runz. Hybrid model approach for wireless sensor networks coverage improvement. In *Proc. of the 8th International Conference on Wireless Networks and Mobile Communications (WINCOM2020), Reims, France*, pages 1–6. IEEE, October 27-28 2020.
- [12] A. Boualem, M. Ayaida, C. De Runz, and Y. Dahmani. An evidential approach for area coverage in mobile wireless sensor networks. *International Journal of Fuzzy System Applications (IJFSA)*, 10(3):30–54, July-September 2021.
- [13] A. Boualem, Y. Dahmani, M. Ayaida, and C. De Runz. A new fuzzy/evidential approach to address the area coverage problem in mobile wireless sensor networks. In *Proc. of the 34th ACM/SIGAPP Symposium on Applied Computing (SAC'19), Limassol, Cyprus*, pages 2430–2433. ACM, New York, NY, USA, April 8–12 2019.
- [14] A. Boualem, Y. Dahmani, C. De Runz, and M. Ayaida. Spiderweb strategy: application for area coverage with mobile sensor nodes in 3d wireless sensor network. *International Journal of Sensor Networks (IJSNet)*, 29(2):121–133, February 2019.

- [15] A. Boualem, Y. Dahmani, and A. Maatoug. Energetic sleep- scheduling via probabilistic interference k-barrier coverage with truth-table technique in sensor network. In *Proc. of the International Conference on Mathematics and Information Technology (ICMIT)*, Adrar, Algeria, pages 255–262. IEEE, December 4-5 2017.
- [16] A. Boualem, Y. Dahmani, A. Maatoug, and C. De Runz. Area coverage optimization in wireless sensor network by semi-random deployment. In *Proc of the 7th International Conference on Sensor Networks (SENSORNETS 2018)*, Funchal, Madeira, Portugal, pages 85–90. ACM, January 22-24 2018.
- [17] P. Brass, Moser, W., and J. Pach. Research problems in discrete geometry. *Springer*, 2005.
- [18] T. L. Brown. *Optimization In Wireless Video Sensor Networks For 3D Indoor Monitoring Video Sensor Networks*. Phd thesis, University of Mississippi, Electronic Theses and Dissertations, Graduate School, 2017.
- [19] A. Cassioli, M. Locatelli, and F. Schoen. Dissimilarity measures for population-based global optimization algorithms. *Computational Optimization and Applications*, 2008.
- [20] J.-M. Etancelin, A. Fabbri, F. Guinand, and MartinRosalie. Dacyclem: a decentralized algorithm for maximizing coverage and lifetime in a mobile wireless sensor network. *Ad Hoc Networks*, Elsevier, 87:174–187, November 2019.
- [21] G. Fejes Tóth. Thinnest covering of a circle by eight, nine, or ten congruent circles. In J. E. Goodman, J. Pach, and E. Welzl, editors, *Mathematical Sciences Research Institute Publications, Combinatorial and Computational Geometry*, Cambridge University Press, volume 52, pages 361–376, 2005.
- [22] L. Fejes Tóth. *Pergamon*, chapter Regular Figures. Macmillan, 1964.
- [23] L. Fejes Tóth. Perfect distribution of points on a sphere. *Periodica Math. Hungar*, pages 25–33, 1971.
- [24] P. Fowler and T. Tarnai. Transition from spherical circle packing to covering: geometrical analogues of chemical isomerization. In *Proceeding of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 452, 452, pages 2043–2064, 1996.
- [25] H. Gupta, Z. Zhou, S. R. Das, and Q. Gu. Connected sensor cover: Self-organization of sensor networks for efficient query execution. In *MobiHoc '03 Proceedings of the 4th ACM international symposium on Mobile ad hoc networking & computing*, Annapolis, Maryland, USA, pages 189–200. ACM, June 01-03 2003.
- [26] A. Heppes. Covering a planar domain with sets of small diameter. *Periodica Mathematica Hungarica*, 2006.
- [27] M. Hifi and R. MĤallah. A dynamic adaptive local search algorithm for the circular packing problem. *European Journal of Operational Research*, 183(3):1280–1294, 2007.
- [28] H. Jandl and K. Wieder. A continuous set covering problem as a quasi differentiale optimization problem. *Optimizat. J. Math. Program and Operat. Res*, 19(6):781–802, 1988.
- [29] B. K. *Optimal covering of circles*. PhD thesis, Budapest in Hungarian, 1979.
- [30] V. Kumar, S. Arya, and H. Ramesh. Hardness of set cover with intersection. In 27th, *Int. Colloquium on Automata, Languages and Programming (ICALP)*, volume 1853 of 0302-9743, pages 624–635. ACM, 2000.
- [31] M. Locatelli and U. Raber. Packing equal circles in a square: a deterministic global optimization approach. *Discrete Applied Mathematics*, 122(1-3):139–166, 2002.
- [32] M. Locatelli and F. Schoen. Efficient algorithms for large scale global optimization: Lennard-jones clusters. *Computational Optimization and Applications*, 26(2):173–190, 2003.
- [33] H. Mahboubi and A. G. Aghdam. Distributed deployment algorithms for coverage improvement in a network of wireless mobile sensors: Relocation by virtual force. *IEEE Transactions on Control of Network Systems*, 4(4):736–748, 2017.
- [34] I. Mahjri, A. Dhraief, A. Belghith, K. Drira, and H. Mathkour. A gps-less framework for localization and coverage maintenance in wireless sensor networks. *SII Transactions on Internet and Information Systems*, 10(1):96–116, 2016.
- [35] H. Melissen. *Packing and covering with circles*. Phd thesis, University of Utrecht, 1997.
- [36] J. Melissen and P. Schuur. Covering a rectangle with six and seven circles. *Discrete Applied Mathematics*, 99:149–156, 2000.
- [37] N. Mladenovi, F. Plastria, and D. Urosevic. Reformulation descent applied to circle packing problems. *Computers & Operations Research*, 32(9):2419–2434, 2005.

- [38] S. M. Nejah Nasri and T. Val. 3d node deployment strategies prediction in wireless sensors network. *International Journal of Electronics*, november 2019.
- [39] K. J. Nurmela. Covering a circle by congruent circular discs. Technical report, Preprint, Department of Computer Science and Engineering, Helsinki University of Technology, 1998.
- [40] P. R. J. Ostergard. *Book review," Computers & Operations Research*, volume 36, page 276–278. Hindawi Publishing Corporation, 2008.
- [41] J.-S. Pan, Q.-W. Chai, S.-C. Chu, and N. Wu. 3-d terrain node coverage of wireless sensor network using enhanced black hole algorithm. *Sensors*, 20(8):1–12, 2020.
- [42] R. Priyadarshi, B. Gupta, and A. Anurag. Deployment techniques in wireless sensor networks: a survey, classification, challenges, and future research issues. *Journal of Supercomputing*, 76:7333–7373, 2020.
- [43] F. P.W. and T. T. Transition from circle packing to covering on a sphere: the odd case of 13 circles. In *Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci. 452*, volume 455 of 455, pages 4131–4143. The Royal Society, December 08 1999.
- [44] M. A. R. S. Sharmin, F. N. Nur, and M. M. Rahman. Area coverage for clustered directional sensor networks using voronoi diagram. In *Proc of the IEEE International WIE Conference on Electrical and Computer Engineering (WIECON-ECE), BUET, Dhaka, Bangladesh*, pages 370–373. IEEE, 19-20 December 2015.
- [45] T. Shu, K. B. Dsouza, V. Bhargava, and C. de Silva. Using geometric centroid of voronoi diagram for coverage and lifetime optimization in mobile wireless sensor networks. In *Proc of the IEEE Canadian Conference of Electrical and Computer Engineering (CCECE)*, pages 1–5. IEEE, May 2019.
- [46] Y. Stoyan and V. Patsuk. Covering a compact polygonal set by identical circles. *Computational Optimization and Applications*, 46:75–92, 2010.
- [47] Y. G. Stoyan. Mathematical methods for geometric design. In *Proceedings of PROLAMAT82, Leningrad, USSR, North-Holland, Amsterdam, The Netherlands*, page 67–86. ACM, May 2003.
- [48] Y. G. Stoyan and G. Yas'kov. A mathematical model and a solution method for the problem of placing various-sized circles into a strip. *European Journal of Operational Research*, 156(3):590–600, 2004.
- [49] W. Sun, M. Tang, L. Zhang, Z. Huo, and L. Shu. A survey of using swarm intelligence algorithms in iot. *Sensors*, 20(5), 4 January 2020.
- [50] P. G. Szabo, M. C. Markót, T. Csendes, E. Specht, L. G. Casado, and I. Garcia. New approaches to circle packing in a square: With program codes. In *Springer Optimization and Its Applications, springer, New York, NY, USA*. ACM, January 2007.
- [51] T. Tarnai and Z. Gaspar. Covering a square by equal circles. *Elem. Math*, 1995.
- [52] K. Tarnaris, I. Preka, D. Kandris, and A. Alexandridis. Anchor node path planning for localization in wireless sensor networks. *Electronics*, 9:1–24, 2020.
- [53] M. Tosun, U. C. Çabuk, V. K. Akram, and O. Dagdeviren. On connectivity-aware distributed mobility models for area coverage in drone networks. In *Proc of the International Conference on Artificial Intelligence and Applied Mathematics in Engineering (ICAIAE 2020), Antalya, Turkey*, pages 1–12, 2020.
- [54] A. Tripathi, H. P. Gupta, T. Dutta, R. Mishra, K. K. Shukla, and S. Jit. Coverage and connectivity in wsns: A survey, research issues and challenges. *Journal of Supercomputing*, 6:26971–26992, 2018.
- [55] V. Ushakov and P. Lebedev. *Algorithms for the construction of an optimal cover for sets in three-dimensional euclidean space*, volume 293. Proc. of the Steklov Institute of Mathematics, 2016.
- [56] Z. Wadud, N. Javaid, M. A. Khan, N. Alrajeh, M. S. Alabed, and N. Guizani. Lifetime maximization via hole alleviation in iot enabling heterogeneous wireless sensor networks. *Sensors (Basel)*, 17(7), 2017.
- [57] J. Xiao, G. Wang, Y. Zhang, and L. Cheng. A distributed multi-agent dynamic area coverage algorithm based on reinforcement learning. *IEEE Access*, 8:33511–33521, January 2020.
- [58] J. Xie, D. Wei, S. Huang, and X. Bu. A sensor deployment approach using improved virtual force algorithm based on area intensity for multisensor networks. *Mathematical Problems in Engineering*, pages 1–9, 2019.
- [59] Z. Yu, R. Tang, K. Yuan, H. Lin, X. Qian, X. Deng, and L. Shiyun. Investigation of parameter effects on virtual-spring-force algorithm for wireless-sensor-network applications. *Sensors (Basel)*, 19(14), July, 12 2019.
- [60] C. T. Zahn. Black box maximization of circular coverage. *Journal of Research of the National Bureau of*

Standards B, 66:181–216, 1962.

- [61] X. Zhao, Y. Cui, Z. Guo, and Z. Hao. An energy-efficient coverage enhancement strategy for wireless sensor networks based on a dynamic partition algorithm for cellular grids and an improved vampire bat optimizer. *Sensors (Basel)*, 20(3), February 2020.
- [62] K. H. Zsolt Gáspár, Tibor Tarnai. partial covering of a circle by equal circles. part i: The mechanical models. *Journal of Computational Geometry*, 5(1):104–125, 2014.