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## Research

# Relevance of a history-based activity for mathematics learning

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## Abstract

This paper reports on an experiment involving 108 sixth-grade French students in which the history of Chinese numeration was used in mathematics education. The aim of this study is to evaluate the relevance of using ancient number systems in mathematics education, which is common in French textbooks. In the experiment a pre-test with usual mathematics exercises and an activity with a post-test comprising history-based ones have been proposed. On this data, a factor analysis shows that the tasks based on the history of mathematics combined well with specific mathematical skills. Then, an analysis of the difficulty levels (1PL model) of the regular items and the history-based items helps identifying some key points for which the history of mathematics is relevant.

**Keywords** Mathematics education · Numeral systems · History of mathematics · Quantitative study

## 1 Introduction

In France, echoing classroom practices evidenced by activity files published over the past few decades [1, 2] and responding to an explicit institutional demand [3] to introduce mathematical history in mathematics education, ancient numeration has become standard in grade 6. Indeed, this historical content is now found in most textbooks. For instance, in 22 French sixth-grade textbooks published between 2005 and 2021, without counting documentary pages or inserts, there are 33 exercises relating to various ancient numeration systems, including Egyptian hieroglyphic numeration (10), Roman numeration (9), Babylonian cuneiform numeration (7), Chinese and Sino-Japanese numeration (5), Mayan numeration (1) and Greek numeration (1). In most cases, the exercises are nicely illustrated and highlight the civilizational era concerned. Only 8 textbooks do not have exercises with ancient number systems. However, beyond the cultural contribution of this content, the presence of ancient numeration systems in school exercises is linked to the teaching of mathematics. Indeed, these ancient systems of number representation can be associated, on the one hand, with learning objectives specific to the contemporary numbering system and to the difficulties it presents to students and, on the other hand, with the role that history plays in the teaching of mathematics. For the first aspect, didactic issues are well identified, as shown in international syntheses such as the International Commission on Mathematical Instruction (ICMI) study [4] or the French report [5] by the Centre national d'étude des systèmes scolaires (CNEC). These syntheses highlight the importance of the construction of the first numbers (1, 2, 3, ...) and of the exchange rule (the one-for-ten groupings in positional systems) as well as the roles of words (difference in construction between oral and written numeration) and the cultural context (presence or absence of a social practice). For the second aspect, many studies have been conducted for three decades on the place of history in mathematics education [6, 7]. From a methodological point of view, teaching

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approaches in can be classified [8] in three categories: the “illumination approaches”, in which learning is supplemented by historical information; the “module approaches” devoted to specific cases in history; and “history-based approaches”, in which history shapes the sequence without appearing explicitly. On the learning goals, history of mathematics [9] can serve as a replacement (for another activity), as a way to create an unusual experience for students, or as cultural enlightenment; additionally, history is seen as a tool or as a learning objective in itself. In this paper, all of these dimensions and the choices made for the experiment conducted in the sixth-grade context are specified. This student-centered research on the place of ancient numerations is in line with other works, such as that of [10] and our previous experimentation on Inca quipus [11] or Babylonian numeration [12, 13]. Thus, in this article, I first present results on the learning of numbers and on its link with addition and subtraction algorithms. Then, after a brief presentation of the ancient Chinese number system, I will return to the main reflection on the use of the history of mathematics in the classroom.

## 2 Didactical and historical frameworks

### 2.1 Numeral system and computation

As Bartolini Bussi and Sun [4] point out in the introduction to the international ICMI study mentioned above, whole numbers and the first elements of arithmetic are the foundation of mathematics learning. Taught from kindergarten onward, integers are progressively mastered in elementary school to serve as a foundation for the notions and methods taught in secondary school and beyond. However, despite the recognized importance of this first learning and the time devoted to it in class, mastering our number system is not easy for students. As noted in the CNESCO report [5], large whole numbers, i.e., those to which students can no longer associate a collection of objects, constitute a difficulty for a significant proportion of students at the end of elementary school. This difficulty is generally associated with an insufficient conceptualization of decimal numbers, or even whole numbers. Therefore, there are didactic issues, which research has explored. In particular, Houdement and Tempier [14] explain that the understanding of numbers in units, tens, hundreds, etc., was seen very early on as a lever for learning and as an indicator of students’ mastery. Whether called multi-units [15], units of numbers [16], or units of numeration [14, 17, 18], these decompositions of numbers exist in oral designation but do not appear explicitly in writing, creating a gap that some students struggle to manage. This difficulty persists throughout schooling, leading to students’ failure in numbering tasks [19–23] and calculations or problem solving [5]. A key to our decimal number system is the switch from one rank (place value) to another by grouping 10 representatives of a number unit (e.g., ten) into 1 representative of the higher number unit (e.g., hundred). This operation in base 10 is also the point of support of the algorithms of calculation in which one finds the necessary mastery of the various ranks by the learners but to which are added competences related to the course of the technique itself. Again, Houdement and Tempier [14] note that the concordance between poor number knowledge and failure in computational tasks was identified early on by a great deal of research [24]. In particular, the authors mention the differences in performance when moving from a two-digit calculation to a three-digit calculation [25]. It is also in calculations where the poor mastery of large numbers appears [26, 27] and where the role of the classes of units, thousands, millions, etc. in which the same ranks (units, tens, hundreds) are repeated is structured [19]. In France, as the official instructions for elementary school emphasize [28], addition and subtraction algorithms rely entirely on a good understanding of our numbering system. This point led CNESCO [29], at the end of the 2015 consensus conference on numbers, operations and the first learning in elementary school, to propose a recommendation entitled “Associating the learning of operating techniques with the understanding of numbers”. This recommendation states that the teaching of procedures used to perform operations in writing (such as carry over in addition) must provide opportunities for students to develop their understanding of numbers. Numeration and calculation algorithms are thus linked, and this is one dimension that the present study proposes to explore through the use of historical materials. However, before going into the details of this experiment, similar to the overview of the didactic results above, it is appropriate to examine questions related to the use of the history of mathematics in the classroom.

In didactic studies, the value of a historical look at the evolution of number systems is commonly recognized [4]. Indeed, knowledge of the developments (simple tally systems, additive systems, multiplicative-additive systems, and positional decimal systems) that preceded our contemporary system highlights both the specific characteristics and the epistemological obstacles [30–32] that our system can present to students. It is therefore not surprising, as it was pointed out in the introduction, that ancient numeration systems are pervasive in mathematics education. The goal is then epistemological in nature by changing what is (supposed to be) familiar into something unfamiliar and thus

challenging students' conventional perception of mathematical knowledge [7]. The experiment described in this article uses an ancient Chinese number system (writing numbers with rods) that is fairly well documented. This system is briefly overviewed as follows.

## 2.2 A Chinese ancient numeral system

In China, rod number writing likely appeared around the second century BC, as found in the *Suàn shù shū* [Book on calculations made with sticks] [33], which dates from this period. This number system also appeared [34] in the Wang Mang period (9–23 A.D.) and lasted at least until the beginning of the eighteenth century. It is a positional decimal system in which different ranks are presented in an alternating horizontal and vertical representation. As Anicotte [33] states in his edition, in the text of the *Book on Calculations*, numbers were written with the words of the common language. For calculations, numbers were represented with sticks on a flat surface, and calculations were carried out by manipulating these sticks, called *suànchóu*. The digits of units, hundreds, and all even powers of ten were represented by arranging sticks vertically, with a horizontal bar for digits above five. For tens, thousands, and all odd powers of ten, the sticks were arranged horizontally, with a vertical bar for digits above five. For example, I=T represents the number 126, and IT represents the number 106. In ancient China, the numbering system did not include zero [33], but a blank space could be left to avoid confusion. Zero appeared as a small circle in much later versions, as seen in the arithmetic triangle [34] published in 1303 by Zhu Shijie (1260–1320). Negative numbers were represented by black rods instead of red ones. In writings after the eleventh century, these negative numbers appear marked with a slash. This system of number representation was widely used from the thirteenth century onward for the resolution of algebraic equations. Even in the seventeenth-eighteenth century [35], Takebe Katahiro (1664–1739), in his *Hatsubi Sanpo Endan Genkai*, annotated version of Seki Takakazu's (1640?–1708) *Hatsubi Sanpo* treatise of 1687, borrows from this notation system.

In the experiment carried out with students, the decimal aspect of the system and the scriptural distinction between the even and odd digit ranks were used. The ambiguity related to the absence of an explicit zero was deliberately accentuated. In the didactic presentation of the system to the students, there was no suggestion to introduce a space between the digits when number units were missing. On the activity sheet and in the exercises, the digits were simply placed side by side; it was up to the students to ensure that the symbols were arranged vertically or horizontally. However, the numerical values were chosen with only one missing rank to make this deduction possible. A presentation of the Chinese rod number system and its epistemological issues can also be found in Bartolini Bussi and Sun [4]. The Chinese numbering system has been chosen for its decimal character and for the explicit presence of the different ranks (units, tens, hundreds, ...). Other ancient systems are quite similar and could allow the creation of quite similar activities. The Egyptian system (hieroglyphs) also presents a base 10 and different symbols for the units, tens... However, it is mainly an additive system which does not put forward the positional aspects. On the contrary, the Babylonian system (cuneiform writing) presents a strong positional dimension, but it is not based on the base 10. For this study, the Chinese system thus offered a double advantage.

## 2.3 Research question

The potential usefulness of history in mathematics education is world-wildly recognized [6, 36–41]. It represents an interesting approach because of the cultural enrichment and epistemological distance it allows. However, while relevance of how students perceive and experience mathematics is well documented [42–47], several questions remain regarding how history in mathematics education affects learning. In particular, in their international synthesis, Clark et al. [7] ask for evidence on what students really learn when mathematics history is used in the classroom. Using the example of the introducing of Chinese numeration in the sixth-grade context, this article explores how history of mathematics can be relevant in a regular school context by allowing the creation of tasks of varying difficulty levels on a specific mathematical skill. The goal, then, is to see how the history can be integrated by providing supports for learning.

In light of elements of didactic analysis, a quantitative study on more than one hundred students has been conducted. The methods used are explained in the following paragraphs about the data and their analysis, but it is useful first to present the activity administered to the students in the experiment.

### 3 Data and methods

#### 3.1 Students' worksheets and activity

The experiment was composed of three parts. The first part included ordinary mathematical exercises and served to support the evaluation of the way the history of mathematics was articulated with usual learning objectives. The second part was the historical activity. It included a presentation of the Chinese number system and examples. Finally, the third part was an evaluation of students' understanding of the Chinese number system presented during the activity. The first exercise sheet was created in a session before the activity. The activity and the second worksheet were completed during the same one-hour session. To allow analysis using item response theory (IRT), all tasks were decomposed, and the responses were dichotomized by a strict rule, with values of 1 for total success and 0 otherwise. In terms of coding, the items on the first form on ordinary mathematical tasks were denoted M1a, M1b, M2a, M2b, etc. (M prefix) depending on the type of task and the sub-questions in the exercises. Similarly, items with a historical dimension in the second exercise sheet were coded H1a, H1b, H2a, H2b, etc. (H prefix). The activity part was carried out in dialogue with the teacher. The two worksheets were performed individually by each student. We now detail the content of these materials by specifying the didactic stakes for the various items.

##### 3.1.1 First worksheet: ordinary mathematics

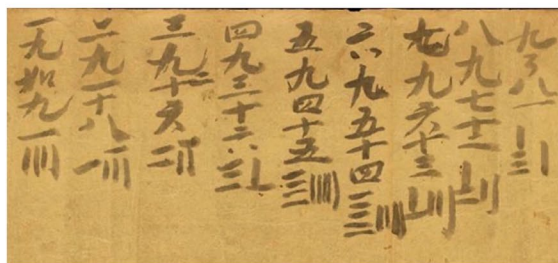
The first worksheet included six questions with a total of 21 items. The first question involved dictation of numbers. It aimed to evaluate students' mastery of the contemporary number system. In accordance with the learning issues mentioned in the previous section, the numbers were chosen to highlight possible gaps. Thus, there were six items with zeros in some ranks (2305, 10 100, 30 095) and large numbers (215 230, 6 800 000, 45 900 030). Next, questions 2 and 3 were related to calculation algorithms. In question 2, students were asked to add three decimal numbers; the point of this question was to check students' understanding of the different ranks in our decimal system. The numbers chosen increased in complexity. The first two decimal numbers ( $3.29 + 1.05$ ) had decimal parts of the same length. Then, the decimal parts had different lengths ( $66.7 + 2.42$ ). Finally, the addition of an integer and a decimal hid the presence of the decimal point ( $786 + 8.6$ ). Question 3 was built on the same model but involved subtraction ( $66.4 - 21.3$ ), ( $24.1 - 0.25$ ) and ( $2043 - 22.2$ ). The objective was to allow the analyses to verify the presence of links between numeration and calculation. The fourth question presented divisions of whole numbers by 10, 100 or 1000 to assess students' understanding of the representation of decimal numbers. Four items of increasing difficulty were presented, (3500:10), (230:100), (451:100) and (75,659:1000). These calculations were to be completed directly in line. The last two questions, 5 and 6, involved the representation of fractions and integers on a graduated axis. Question 5 included only one item in which students had to determine the fraction of the colored part of a flower ( $\frac{2}{5}$ ). Question 6 presented two types of axes. On the first axis, students had to place 25 and 58 (simple axis, graduated in units, tens explained), and for the second, they had to place  $\frac{3}{5}$  and  $\frac{12}{5}$  (axis graduated in fifths, units explained). Questions 4, 5 and 6 tested, above all, the mode of representation of numbers.

##### 3.1.2 The activity on ancient Chinese numeral system

The discovery activity part was presented on two pages. The first page explained the historical context and the functioning of the Chinese number system. The two sets of digits, those for even and those for odd ranks, were given. Two examples, in addition to those possibly proposed by the teacher during implementation in class, were written at the bottom of the sheet. These were two numbers, 167 and 107, for which the difficulty linked to the absence of the digit zero appeared. This difficulty was thus made explicit from the start, in addition to the alternation of symbols between the digit ranks. On the second page, reflecting what is widely found in school textbooks using an old numeration system, students had to switch from one form of writing to another [46] in several examples. Initially, this involved switching from contemporary decimal writing to the ancient Chinese system with increasingly larger numbers (12, 33, 46, 332, 467, and 5678). Reverse encoding was then proposed for the numbers 51, 81, and 457. This transition from the Chinese system to our numbering system was an opportunity to return to the absence of zero with two

**Fig. 1** Part of the activity worksheet. Translation: The manuscript below is a mathematical text from the seventh century. The last line gives a sequence of numbers. Write this list of numbers with the French system. The numbers are: Do you recognize this sequence of numbers?

Le manuscrit ci-dessous est un texte mathématique du 7<sup>ème</sup> siècle. La dernière ligne donne une suite de nombres. Écris cette liste de nombres avec le système français.



Source : British Museum (Or.8210) International Dunhuang Project ([idp.bnf.fr](http://idp.bnf.fr))

|                                       |  |
|---------------------------------------|--|
| Les nombres sont :                    |  |
| Reconnais-tu cette suite de nombres ? |  |

numbers, 40 and 409. To avoid ambiguity, it was clearly stated that these two numbers were less than 1000. The last part of the activity involved reading an original source from the seventh century written in the Chinese system (Fig. 1).

The numbers 9, 18, 27, 36, 45, 54, 63, 72 and 81 appeared in that source, and the students had to identify them and recognize the results of the multiplication table by 9. Again, the whole discovery activity was conducted in dialogue in the classroom; the students could converse with each other and with the teacher.

### 3.1.3 The second worksheet: evaluation

After the activity, an evaluation was given to the students. This worksheet repeated the tasks seen during the discovery of the Chinese number system. Throughout the worksheet, the ranges in which the numbers fell were always specified, and throughout the assessment, students could consult the activity sheet. The assessment was in the form of a sequence of five questions, each with two to four items, for a total of 13 items. In question 1, students were asked to write the numbers 73, 221, and 6789 in Chinese, and in question 2, they were asked to rewrite the numbers 42, 50, 346, and 306 in our system. The numbers were increasing in size and intentionally included zeros. Therefore, these first two exercises involved the direct application of what students had already done. Exercises 3, 4, and 5, on the other hand, presented new challenges to students, relating to the notions of the successor and predecessor of an integer. In connection with the notions of addition and subtraction, the objective was to determine whether the use of another decimal system also allows for work on the fundamental one-for-ten exchange rule. In question 3, the students had to write, using the Chinese system, the successor of 23 and then that of 29. The situation was similar in Exercises 4 and 5, which had predecessors of 23 and 80 and of 6789 and 6780, respectively.

## 3.2 Data and processing

The experiment was conducted in four sixth-grade classes, each with a different teacher. In all, 108 students (63 girls [58%]; 45 boys [42%]) participated all or part of the experiment. Mainly because of the pandemic, some students were absent from some parts of the experiment. It should also be noted that for questions 3, 4 and 5 (regarding predecessors and successors), some students may have had difficulty understanding the instructions; they wrote the numbers in our system instead of in the Chinese system. In both cases, the items were considered not addressed (NA), i.e., missing data. In total, out of 3672 observations, there are 196 missing data points, i.e., approximately 5%, which is compatible with statistical analysis. Eighty-six students completed all exercises and activities. This point leads to specify the limits of this study because the first thing to note, in connection with what has just been said, is that even if it is representative of what can be found in textbooks, the activity remains unique. As a result, it undoubtedly benefits from a novelty effect that has not been measured. The second limitation of this experiment concerns the representativeness of the sample. Indeed, although involving more than a hundred students, the experimentation took place in the same school. The teachers were different each time and the school is considered to be ordinary, but biases could be due to the profiles of the students tested.

**Table 1** Success rate  $m$  by item and standard deviation  $sd$ 

| Item | M1a   | M1b   | M1c   | M1d   | M1e   | M1f   | M2a   | M2b   | M2c   | M3a   | M3b   | M3c   |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $m$  | 0.971 | 0.962 | 0.876 | 0.943 | 0.771 | 0.781 | 0.905 | 0.848 | 0.667 | 0.895 | 0.609 | 0.562 |
| $sd$ | 0.167 | 0.192 | 0.331 | 0.233 | 0.422 | 0.415 | 0.295 | 0.361 | 0.474 | 0.308 | 0.490 | 0.498 |
| Item | M4a   | M4b   | M4c   | M4d   | M5a   | M6a   | M6b   | M6c   | M6d   | H1a   | H1b   | H1c   |
| $m$  | 0.562 | 0.428 | 0.409 | 0.429 | 0.724 | 0.990 | 0.943 | 0.854 | 0.438 | 0.922 | 0.913 | 0.854 |
| $sd$ | 0.498 | 0.497 | 0.494 | 0.497 | 0.449 | 0.097 | 0.233 | 0.501 | 0.498 | 0.269 | 0.284 | 0.354 |
| Item | H2a   | H2b   | H2c   | H2d   | H3a   | H3b   | H4a   | H4b   | H5a   | H5b   |       |       |
| $m$  | 0.903 | 0.718 | 0.777 | 0.476 | 0.785 | 0.731 | 0.828 | 0.419 | 0.595 | 0.315 |       |       |
| $sd$ | 0.297 | 0.452 | 0.418 | 0.502 | 0.413 | 0.446 | 0.379 | 0.496 | 0.493 | 0.467 |       |       |

**Table 2** Extraction of two factors by factor analysis with oblimin rotation

| Item | MR1   | MR2 | Item | MR1   | MR2   | Item | MR1   | MR2 |
|------|-------|-----|------|-------|-------|------|-------|-----|
| M1b  | 0.687 |     | M4a  |       | 0.762 | H1c  | 0.624 |     |
| M1c  | 0.484 |     | M4b  |       | 0.854 | H2b  | 0.493 |     |
| M1d  | 0.784 |     | M4c  |       | 0.960 | H2d  | 0.584 |     |
| M1e  | 0.868 |     | M4d  |       | 0.898 | H3a  | 0.596 |     |
| M1f  | 0.890 |     | M5a  |       | 0.466 | H3b  | 0.835 |     |
| M2a  | 0.621 |     | M6b  | 0.577 |       | H4a  | 0.714 |     |
| M2b  | 0.676 |     | M6c  |       | 0.618 | H4b  | 0.372 |     |
| M2c  | 0.644 |     | M6d  |       | 0.657 | H5a  | 0.713 |     |
| M3b  | 0.497 |     | H1a  | 0.721 |       | H5b  | 0.541 |     |
| M3c  | 0.597 |     | H1b  | 0.693 |       |      |       |     |

The data were processed in the *R* environment [48] with the *psych* package [49] for data consistency checking and factor analyses, in the *ltm* package [50] for 1PL modelling. The *ltm* package, known for its performance [51, 52], was chosen because of its algorithms specifically dedicated to dichotomous items and its missing data model fitting that takes into account the observed part of the sample according to the missing at random (MAR) presupposition [53].

## 4 Results

### 4.1 Items reliability and filtering

For the 34 items, the final average score was 24.98 (standard deviation 6.56), which means that the experiment was globally successful for the students. This result is in line with the objective of creating a classroom activity that is integrated into regular practice, with a large amount of student involvement, and not a single test with a certification purpose. However, each group of questions has both easy and more difficult items. Table 1 presents the success rates item by item.

The data do not show a difference between girls and boys. Student's *t*-test ( $t = 0.061832$ ;  $df = 78.767$ ;  $p\text{-value} = 0.9509$ ) yields a *p*-value much higher than 0.05 and is thus not significant. The items were filtered using item-inclusive and item-exclusive point-biserial correlation coefficients. The literature [54–56] suggests a cutoff value of 0.2, but for consistency in modeling, a slightly higher cutoff of 0.25 was chosen. Items with any of the values below the threshold were removed. M1a, M3a, M6a and H2a were excluded. Then, after checking all the remaining items, H2c, whose value was under the threshold was also removed. For the remaining 29 items, the consistency is good and sufficient interitem correlation is ensured by a Cronbach [57] alpha of 0.91, a KMO [58, 59] index of 0.74.

## 4.2 Item content specificity

To determine how the items cluster around certain mathematical skills or learning objects, a factor analysis was conducted. Since the parallel analysis suggested two factors, these factors were extracted with an *oblimin* rotation that allowed the best separation of variables (Table 2).

The first axis accounts for 33.3% of the variance and the second 15.8%, i.e., 49.0% in total. With a threshold value for the loadings of 0.3 [60], the first factor MR1 consists mainly of items M1a to M3c and all the H items. The second factor MR2 consists only of items M4a to M6d. It can be noted that with the exception of item M6b, the items of questions 4, 5, and 6 on the ordinary mathematics worksheet do not group together with the items of the Chinese numeration worksheet. It can therefore be assumed that the targeted skill is not the same. Since the didactic analysis shows that items M1x, M2x, and M3x are based on the articulation between the mastery of decimal numbering and the calculation of addition and subtraction, it can be concluded that the items H1x, H2x, H3x, H4x and H5x concern the same competence. Items M4x, M5x and M6x include a dimension related to the representation of numbers, which does not seem to work with the evaluation of the historical part, although it could have been expected to be activated by the use of the new symbols of Chinese numeration. For the rest of the analyses, only items M1x, M2x, and M3x and the set of H items are kept, that is, 21 items. For this new set, the data consistency is stable, with  $\alpha = 0.89$ ,  $KMO = 0.74$ . The parallel analysis suggests a single principal component and a single factor. Unidimensionality [61, 62] is further confirmed by the ratio between the first eigenvalue and the second eigenvalue being greater than 4 (here  $10.59/1.92 = 5.51$ ). Modeling with a one parameter logistic model (1PL) is therefore conceivable [63].

## 4.3 Item content specificity

To more precisely identify the groupings of the items, an ascending hierarchical classification was carried out considering the difficulty levels of the items based on a 1PL model (model relevance verified by bootstrapping [50] using the dedicated function in the *ltm* package). The classification helps identifying three main groups, each one broken down into two subgroups, i.e., six sets of items (Table 4).

To study the different groupings of items and their consequences on learning, each item has been assigned a coding according to its main didactic issues. We will thus distinguish five symbols: (=), (p), (1), (+), and (−). The two codings (=) and (p) reflect the positional dimension of the numeral systems used in the experiment. The code (=) characterizes items for which the writing of the different digits can be done in a simple way, in the sense that there is only one digit (different from zero) in each rank or blocks of three digits for each class (units, thousands, millions). In both cases, the number can be written by joining the different digits or groups of digits. On the other hand, the code (p) indicates items for which the syntax rules of numeration must be fully mastered. These items have in particular empty ranks which are thus either marked with a zero (usual system) or with an absence of digit (Chinese system). The three other codes (1), (+), and (−) concern the role of groupings by ten in the writing of numbers. Code (1) simply indicates items where a unit is added or subtracted, without using the one-for-ten exchange rule. When the exchange rule is used, two situations can be distinguished. The first one, marked by (+), refers to items in which the base 10 is used in the way of grouping 10 units of a certain rank into 1 unit of the higher rank. Conversely, when the item requires re-decomposing 1 unit of a rank into 10 units of the lower rank, they are marked with the code (−). Grouping and decomposition are the two main aspects of the implementation of the decimal dimension common to the numeral systems used. The coding of the items will allow us to highlight the grouping of certain items on common learning objects (Table 4) as well as several breaks in the continuum of difficulty levels measured by the modeling.

On the mastery of the positional system, the items marked with the code (=) are concentrated in the sub-group 1.1 whereas the items identified by the code (p) will, on the contrary, be concentrated in the other groups 2 and 3. The break line appears in subgroup 1.2 where the two types of items (=) and (p) coexist. The experimentation does not allow the demonstration of a perfectly clear separation on this hierarchy between items requiring a simple implementation of the various ranks and those with a more complex use of the positional system. However, one can note the predominance of two blocks with on one side group 1 (or even the sub-group 1.1) and on the other side groups 2 and 3 (or even 1.2, 2, and 3). In all cases, the items fall under both the ordinary mathematical part and the part with a historical dimension.

The break between the learning implemented in group 1 and those of groups 2 and 3 is also valid for the addition or removal of a unit to numbers written in Chinese. The items coded (1) are indeed in these two groups of higher

**Table 4** Grouping of items and didactic analysis

| Group | Items M                           | Items H                               | Difficulty   | Didactical issues  |
|-------|-----------------------------------|---------------------------------------|--------------|--|
| 1.1   | (=) M1b<br>(=) M1d<br>(=) (+) M2a | (=) H1a<br>(=) H1b                    | -2.6 to -1.9 | Writing the digits of the different ranks in a simple case (H1a, H1b) with the possibility of simply joining the values of the units and thousands classes (M1b, M1d)<br>Addition in a simple case with the same number of digits and decimals, and the need for the one-for-ten exchange rule (M2a)   |
| 1.2   | (p) M1c<br>(p) (+) M2b            | (=) H1c                               | -1.7 to -1.5 | Writing of the digits of the different ranks in a simple case (H1c) and in a complex case with impossibility of joining the values of the classes (M1c)<br>Additions in a complex case with the same number of digits but decimal parts of different lengths, and necessity of the one-for-ten exchange rule (M2b)                                 |
| 2.1   | (p) M1e<br>(p) M1f                | (1) H3a<br>(1) H4a                    | -1.3 to -1.1 | Write the digits of the different ranks of large numbers in a simple case (M1e) and in a complex case with the impossibility of simply joining the values of the classes (M1f)<br>Addition and subtraction of a unit in a simple case (H3a, H4a)   |
| 2.2   | (p) (+) M2c                       | (p) H2b<br>(+) H3b                    | -0.8 to -0.6 | Writing the digits of the different ranks in a complex case with impossibility of simply joining the values (H2b)<br>Addition in a complex case with different number of digits, decimal parts of different lengths, and need of the one-for-ten exchange rule (M2c)<br>Addition of a unit in a case requiring the one-for-ten exchange rule (H3b) |
| 3.1   | (p) (-) M3b<br>(p) (-) M3c        | (1) H5a                               | -0.4 to -0.2 | Subtraction in a complex case with decimal parts of different lengths (M3b, M3c), and need of the one-for-ten exchange rule<br>Subtraction of a unit in a simple case but on a large number (H5a)  |
| 3.2   |                                   | (p) H2d<br>(p) (-) H4b<br>(p) (-) H5b | 0.1 to 0.7   | Write the numbers of the different ranks in a complex case with the impossibility of simply joining the values (H2d)<br>Addition and subtraction of a unit in a complex case requiring the one-for-ten exchange rule (H4b, H5b)  |

difficulty than in group 1. It can also be noticed that this separation coincides with a break in the historical items in the direction of the translation between the two numbering systems. The items concerning the translation from the usual writing to the Chinese writing appear in group 1 whereas those mobilizing the opposite direction are positioned in groups 2 and 3. We can suppose that the addition or removal of a unit in coded items (1) requires first a conversion from the Chinese system to the usual system, but this aspect has not been specifically studied.

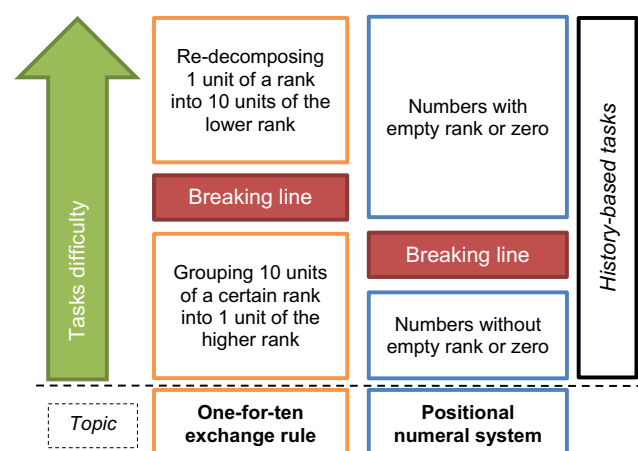
On the mastery of the exchange rule, the situation is clearer. As can be seen in Table 4, the items marked with the sign (–) are concentrated in group 3. These items are also the most difficult for the students (difficulty of – 0.4 to 0.7), both for the mathematical part and for the historical part. The application of the one-for-ten exchange rule in its decomposition way thus creates a new level of difficulty compared to the decimal re-grouping way. This is true both in its expression in the calculations performed and in the manipulation of the Chinese numeral system involving the notions of predecessor and successor of a number.

The correspondence between the groupings of items and the learning objectives confirms the adequacy of the history of mathematics to precise didactic objectives, in this case, the mastery of the decimal and positional dimensions (Fig. 2).

## 5 Discussion

Above analyses of the activity show that the different tasks proposed in the part on ancient numeration have mathematical objectives similar to those of some items in the first part. Among these, learning related to the use of the one-for-ten exchange rule, common to both numbering systems, is clearly emphasized. This fundamental rule of the decimal numeral system is also what makes the addition and subtraction algorithms work, particularly when there is a carry over. Even if the presence of symbols and ancient sources creates an unusual situation [64], mathematics learning is above all based on what is common in the ancient Chinese and contemporary systems of numeration. This point makes explicit the interest of a thorough didactic analysis to properly define a targeted dual objective [65] for an activity with historical support. The school anchoring of the activity thus offers a response to several objections to the use of history pointed out by literature [66, 67] such as the wish to teach a subject first before its historical dimension or the lack of suitable resources. As seen in the analyses, such tasks can be of varying difficulty based on variation in the numerical values chosen (presence or absence of zero, presence or absence of carry over in operations, large numbers) and in the historical dimension (new symbols, new rules for writing numbers). While writing a number with fewer than four digits does not pose a problem for students in the contemporary system, writing even small numbers can be difficult in the ancient Chinese system. For example, in the contemporary system, the writing of 10,100 (item M1b) has a modeled difficulty of – 2.599, while in the Chinese system, the writing of 6789 (item H1c) is – 1.502 (about 1 logit difference). The gap is even more pronounced in moving from the rod number system to the contemporary system, where the modeled difficulty of writing the number 306 (item H2d) is 0.069 (2 logit difference). There is thus a shift in the level of cognitive difficulty introduced by the historical content, as presumed by Agterberg et al. [68] and objectified in this study.

**Fig. 2** Tasks difficulty vs mathematical learnings in history-based tasks



## 6 Conclusion

This paper explored how an activity involving the history of mathematics is relevant for learning. In relation to existing classroom practices or those suggested by textbooks and official instructions, the aim was to know precisely what this type of activity allows students to learn. Different statistical analyses on data collected from 108 students revealed that the history-inspired items worked together with some regular mathematics items but not others. Thus, the ancient Chinese numeration allowed work on the different ranks in the writing of numbers and on the role of groupings by 10 in the decimal systems. On the other hand, the introduction of new symbols in this ancient numeration did not seem to be associated with modes of graphic representation of numbers such as graduated axes or colored parts for fractions. The items were therefore filtered according to their consistency for the whole test and to this grouping to a precise didactic objective. The tasks proposed in the historical part of the activity were of various difficulties and fit well within the ordinary mathematical tasks. These two aspects justify the relevance of the history approach in a school context. In all analyses, I stayed as close to the statistical model as possible. The value of this method is to draw conclusions that are as independent as possible of a particular implementation. The levels of difficulty of the various tasks and the learning objectives are then ensured by the model and not by particular empirical values or subjective analyses. One can therefore be more confident in the fact that the proposed activity, and probably others built on the same model, will be as relevant and precisely focused, regardless of the context.

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**Declarations**

**Ethics approval and consent to participate** Not applicable.

**Consent for publication** Not applicable.

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