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An exactly solvable predator prey model with resetting

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Abstract

We study a simple model of a diffusing particle (the prey) that on encounter with one of a swarm of diffusing predators can either perish or be reset to its original position at the origin. We show that the survival probability of the prey up to time t decays algebraically as $\sim t^{-\theta(p,\gamma)}$ where the exponent θ depends continuously on two parameters of the model, with p denoting the probability that a prey survives upon encounter with a predator and $\gamma = D_A/(D_A + D_B)$ where D_A and D_B are the diffusion constants of the prey and the predator respectively. We also compute exactly the probability distribution $P(N|t_c)$ of the total number of encounters till the capture time t_c and show that it exhibits an anomalous large deviation form $P(N|t_c) \sim t_c^{-\Phi\left(\frac{N}{\ln t_c} = z\right)}$ for large t_c . The rate function $\Phi(z)$ is computed explicitly. Numerical simulations are in excellent agreement with our analytical results.

Keywords: resetting, survival probability, diffusion, predator–prey model

(Some figures may appear in colour only in the online journal)

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1. Introduction

It is by now well established that resetting a stochastic process to its initial condition can fundamentally change the behaviour of the process. For example, the introduction of resetting can generate nontrivial nonequilibrium stationary states and can strongly affect first passage properties [1].

An archetypal example of a stochastic process under resetting is a diffusing particle that is reset to the origin after random waiting times with exponential distribution (Poissonian resetting) [2–5]. It has been shown that the mean first-passage time to some target is rendered finite, rather than infinite as in the absence of resetting. Moreover, under resetting the survival probability of the diffusive particle in the presence of an absorbing target decays exponentially in time, whereas without resetting the decay is a well-known power law $\sim t^{-1/2}$. Various aspects and generalisations of diffusion under resetting have been studied both theoretically (see for example [6–34]) and experimentally in optical traps [35–37].

Our aim in this work is to illustrate how resetting can also create non-trivial power-law decays for a survival probability, with exponents that vary continuously on the parameters of the model. In the case of diffusion with resetting a power-law decay occurs for the average survival probability of a static target at the origin in the presence of a finite density of diffusive particles [2]. The diffusive particles each reset to their own initial positions. Interestingly, the typical survival probability decays exponentially and the power-law decay of the average is due to rare configurations of the initial positions [2]. It is of interest to explore other mechanisms to generate power-law decays.

In this work we consider a simple model of a diffusing particle (the prey) that on encounter with one of a swarm of diffusing predators can either perish or be reset to its original position at the origin. Without resetting, the first-passage properties of a swarm of particles was studied in reference [38]. Here, we consider the additional effect on this swarm of particles (predators) when the target (prey) has a resetting dynamics. Furthermore, the reset is triggered by some internal dynamical event rather than an external process, as in Poissonian resetting. Related scenarios that have previously been considered are two diffusing particles, which reset to their initial positions on contact [39], and first-passage resetting where a reset event occurs when the diffusing particle reaches a specified threshold [40].

As we shall see, our predator prey model effectively reduces to a diffusive process that on reaching the origin has a probability of resetting to a time-dependent, Gaussian distribution. The resets slow down the absorption of the process and turns the temporal decay of the survival probability into a power-law form with a non-trivial exponent that depends continuously on the parameters of the model. This leads us naturally to investigate the statistics of the number of resetting events up until eventual capture. We compute the joint distribution of the number of resets N and capture time t_c . It turns out that this has an interesting scaling with the mean and variance of the number of resetting events depending logarithmically on the capture time. Moreover, we are able to compute the large deviation function for the probability of the number of resets, conditioned on the capture time.

The paper is organised as follows. In section 2 we define the predator–prey model and compute the Laplace transforms of the survival probability of the prey and the capture time distribution. From these we extract the asymptotic power-law decay. In section 3 we consider the statistics of the number of encounters until the time that capture occurs and compute the Laplace transform of the probability of N encounters given the capture time t_c . From this we extract the asymptotics of the moments of N . In section 4 we compute the large deviation form

of $P(N|t_c)$ and show that it translates into an asymptotic power-law decay for large t_c

$$P(N|t_c) \sim t_c^{-\Phi\left(\frac{N}{m t_c} = z\right)}, \quad (1)$$

where the large deviation function $\Phi(z)$ acts as an effective exponent.

2. A simple predator–prey model with resetting

We consider a single prey and a swarm of predators diffusing on a line with diffusion constants D_A (for the prey) and D_B (for each of the predators). We assume that the population of the predators in the swarm is infinite and we label them $n = 1, 2, 3, \dots$. Initially, only the predator number $n = 1$ is ‘active’ in the sense that only this predator can detect the prey. The other predators are ‘passive’, i.e., even if they cross the path of the prey, they do not detect it. When the prey and the current active predator encounter each other, with probability p the active predator retires and the prey escapes to its retreat or safe haven located at the origin. With the complementary probability $1 - p$, the predator wins, in which case the prey dies. In the former case, when the prey escapes, another predator (say $n = 2$) becomes the current active predator and the pursuit continues. We ask the simple question: what is the probability of the prey surviving up to some time t ?

2.1. Model definition

More precisely, let $x_A(t)$ be the position of the prey at time t and $x_B(t)$ denote the position of the current active predator. Each of them performs independent Brownian motions with diffusion constants D_A and D_B respectively, i.e., their positions evolve via

$$\frac{dx_A(t)}{dt} = \eta_A(t) \quad (2)$$

$$\frac{dx_B(t)}{dt} = \eta_B(t), \quad (3)$$

where $\eta_A(t)$ and $\eta_B(t)$ are independent Gaussian white noises with zero mean and the correlators:

$$\langle \eta_A(t)\eta_A(t') \rangle = 2D_A\delta(t - t'), \quad \langle \eta_B(t)\eta_B(t') \rangle = 2D_B\delta(t - t'), \quad (4)$$

$$\langle \eta_A(t)\eta_B(t') \rangle = 0. \quad (5)$$

The prey has a preferred safe position, say the origin, and immediately retreats there after surviving an encounter with an active predator. For simplicity, we take the initial condition as all particles (predators and prey) located at the origin. Our goal is to compute the survival probability, $Q(t_0, t)$, of the prey over a time window $[t_0, t]$.

To make progress, we consider the relative coordinate $x(t) = x_B(t) - x_A(t)$. Then $x(t)$ also performs Brownian motion between any two successive encounters according to

$$\frac{dx(t)}{dt} = \eta(t) \equiv \eta_A(t) - \eta_B(t), \quad (6)$$

where the relative noise $\eta(t)$ is again a Gaussian white noise with zero mean and its correlator, using equation (4), is given by

$$\langle \eta(t)\eta(t') \rangle = 2(D_A + D_B)\delta(t - t'). \tag{7}$$

When the prey encounters the active predator (say at time t_i), the relative coordinate $x(t_i)$ reaches 0. The ‘relative’ particle is then absorbed with probability $(1 - p)$, corresponding to the situation where the active predator wins. With the complementary probability p , the position of the prey is reset to the origin and a new predator is designated as active. Now, as all predators perform independent Brownian motions with diffusion constant D_B , the position distribution of any of the passive predators within the swarm at time t is simply $\frac{e^{-x^2/(4D_B t)}}{\sqrt{4\pi D_B t}}$. Therefore, since $x_A(t_i)$ resets to 0 after each encounter with probability p , the relative coordinate is reset from 0 to $x_B(t_i)$ immediately after the encounter where $x_B(t_i)$ is a Gaussian distributed random variable:

$$\text{Prob. } (x_B(t_i) \in [z, z + dz]) = \frac{1}{\sqrt{4\pi D_B t_i}} e^{-z^2/4D_B t_i} dz. \tag{8}$$

Crucially, the reset of the relative co-ordinate to $x_B(t_i)$ is independent of the position of the previous active predator; this makes manifest the renewal property of the process, which we will use in the following.

2.2. Survival probability of prey

We first consider what happens between two successive encounters. Suppose that the i th encounter happens at time t_i . Let $Q_0(t_i, t)$ denote the probability that the predator and the prey do not encounter each other up to time $t > t_i$, after their last encounter at t_i . This can be simply computed as

$$Q_0(t_i, t) = \int_{-\infty}^{\infty} dz \operatorname{erf} \left(\frac{|z|}{\sqrt{4(D_A + D_B)(t - t_i)}} \right) \frac{1}{\sqrt{4\pi D_B t_i}} e^{-z^2/4D_B t_i} \tag{9}$$

$$= \frac{2}{\pi} \tan^{-1} \left[\sqrt{\frac{\gamma}{\frac{t}{t_i} - 1}} \right], \tag{10}$$

where we denote by γ the ratio

$$\gamma = \frac{D_B}{D_A + D_B}. \tag{11}$$

To understand (9), note that the integral is the average over z , the position of the relative coordinate just after the last encounter at t_i , which has the Gaussian distribution (8). The other factor in the integrand is the probability that the Brownian motion (6) does not cross the origin within time $t - t_i$. We use the well known result [41, 42] that for Brownian motion with diffusion constant D , starting at x_0 , the probability of not reaching the origin within time τ is $\operatorname{erf} \left(|x_0|/\sqrt{4D\tau} \right)$, where the error function is defined as

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z du e^{-u^2}. \tag{12}$$

The resulting integral in (9) may be explicitly evaluated to yield (10). Finally, one can easily check that when $t_i \rightarrow t$, $Q_0(t_i, t) \rightarrow 1$.

We now consider the first-passage probability density $F_0(t_i, t)$, i.e., the probability that the $(i + 1)$ th encounter (the first encounter after the reset at time t_i) takes place between time t and $t + dt$. This is given by

$$F_0(t_i, t)dt = -\partial_t Q_0(t_i, t)dt. \tag{13}$$

We note that $Q_0(t_i, t)$ and $F_0(t_i, t)$ depend on both epochs t_i and t and hence are non-stationary. However, they can be made stationary by introducing the Lamperti transformation [42], i.e., defining the change of time variable $t = e^T$ or equivalently

$$T = \ln t. \tag{14}$$

Then, in terms of the Lamperti time T , we have

$$Q_0(t_i, t) = q_0(T - T_i) \tag{15}$$

$$F_0(t_i, t)dt = f_0(T - T_i)dT, \tag{16}$$

where $q_0(T)$ and $f_0(T)$, using equations (9) and (13), are given explicitly as

$$q_0(T) = \frac{2}{\pi} \tan^{-1} \left[\sqrt{\frac{\gamma}{e^T - 1}} \right] \tag{17}$$

$$f_0(T) = -\frac{dq_0(T)}{dT} = \frac{\sqrt{\gamma}}{\pi} \frac{e^T}{(e^T - 1 + \gamma)\sqrt{e^T - 1}}. \tag{18}$$

Again, one can easily check that when $T \rightarrow 0$, $q_0(T) \rightarrow 1$.

We next consider the full process over a time window $[t_0, t]$. At the beginning of the time window, t_0 , the process has Gaussian distribution (8) with $t_i = t_0$, and there may be any number of possible encounters within the window. The survival probability, $Q(t_0, t)$, then satisfies the (first) renewal equation

$$Q(t_0, t) = Q_0(t_0, t) + p \int_{t_0}^t dt' F_0(t_0, t') Q(t', t). \tag{19}$$

The first term in (19), $Q_0(t_0, t)$, is the survival probability without any encounters between predator and prey, i.e., it is simply the survival probability for Brownian motion with diffusion constant $D_a + D_b$ and an absorbing target at the origin, averaged over initial Gaussian distribution (8) at time t_0 . The second term in (19) integrates over the time, t' , of the first encounter, the factor of p being the probability that the prey survives this encounter. The integrand thus contains the probability density for the time t' of the first encounter, $F_0(t_0, t')$ multiplied by the survival probability, $Q(t', t)$, over the time window $[t', t]$.

In terms of the Lamperti time $T = \ln(t/t_0)$, (19) simplifies to

$$q(T) = q_0(T) + p \int_0^T dT' f_0(T') q(T - T'). \tag{20}$$

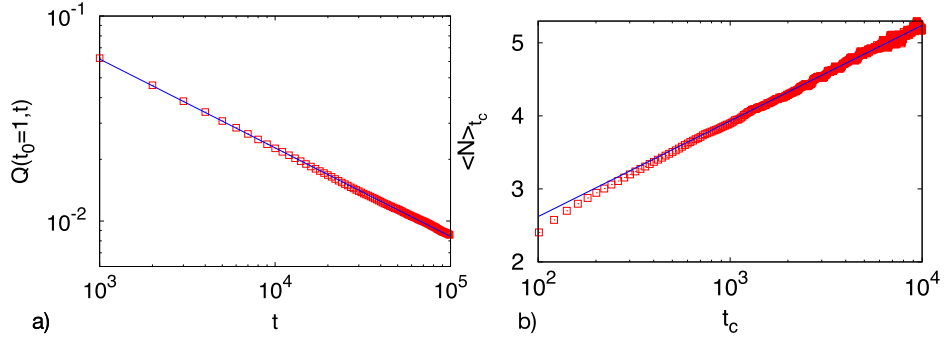


Figure 1. (a) Log–log plot of the survival probability $Q(t_0 = 1, t)$ vs t for $p = 0.25$ and $\gamma = 1/2$ (this corresponds to $D_A = D_B$). The symbols correspond to the numerical simulations while the solid blue line corresponds to the fitting function $f(t) = at^{-\theta}$ where θ is the exact predicted value in equation (27), where a is just a fitting amplitude. (b) Log–linear plot of the average number of encounters $\langle N \rangle_{t_c}$ given the capture time t_c vs t_c for $p = 0.8$ and $\gamma = 1/2$. The symbols correspond to the numerical simulations while the solid blue line corresponds to the fitting function $g(t_c) = A_1 \ln t_c$ where A_1 is the exact predicted value given in equation (47).

Taking the Laplace transform with respect to T , i.e.,

$$\tilde{q}(s) = \int_0^\infty dT q(T) e^{-sT} \tag{21}$$

and using the convolution property of the renewal equation (20), we obtain

$$\tilde{q}(s) = \frac{\tilde{q}_0(s)}{1 - pf_0(s)}, \tag{22}$$

where the Laplace transform of $f_0(T)$ is

$$\tilde{f}_0(s) = \int_0^\infty dT f_0(T) e^{-sT}. \tag{23}$$

Using the expression in equation (18), $\tilde{f}_0(s)$ can be explicitly computed as

$$\tilde{f}_0(s) = \frac{\sqrt{\gamma}}{\sqrt{\pi}} \frac{\Gamma(1/2 + s)}{\Gamma(1 + s)} {}_2F_1 [1, s + 1/2, s + 1; 1 - \gamma], \tag{24}$$

where ${}_2F_1 [a, b, c; z]$ is the usual hypergeometric function (see appendix A).

The asymptotic large T behaviour of $q(T)$ is determined by the singularity of $\tilde{q}(s)$ with largest real part. If this singularity is a pole at $s = -\theta$ then

$$q(T) \sim e^{-\theta T}. \tag{25}$$

Reverting to real time we then obtain

$$Q(t_0, t) \sim (t/t_0)^{-\theta} \quad \text{for } t \gg t_0. \tag{26}$$

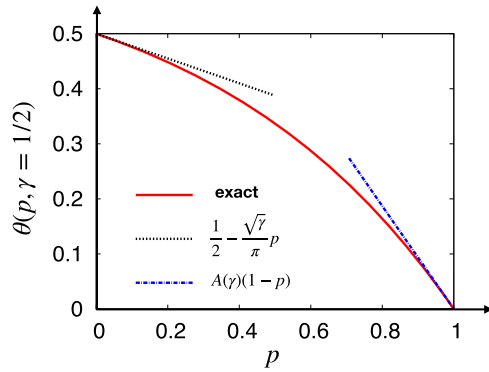


Figure 2. Plot of $\theta(p, \gamma)$ vs p for $\gamma = 1/2$. The solid red line corresponds to the value of $\theta(p, \gamma)$ obtained by inverting numerically the exact relation in (27). The dotted line corresponds to the first two terms in the small p expansion of $\theta(p, \gamma = 1/2)$ given in the first line of equation (29), while the blue dashed line is the leading behaviour of $\theta(p, \gamma = 1/2)$ for p close to 1 given in the second line of (29) with $A(\gamma = 1/2) = 0.934\,929\dots$

Thus we identify the exponent θ with the dominant pole in $\tilde{q}(s)$. From equation (22), we find that the θ is given by the smallest positive root of the transcendental equation, $1 - p\tilde{f}_0(-\theta) = 0$, i.e.,

$$\frac{1}{p} = \frac{\sqrt{\gamma}}{\sqrt{\pi}} \frac{\Gamma(1/2 - \theta)}{\Gamma(1 - \theta)} {}_2F_1 [1, 1/2 - \theta, 1 - \theta; 1 - \gamma]. \tag{27}$$

Hence, the exponent $\theta(p, \gamma)$ depends continuously on two parameters, namely, $0 \leq p \leq 1$ and $0 \leq \gamma \leq 1$ where γ is given by (11). In figure 1 we plot results from numerical simulations, which show excellent agreement with the analytical predictions.

We now consider some limiting cases where we can develop explicit expressions for the exponent θ . It is easy to check that for $\gamma = 1$, equation (27) reduces to

$$\frac{1}{p} = \frac{1}{\sqrt{\pi}} \frac{\Gamma(1/2 - \theta)}{\Gamma(1 - \theta)}. \tag{28}$$

In fact, in this limit $\gamma \rightarrow 1$, it turns out that our model effectively reduces to a model of adaptive persistence studied in a completely different context in reference [43] and our expression for the exponent θ in equation (28) coincides with that of reference [43] (with p here replaced by $p/2$ in that paper). From the formula (27) it is easy to work out the limiting behaviours of the exponent θ as $p \rightarrow 0$ and $p \rightarrow 1$. One finds

$$\theta(p, \gamma) \rightarrow \begin{cases} \frac{1}{2} - \frac{\sqrt{\gamma}}{\pi} p + O(p^2) & \text{as } p \rightarrow 0 \\ A(\gamma)(1 - p) + O((1 - p)^2) & \text{as } p \rightarrow 1, \end{cases} \tag{29}$$

where the amplitude $A(\gamma)$ in the second line in equation (29) can in principle be computed explicitly in terms of hypergeometric function and its derivatives but we do not provide the details here. In figure 2, we show a plot of $\theta(p, \gamma)$ as a function of p for $\gamma = 1/2$ obtained from equation (27), where we also indicate the asymptotic behaviours for $p \rightarrow 0$ and $p \rightarrow 1$ given in equation (29).

2.3. Capture time distribution

The distribution of the capture time t_c of the prey can also be simply obtained. In the Lamperti time $T_c = \ln t_c$, this is

$$f(T_c) = -q'(T = T_c), \tag{30}$$

where $q(T)$ is given in equation (20). Hence its Laplace transform is given by

$$\tilde{f}(s) = \int_0^\infty dT_c f(T_c) e^{-sT_c} = 1 - s\tilde{q}(s) \tag{31}$$

where $\tilde{q}(s)$ is given in equation (22). This gives

$$\tilde{f}(s) = 1 - s \frac{\tilde{q}_0(s)}{1 - pf_0(s)} = \frac{(1 - p)\tilde{f}_0(s)}{1 - p\tilde{f}_0(s)}, \tag{32}$$

where we used $\tilde{f}_0(s) = 1 - s\tilde{q}_0(s)$. Note that $\tilde{f}_0(s)$ is given explicitly in equation (24). One may check that $f(T_c)$ is normalized to unity by substituting $s = 0$ in equation (32), and using $\tilde{f}_0(0) = 1$.

As the expression in equation (32) also has a pole at s satisfying $1 - p\tilde{f}_0(s)$, it follows that the asymptotic behaviour is $f(T_c) \sim e^{-\theta T_c}$ for large T_c with the same θ given in equation (27). In real time $t_c = e^{T_c}$, the capture time distribution $\text{Prob}(t_c) = F(t_c)$ then decays for large t_c as a power law

$$F(t_c) = f(T_c) \frac{dT_c}{dt_c} \sim (t_c)^{-\theta-1}. \tag{33}$$

3. Statistics of the number of encounters

In this section we address another natural question: how many encounters with predators are needed to catch the prey at a given capture time t_c ? We begin by considering the joint probability of N encounters with capture at the final encounter. The renewal property of the process again facilitates the computation.

3.1. Joint probability of number of encounters and capture time

Consider a typical trajectory of the process till the capture time t_c of the prey, starting at t_0 . Let the predator and prey encounter each other at times $\{t_0, t_1, t_2, \dots, t_N = t_c\}$, i.e., there are a total of N encounters till the final capture time. Note that the number of encounters is precisely the number of predators needed to catch the prey at time t_c .

As usual, it is convenient to work in Lamperti time $T_i = \ln(t_i/t_0)$. Then the relative process starts at time $T = 0$, and encounters occur (when x reaches 0) at epochs $\{T_1, T_2, \dots, T_N = T_c\}$. Let us also denote the intervals between encounters as

$$\tau_i = T_i - T_{i-1} \tag{34}$$

with $T_0 = 0$ and

$$T_c = \sum_{i=1}^N \tau_i. \tag{35}$$

Thus a ‘configuration’ is specified by the vector $\vec{\tau} = \{\tau_1, \tau_2, \dots, \tau_N\}$, their number N and the final capture time T_c , which are all random variables. The probability of a configuration is given by $P(\vec{\tau}, N, T_c)$, the joint distribution of $\vec{\tau}, N$, and T_c . This joint distribution can be explicitly written as

$$P(\vec{\tau}, N, T_c) = (1 - p) p^{N-1} f_0(\tau_1) f_0(\tau_2) \dots f_0(\tau_N) \delta(\tau_1 + \tau_2 + \dots + \tau_N - T_c), \quad (36)$$

where $f_0(\tau)$, given in equation (18), denotes the distribution of the time interval between two successive encounters. Due to the renewal property, the successive intervals between encounters are statistically independent, except that they must add up to T_c , providing a global constraint enforced by the delta function in equation (36). The factor $(1 - p)p^{N-1}$ in equation (36) reflects the fact that each of the first $(N - 1)$ encounters, where the prey survives, happens with probability p and the final one, where capture occurs, happens with probability $(1 - p)$.

By integrating over τ_i 's in equation (36), one obtains the joint distribution $P(N, T_c)$ of the number of encounters N and the capture time T_c

$$P(N, T_c) = \frac{(1 - p)}{p} \int_0^\infty \prod_{i=1}^N d\tau_i \left[\prod_{i=1}^N p f_0(\tau_i) \right] \delta(\tau_1 + \tau_2 + \dots + \tau_N - T_c). \quad (37)$$

Taking the Laplace transform with respect to T_c , one obtains

$$\tilde{P}(N, s) = \int_0^\infty dT_c P(N, T_c) e^{-sT_c} = \frac{(1 - p)}{p} [p\tilde{f}_0(s)]^N, \quad (38)$$

where $\tilde{f}_0(s)$ is given by (24). Equation (38) is the main result of this subsection.

One can check that (38) can be used to recover results of section 2. Summing $P(N, T_c)$ over $N = 1, 2, \dots$, yields the marginal distribution of the capture time

$$f(t_c) = \sum_{N=1}^\infty P(N, T_c). \quad (39)$$

The Laplace transform of this distribution is obtained by summing equation (38) over N :

$$\tilde{f}(s) = \frac{(1 - p)\tilde{f}_0(s)}{1 - p\tilde{f}_0(s)}, \quad (40)$$

which precisely coincides with equation (32).

3.2. Moments of N conditioned on the capture time

From the result (38) it is easy to compute the conditional distribution of the number of encounters $P(N|T_c)$, given the capture time T_c . This can be obtained via

$$P(N|T_c) = \frac{P(N, T_c)}{f(T_c)}, \quad (41)$$

where the Laplace transforms of $P(N, T_c)$ and $f(T_c)$ are given respectively in equations (38) and (40). Formally inverting these Laplace transforms separately, we can then write

$$P(N|T_c) = \frac{\mathcal{L}_{s \rightarrow T_c}^{-1} \left[\left(p \tilde{f}_0(s) \right)^N \right]}{p \mathcal{L}_{s \rightarrow T_c}^{-1} \left[\frac{\tilde{f}_0(s)}{1 - p \tilde{f}_0(s)} \right]}, \tag{42}$$

where the notation $\mathcal{L}_{s \rightarrow T_c}^{-1}$ means the Bromwich integral in the complex s plane

$$\mathcal{L}_{s \rightarrow T_c}^{-1} [\tilde{U}(s)] = \int_{\Gamma} \frac{ds}{2\pi i} e^{sT_c} \tilde{U}(s). \tag{43}$$

The mean number of encounters $\langle N \rangle_{T_c}$, given the capture time T_c , can be computed from

$$\langle N \rangle_{T_c} = \sum_{N=1}^{\infty} N P(N|T_c) = \frac{\mathcal{L}_{s \rightarrow T_c}^{-1} \left[\frac{\tilde{f}_0(s)}{(1 - p \tilde{f}_0(s))^2} \right]}{\mathcal{L}_{s \rightarrow T_c}^{-1} \left[\frac{\tilde{f}_0(s)}{1 - p \tilde{f}_0(s)} \right]}, \tag{44}$$

where we used equation (42) and the identity $\sum_{N=1}^{\infty} N x^N = x/(1-x)^2$. Similarly, the second moment can be obtained as

$$\langle N^2 \rangle_{T_c} = \sum_{N=1}^{\infty} N^2 P(N|T_c) = \frac{\mathcal{L}_{s \rightarrow T_c}^{-1} \left[\frac{\tilde{f}_0(s)(1 + p \tilde{f}_0(s))}{(1 - p \tilde{f}_0(s))^3} \right]}{\mathcal{L}_{s \rightarrow T_c}^{-1} \left[\frac{\tilde{f}_0(s)}{1 - p \tilde{f}_0(s)} \right]}. \tag{45}$$

Note that all the Bromwich integrals appearing in equations (44) and (45) are dominated exponentially, for large T_c , by the contribution from the pole at $s = s^* = -\theta$, where $1 - p \tilde{f}_0(s^*) = 0$. Thus we need to just calculate the residues at this pole of all the Bromwich integrals. This can be done explicitly. Skipping details, we get

$$\langle N \rangle_{T_c} = A_1 T_c + A_0 + o(1) \tag{46}$$

with the prefactors

$$A_1 = -\frac{1}{p a_1} = -\frac{1}{p f_0'(s^*)}, \quad A_0 = \frac{2a_2}{p a_1^2} - 1, \tag{47}$$

where

$$a_n = \frac{1}{n!} \tilde{f}_0^{(n)}(s^*) \tag{48}$$

is the n th derivative of $\tilde{f}_0(s)$, given explicitly in equation (24), evaluated at the pole $s = s^*$ and divided by $n!$. Note that $a_1 = \tilde{f}_0'(s^*) < 0$. Similarly, we get

$$\langle N^2 \rangle_{T_c} = B_2 T_c^2 + B_1 T_c + B_0 + o(1), \tag{49}$$

where

$$B_2 = \frac{1}{p^2 a_1^2}, \tag{50}$$

$$B_1 = \frac{3}{p^2 a_1^2} \left(p a_1 - \frac{2 a_2}{a_1} \right), \tag{51}$$

$$B_0 = \frac{1}{p^2 a_1^2} \left[p^2 a_1^2 - 6 p a_2 + 6 \left(\frac{2 a_2^2}{a_1^2} - \frac{a_3}{a_1} \right) \right]. \tag{52}$$

Thus the variance is given by

$$\sigma_N^2 = \langle N^2 \rangle_{T_c} - \langle N \rangle_{T_c}^2 = (B_2 - A_1^2) T_c^2 + (B_1 - 2 A_1 A_0) T_c + (B_0 - A_0^2) + o(1), \tag{53}$$

where A_i 's and B_i 's are given above. Since, $B_2 = A_1^2$, the leading order term $O(T_c^2)$ vanishes in the variance and one gets for large T_c

$$\sigma_N^2 = B T_c + O(1), \tag{54}$$

where the prefactor B is given explicitly by

$$B = B_1 - 2 A_1 A_0 = \frac{1}{p a_1} - \frac{2 a_2}{p^2 a_1^3} = \frac{1}{p \tilde{f}'_0(s^*)} - \frac{\tilde{f}''_0(s^*)}{p^2 (\tilde{f}'_0(s^*))^3}, \tag{55}$$

with $\tilde{f}_0(s)$ given explicitly in equation (24) and s^* is determined from $\tilde{f}_0(s^*) = 1/p$.

For general (p, γ) the amplitude B in equation (55) has a complicated expression, in particular since there is no explicit expression for s^* . For $\gamma = 1$, $\tilde{f}_0(s)$ simplifies to

$$\tilde{f}_0(s) = \frac{1}{\sqrt{\pi}} \frac{\Gamma(s + 1/2)}{\Gamma(s + 1)}. \tag{56}$$

In this case, the expression for B in equation (55) can be explicitly evaluated as

$$B = \frac{\psi^{(1)}(s^* + 1) - \psi^{(1)}\left(s^* + \frac{1}{2}\right)}{\left(\psi^{(0)}(s^* + 1) - \psi^{(0)}\left(s^* + \frac{1}{2}\right)\right)^3}, \tag{57}$$

where $\psi^{(m)}(z)$ is the polygamma function of index m , i.e., the $(m + 1)$ th derivative of $\ln \Gamma(z)$ and s^* is again determined from $\tilde{f}_0(s^*) = 1/p$ with $\tilde{f}_0(s)$ given in equation (56).

Note that in terms of the original time $t_c = e^{T_c}$, the results for the mean and the variance of N , respectively in equations. (46) and (54) translate, for large t_c , to

$$\langle N \rangle_{t_c} = A_1 \ln t_c + O(1), \quad \text{and} \quad \sigma_N^2 = B \ln t_c + O(1), \tag{58}$$

indicating a rather slow logarithmic growth in the original capture time t_c . Note that the mean number of resets grows logarithmically with time, as opposed to linearly in standard resetting processes where resetting occurs at a constant rate. The reason behind this slow growth can be traced back to the fact that the interval between successive resets/encounters is power-law distributed in this problem.

3.3. *The large deviation form of the conditional distribution $P(N|T_c)$*

We now consider the full conditional distribution $P(N|T_c)$ in equation (42) in the limit when N is large, T_c is large, but with the ratio $z = N/T_c$ fixed. We will derive a large deviation form for $P(N|T_c)$.

The denominator in equation (42) can again be evaluated for large T_c by computing the residue at the pole $s = s^*$ and one gets, to leading order for large T_c

$$p\mathcal{L}_{s \rightarrow T_c}^{-1} \left[\frac{\tilde{f}_0(s)}{1 - p\tilde{f}_0(s)} \right] \approx \frac{1}{-p\tilde{f}'_0(s^*)} e^{s^*T_c}, \tag{59}$$

where s^* is given by $\tilde{f}_0(s^*) = 1/p$. The numerator in equation (42) can be evaluated, for large N , by the saddle point method. Ignoring pre-exponential terms, this gives

$$\mathcal{L}_{s \rightarrow T_c}^{-1} \left[\left(p\tilde{f}_0(s) \right)^N \right] \sim \exp \left[T_c \min_s [s + z \ln(p\tilde{f}_0(s))] \right], \tag{60}$$

where the minimum occurs at the saddle point with real $s = s_0$ and $z = N/T_c$. Taking the ratio of the numerator and denominator in equation (42), we then get for fixed z , the following large deviation behaviour in the limit $N \rightarrow \infty, T_c \rightarrow \infty$ but with $z = N/T_c$ fixed

$$P(N|T_c) \sim e^{-T_c \Phi(z = \frac{N}{T_c})}, \tag{61}$$

where the rate function $\Phi(z)$ is given by

$$\Phi(z) = -\min_s [s + z \ln(p\tilde{f}_0(s))] + s^* = -\min_s [s + z \ln(p\tilde{f}_0(s))] - \theta, \tag{62}$$

where we have used $s^* = -\theta$, which is the solution of $1 - p\tilde{f}_0(s^*) = 0$.

While it is difficult to compute this rate function $\Phi(z)$ exactly for all z , one can derive its asymptotic behaviours as $z \rightarrow 0$ and $z \rightarrow \infty$ and also its behaviour near its minimum. Suppose that the minimum over s in equation (62) occurs at $s = s_0$. Defining $W(s) = s + z \ln(p\tilde{f}_0(s))$, and setting $W'(s_0) = 0$ gives

$$-\left. \frac{\tilde{f}'_0(s)}{\tilde{f}_0(s)} \right|_{s=s_0} = \frac{1}{z}, \tag{63}$$

which determines s_0 upon using $\tilde{f}_0(s)$ from equation (24). In figure 3, we give a plot of the function $-\tilde{f}'_0(s)/\tilde{f}_0(s)$ vs s for $\gamma = 1$. From this figure 3, it is clear that when $z \rightarrow 0, s_0 \rightarrow -1/2$, while when $z \rightarrow \infty, s_0 \rightarrow \infty$.

3.3.1. *The limit $z \rightarrow 0$.* We first consider the limit $z \rightarrow 0$. From figure 3, we see that $s_0 \rightarrow -1/2$ as $z \rightarrow 0$ (consequently $1/z$ diverges) and hence we set $s_0 = -1/2 + \epsilon$. Expanding the function for small ϵ , we get from equation (63)

$$z = \epsilon + O(\epsilon^2). \tag{64}$$

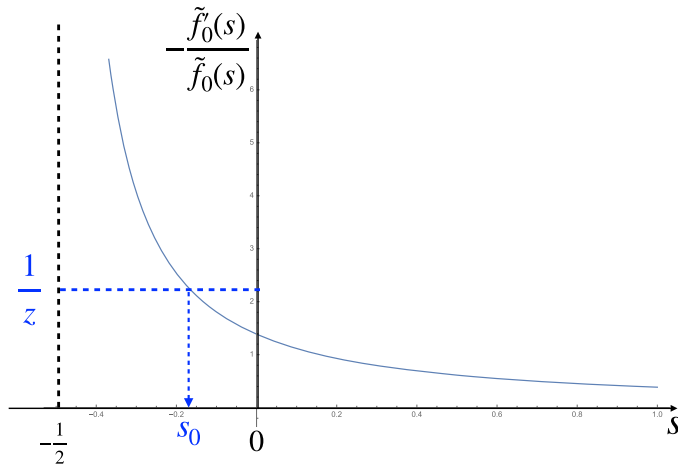


Figure 3. Plot of $-\tilde{f}'_0(s)/\tilde{f}_0(s)$ vs s for $\gamma = 1$, using $\tilde{f}_0(s)$ from equation (56). The function diverges as $s \rightarrow -1/2$ and decreases monotonically for increasing s . The location of the saddle point s_0 is given by the value of s at which this curve crosses the value $1/z$, as indicated in the figure. For other values of $0 < \gamma < 1$ the behaviour is qualitatively similar.

Inverting, for small z , yields

$$\epsilon = z + O(z^2). \tag{65}$$

We also find that, to leading order in ϵ

$$\tilde{f}_0\left(s_0 = -\frac{1}{2} + \epsilon\right) = \frac{\sqrt{\gamma}}{\pi\epsilon} + O(1). \tag{66}$$

We substitute $s_0 = -1/2 + \epsilon$, $\epsilon = z + O(z^2)$ and $\tilde{f}_0(s_0 = -1/2 + \epsilon)$ from equation (66) in the expression for $\Phi(z)$ in equation (62). This gives, as $z \rightarrow 0$,

$$\Phi(z) = \frac{1}{2} - \theta + z \ln z - z \ln\left(\frac{pe\sqrt{\gamma}}{\pi}\right) + O(z^2). \tag{67}$$

Thus, as $z \rightarrow 0$, the rate function approaches a constant $\Phi(z) \rightarrow 1/2 - \theta > 0$. This can be understood by setting, for example, $N = 1$ in equation (42). For $N = 1$, the numerator behaves as $\sim e^{-T/2}$ since $f_0(T) \sim e^{-T/2}$ from equation (18). The denominator in equation (42) scales, for large T , as $\sim e^{-\theta T}$. Hence the ratio, for $N = 1$, scales as $\sim e^{-(1/2-\theta)T}$, indicating, from the large deviation form in equation (61), that $\Phi(0) = 1/2 - \theta$. Note that this leading term does depend on both p and γ . However, the first sub-leading term $z \ln z$ is universal, i.e., independent of p and γ .

3.3.2. The limit $z \rightarrow \infty$. We now turn to the opposite $z \rightarrow \infty$ limit. From figure 3, it is clear that $s_0 \rightarrow \infty$ for large z . Expanding $\tilde{f}_0(s)$ in equation (24) for large s , we find

$$\tilde{f}_0(s) = \frac{1}{\sqrt{\pi\gamma s}} + \frac{3\gamma - 4}{8\sqrt{\pi}\gamma^{3/2}s^{3/2}} + O(1/s^{5/2}). \tag{68}$$

Substituting this asymptotic behaviour in equation (63) gives, for large z

$$z = 2s_0 + \frac{4 - 3\gamma}{2\gamma} + O(1/s_0). \tag{69}$$

Inverting this relation, we get

$$s_0 = \frac{z}{2} - \frac{4 - 3\gamma}{4\gamma} + O(1/z). \tag{70}$$

Substituting these behaviours in the expression for the rate function $\Phi(z)$ in (62), gives, for large z

$$\Phi(z) = \frac{z}{2} \ln z - z \ln \left(p \sqrt{\frac{2e}{\gamma\pi}} \right) + \frac{4 - 3\gamma}{4\gamma} - \theta + O(1/z). \tag{71}$$

Note that the leading term is universal, i.e., independent of p and γ .

3.3.3. The behaviour close to the minimum of $\Phi(z)$. From the expression of $\Phi(z)$ in equation (62), it is clear that when $s_0 \rightarrow s^*$ where $f_0(s^*) = 1/p$, the rate function approaches $-s^* - \theta = 0$, which is the minimum value of $\Phi(z)$, since the rate function is necessarily non-negative. We therefore set $s_0 = s^* + \epsilon$, with $|\epsilon| \ll 1$ in order to study the behaviour of $\Phi(z)$ in the vicinity of its minimum. Expanding the saddle point equation (63) for small $|\epsilon|$ and using $pf_0(s^*) = 1$, we get

$$\frac{1}{z} = -p\tilde{f}'_0(s^*) + p^2 \left((\tilde{f}'_0(s^*))^2 - \frac{1}{p}\tilde{f}''_0(s^*) \right) \epsilon + O(\epsilon^2). \tag{72}$$

This indicates that, exactly at $\epsilon = 0$, $z \rightarrow z^* = -1/(p\tilde{f}'_0(s^*)) = A_1$ where A_1 has already been defined in equation (47). Setting $z = A_1 + \delta$ and inverting the relation (72) we obtain

$$\epsilon = -\frac{1}{CA_1^2}\delta + O(\delta^2), \quad \text{where } C = (p\tilde{f}'_0(s^*))^2 - p\tilde{f}''_0(s^*). \tag{73}$$

We now expand $\Phi(z)$ in equation (62) around $z = A_1$ and express it as a function of δ only. To leading order we get

$$\Phi(z) = \frac{1}{2C^2A_1^5} \left(1 - 2CA_1^2 - pA_1^2\tilde{f}''_0(s) \right) \delta^2 + O(\delta^3). \tag{74}$$

Substituting the expression for A_1 and C and simplifying, we find a leading quadratic behaviour of $\Phi(z)$ around $z = A_1$

$$\Phi(z) \approx \frac{1}{2B}(z - A_1)^2 \quad \text{as } z \rightarrow A_1, \tag{75}$$

where A_1 and B are respectively in equations (47) and (55). Note that this quadratic behaviour of the rate function indicates that the typical fluctuations of N , for fixed T_c , are Gaussian distributed with mean $\langle N \rangle_{T_c} \approx A_1 T_c$ and variance $\sigma_N^2 \approx B T_c$, as we had already derived in section 3.2. Thus the mean and the variance from the expansion of the large deviation function near its minimum coincide perfectly with their expressions obtained directly in equations (46) and (54) respectively.

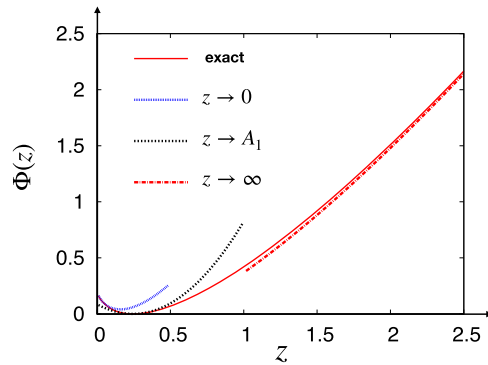


Figure 4. The rate function $\Phi(z)$ vs z for $\gamma = 1$ and $p = 1/2$ for which $\theta = 0.300568\dots$. The asymptotic behaviours of $\Phi(z)$ for $z \rightarrow 0$, $z \rightarrow \infty$, as well as the quadratic behaviour in equation (76), namely $\Phi(z) \approx \frac{1}{2B}(z - A_1)^2$ near $z = A_1 = 0.244956\dots$ (and with $B = 0.346483\dots$), are also plotted.

The asymptotic behaviours of $\Phi(z)$ can be summarised as follows

$$\Phi(z) = \begin{cases} \frac{1}{2} - \theta + z \ln z - z \ln \left(\frac{pe\sqrt{\gamma}}{\pi} \right) + O(z^2) & z \rightarrow 0 \\ \frac{1}{2B}(z - A_1)^2 & z \rightarrow A_1 \\ \frac{z}{2} \ln z - z \ln \left(p\sqrt{\frac{2e}{\gamma\pi}} \right) + \frac{4 - 3\gamma}{4\gamma} - \theta + O(1/z) & z \rightarrow \infty, \end{cases} \quad (76)$$

where A_1 and B are respectively in equations (47) and (55). A plot of this function is provided in figure 4 for $\gamma = 1$ and $p = 1/2$, for which $\theta = 0.300568\dots$. In this case, from equations (47) and (55), we get $A_1 = 0.244956\dots$ and $B = 0.346483\dots$. In this plot, we also provide the asymptotic behaviours for $z \rightarrow 0$, $z \rightarrow \infty$, as well as the quadratic behaviour near the minimum $z = A_1 = 0.244956\dots$ in equation (76), finding excellent agreement.

Finally, reverting back to the original time $t_c = e^{T_c}$, we see that the large deviation form in equation (61) translates into an anomalous form in t_c , namely,

$$P(N|t_c) \sim t_c^{-\Phi\left(\frac{N}{\ln t_c} = z\right)}. \quad (77)$$

Thus $\Phi(z)$ acts like an ‘effective’ exponent if one observes $P(N|t_c)$ as a function of t_c for large t_c and fixed N . This type of ‘anomalous large deviation behaviour’ was also found in other contexts such as in the study of the persistence/survival probability of a stationary Gaussian process [42, 44–47].

4. Conclusion

In this work we have considered a simple predator–prey model where on encounters with an active predator, the prey has a probability of surviving and escaping back to the origin. The model exhibits a survival probability decaying as a power law with a nontrivial exponent $\theta(p, \gamma)$. This exponent depends continuously on the escape probability p and the parameter γ

(11) which is a ratio involving the diffusion constants of predator and prey. Moreover, we have studied the distribution of the number of encounters N conditioned on capture at time t_c .

The presence of a swarm of predators from which a new active predator is selected after each encounter implies a renewal form for the survival probability (19). This renewal property facilitates computation of the survival probability and other quantities we have considered.

It would be natural to consider a single predator and prey, with the same escape probability p after an encounter but with the relative separation then being reset to the position of the prey. However, in that case the corresponding equation for the survival probability would contain a memory of the position of the predator at the encounter, and such memory dependence poses an open problem. In this case, we expect again that the exponent θ will depend continuously on p and γ , though its expression will be different from the result in equation (27). Our preliminary numerical simulations confirm this conclusion. Another natural modification of the model would be to consider multiple predators such that once the prey survives an encounter and returns to its nest, the status ‘active’ is accorded to the predator who is currently closest to the prey at the origin following its return. It would be interesting to explore these cases and other generalisations of the model.

Stripping away the context predator and prey, the process reduces to a positive diffusive field (6), i.e., the separation, which on reaching the origin is reset with probability p to a value drawn from the time-dependent distribution (8). In this light, the connection with the model of [43] and various problems of persistence of a fluctuating field [42, 44–47] becomes apparent.

The origin of the slow decay of the survival probability is that at each resetting event the separation of predator and prey is reset to a time-dependent distribution (8) which is a Gaussian whose width increases with time as \sqrt{t} . Thus at each reset the typical separation increases. It would be of interest to see if other choices of the resetting distribution of the separation could lead to nontrivial forms for the survival probability, e.g., a stretched exponential. Another interesting extension would be to consider ‘interacting’ predators with a short-range interaction. If the positions of such interacting walkers typically scale as \sqrt{t} , we believe that much of the behaviour found here for noninteracting predators would still hold, e.g., the exponent characterizing the decay of the survival probability will still be a continuous function of the parameters of the model. Similarly, the large deviation form for the number of encounters in equation (1), given the capture time t_c , is also expected to hold albeit with a different rate function $\Phi(z)$. An example of such short-ranged interacting predators where the positions still scale diffusively as \sqrt{t} is given by the nonintersecting Brownian motions (also known as vicious walkers). It would be interesting to compute the survival probability exponent θ or the rate function $\Phi(z)$ analytically for this example.

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Data availability statement

No new data were created or analysed in this study.

Appendix A. Derivation of formula (24)

Here we compute the Laplace transform of the first passage time distribution

$$\tilde{f}_0(s) = \int_0^\infty dT f_0(T) e^{-sT}. \quad (\text{A.1})$$

Using expression equation (18) for $f_0(T)$, we obtain

$$\tilde{f}_0(s) = \frac{\gamma^{1/2}}{\pi} \int_0^\infty dT \frac{e^T}{(e^T - 1)^{1/2}} \frac{e^{-sT}}{(e^T - 1 + \gamma)}. \quad (\text{A.2})$$

Changing integration variable to $x = e^{-T}$ yields

$$\tilde{f}_0(s) = \frac{\gamma^{1/2}}{\pi} \int_0^1 dx \frac{x^{s-1/2}}{(1-x)^{1/2}(1-(1-\gamma)x)}. \quad (\text{A.3})$$

We now compare (A.3) with the integral representation of the hypergeometric function

$$B(b, c-b) {}_2F_1[a, b, c; z] = \int_0^1 dx x^{b-1} (1-x)^{c-b-1} (1-zx)^{-a}, \quad (\text{A.4})$$

where the Beta function is defined, as usual, by

$$B(u, v) = \frac{\Gamma(u)\Gamma(v)}{\Gamma(u+v)}, \quad (\text{A.5})$$

and $\Gamma(u)$ is the usual gamma function. We identify $a = 1$, $b = s + 1/2$, $c = s + 1$, $z = 1 - \gamma$ and hence deduce

$$\tilde{f}_0(s) = \frac{\gamma^{1/2}}{\pi^{1/2}} \frac{\Gamma(1/2 + s)}{\Gamma(1 + s)} {}_2F_1[1, s + 1/2, s + 1; 1 - \gamma]. \quad (\text{A.6})$$

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