

Control Performance Improvement in Dynamically Decoupled Manipulators

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Abstract - The control of industrial robot manipulators presents a difficult problem for control engineers due to the complexity of robot dynamics models. Nonlinear controls based on feedback linearization are developed to meet control requirements. Model-based nonlinear control is highly sensitive to parameter errors and leads to problems of robustness for tracking trajectories at high speeds, and there is the additional problem of a heavy computational burden to consider in the design of nonlinear controllers. In this paper, a mechatronics design approach is proposed, which aims to facilitate controller design by redesigning the mechanical structure. The problem is approached in two steps: first, the dynamic decoupling conditions of manipulators are described and discussed, involving redistribution of the moving mass, which leads to the decoupling of motion equations. A classical linear control technique is then used to track the desired efficient bang-bang profile trajectory. An analysis of the results from a simulation of this approach demonstrates its effectiveness in controller design. The proposed improvement in control performance is illustrated via a two-revolute joint spatial manipulator.

1 Introduction

In recent years, industrial robots have been widely used for high-precision applications [1]. Different types of industrial robots are developed to carry out various tasks such as painting, welding, manufacturing, assembling and so on. The requirements for industrial robot performance are accuracy, speed and versatility of manipulation. To meet this demand, robot control is a key element for robot manufacturers, and a great deal of development work is carried out to increase the performance of robots, reduce their cost and introduce new functionalities. In terms of controller design, robustness, tracking accuracy, energy consumption and computational burden must be taken into account [2, 3]. However, robot manipulator dynamics are known to be highly nonlinear and coupled, which results in poorer control performance at high speeds [4, 5]. The complicated dynamics are the result of varying inertia and interactions between the different joints, and the nonlinear coupled dynamics of the robot manipulator also increase the energy consumption and computational burden of the controller [4]. Various nonlinear control strategies have been developed to resolve this problem, such as feedback linearization [6].

Feedback linearization is a tool widely used for controlling non-linear systems, and operates by canceling the nonlinearities in robot dynamics [7, 8]. With this method, the closed-loop system becomes nominally linear and classical linear system control techniques can then be applied in the controller design, with relative efficiency. However, feedback linearization requires an accurate model and high sampling rate for successful implementation [9]. In addition, the computational burden caused by nonlinear and coupled terms cannot be avoided with complex robot structures in feedback control [10, 11].

For example, computed torque (CT) control is a well-known control law that consists of a proportional-derivative (PD) term and the feedback dynamic compensation term calculated from the actual velocity and desired acceleration signals [12]. CT control offers a wide range of advantages, such as high tracking accuracy, low energy consumption, and more compliant control. However, its major drawback is the precise analytical dynamic models required for torque generation, which cannot always be obtained due to modeling errors and unknown disturbances [13]. Inaccurate models will lead to low tracking performance in practice, and a complex robot structure also requires an enormous calculations in the CT controller.

The nonlinear model can be said to increase the difficulty in designing suitable controllers for robot manipulators. Some researchers suggest canceling the nonlinearities by removing the complex inertia terms in dynamic models, as these are negligible at low speeds [14, 15]. However, at high motion speeds, the errors in the model resulting from this approach become significant and would probably reduce stability, leading to loss of effectiveness in tracking performance.

As described above, it is challenging to find a simple and effective strategy for controlling robot manipulators due to the presence of model nonlinearities and errors. To resolve this problem, a mechatronic design methodology known as Design For Control (DFC) was proposed by [16]. The method points out that the performance of the mechatronic system relies not only on the design of the controller but also on the design of its mechanical structure. Controller design is usually considered subsequent to mechanical design, but an appropriate mechanical structure design will lead to a simple dynamic model which makes design of the controller easier. In this paper, an effective design approach is proposed to improve the control performance of a 2R robot manipulator, based on development of the mechanical structure design. With dynamic decoupling, a simplified system is obtained and effective conventional control techniques, rather than complex nonlinear control techniques, are applied. There are three ways to create dynamically decoupled manipulators through mechanical transformation: i) via mass redistribution; ii) via actuator relocation; iii) via the addition of auxiliary links.

To eliminate the coupling and nonlinear torques via mass redistribution, the inertia matrix must be diagonalized and made invariant for all arm configurations [17–20]. The linearization and dynamic decoupling of 2-DOF manipulators via mass redistribution has been considered previously [20]. In this study, all arm constructions yielding decoupled inertia matrices were identified. The proposed approach was applied to serial manipulators in which the axes of joints were not parallel; with parallel axes this approach allows linearization of the dynamic equations but not their dynamic decoupling [21] and cannot therefore, be used with planar serial manipulators. As a result, the inertia matrix cannot be decoupled unless the joint axes are orthogonal to each other in serial manipulator arms with an open kinematic chain structure.

As regards the decoupling of dynamic equations via actuator relocation, a review [18] has shown that the remote-actuation design concept is not optimal from the point of view of precise reproduction of the end-effector tasks, because it accumulates all errors due to intermediate transmissions. It is evidently much better to connect actuators directly to the links than to use transmission mechanisms. The clearance, flexibility, manufacturing and assembly errors of the added transmission mechanisms have a negative impact on the robot's precision.

The linearization of dynamic equations and their decoupling by adding auxiliary links to redesign the manipulator has also been developed [22–24]: dynamic decoupling of the manipulator by connecting a two-link group to the initial structure, forming a Scott-Russell mechanism, was proposed by [25]. Noted that dynamic decoupling via redesign of the manipulator by adding auxiliary links is a promising new approach in robotics.

As discussed above, taking into account the structural features of the 2R spatial serial manipulator, i.e., orthogonality of the joint axes, it can be confidently asserted that dynamic decoupling via mass redistribution is the most expedient approach. This paper is organized as follows: section 2 contains a review of the dynamic decoupling conditions of 2R spatial serial manipulators based on mass redistribution, and the generation of motion by “bang-bang” profile, enabling reduction of the maximal input torque values. In section 3, different conventional linear controllers are used to stabilize the decoupled system and track the desired trajectory. Numerical simulations are also carried out. Section 4 presents conclusions and perspectives.

2 Decoupled dynamics of the 2R spatial serial manipulator and its motion generation via bang-bang profile

Before addressing the problem of control performance improvement, the dynamic decoupling conditions of the 2R spatial serial manipulator (Fig.1) are reviewed below.

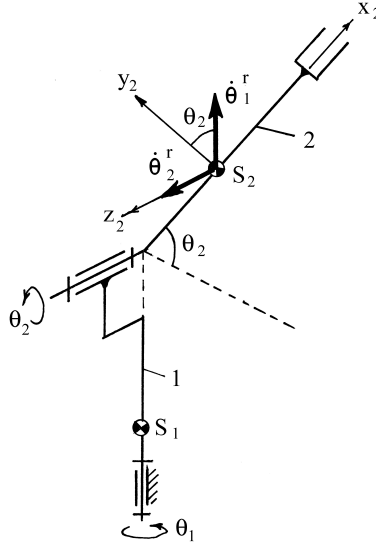


Figure 1. 2R spatial serial manipulator

The manipulator consists of two orthogonal links, 1 and 2, with rotating angles θ_1 and θ_2 . We will distinguish the relative angular velocities of vectors $\dot{\theta}_1^r$ and $\dot{\theta}_2^r$ with $\ddot{\theta}_1^r$, $\ddot{\theta}_2^r$, and the vectors $\dot{\theta}_1$ and the absolute angular velocities $\dot{\theta}_2$ with $\dot{\theta}_1 = \dot{\theta}_1^r$ and $\dot{\theta}_2 = \dot{\theta}_1^r + \dot{\theta}_2^r$.

In the study [17], it was reported that the 2R spatial serial manipulator can be dynamically decoupled completely if the following conditions are satisfied:

- the potential energy of the manipulator is constant (or canceled), i.e. the manipulator is statically balanced;
- $I_{x_2} = I_{y_2} = I^*$, where I_{x_2} and I_{y_2} are the axial moments of inertia of link 2 relative to the corresponding coordinate axes of the system associated with link 2;

As a consequence of such a redistribution of moving masses, the motion equations become linear and decoupled:

$$\tau_1 = (I_1 + I^*)\ddot{\theta}_1^* \quad (1)$$

$$\tau_2 = I_{z_2}\ddot{\theta}_2^* \quad (2)$$

The non-linear dynamic system is thus transformed into a double integrator model and the state space equation of each link can be defined:

$$\dot{x} = Ax + Bu \quad (3)$$

with

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1/I \end{bmatrix} \quad (4)$$

where, I is calculated with inertia moment of each link in Eq.8 and Eq.9. $I = (I_1 + I^*)$ for the first link and $I = I_{z_2}$ for the second link.

As discussed in [17], to generate motion in the dynamically decoupled 2R spatial serial manipulator, it is preferable to apply the “bang-bang” profile (Fig.2), which enables reduction of the maximal input torque values.

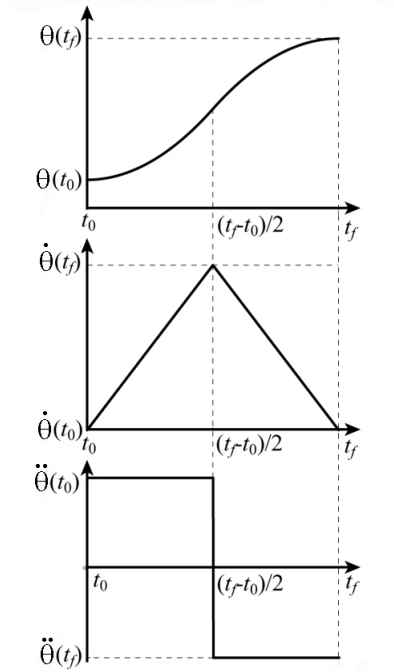


Figure 2. “Bang-bang” profile used for generation of motion in the dynamically decoupled 2R spatial serial manipulator.

Bang-Bang motion profile:

$$\theta(t) = \begin{cases} \theta(t_0) + 2\left(\frac{t}{t_f}\right)^2\theta_f, & t_0 < t < t_f/2 \\ \theta(t_0) + \left[-1 + 4\left(\frac{t}{t_f}\right) - 2\left(\frac{t}{t_f}\right)^2\right]\theta_f, & t_f/2 < t < t_f \end{cases} \quad (5)$$

3 Design of Linear Controller

3.1 Performance Indices

For the controller design, performance indices are introduced to quantify and evaluate system performance. Two performance indices were considered in our case: the integral of the square of the error (ISE) and the maximum input torque. The form of the ISE is defined as:

$$\text{ISE} = \int_{t_0}^{t_f} e^2 dt \quad (6)$$

The ISE discriminates between excessively over-damped and excessively under-damped systems. Tracking accuracy can be evaluated using this criterion. The other criterion introduced was the maximum input torque, which represents input energy. The actuator capacity requirement also depends on the maximum input torque.

3.2 Pole Placement

Since the double integrator model system is unstable, the control technique should increase the stability of the system. Therefore, the speed of response and tracking error requirement should be improved in order to track the bang-bang profile. Pole placement is a well-established design method for linear control systems. We assumed that all state variables were measurable and also available for feedback. It will be shown that if the system considered is completely state controllable, then pole placement is through an appropriate state feedback gain matrix. The system must be completely state-controllable for arbitrary pole placement. The controllability matrix is given by:

$$C = [B \quad AB \quad A^2B \quad \dots \quad A^{n-1}B] = \begin{bmatrix} 0 & 1/I \\ 1/I & 0 \end{bmatrix} \quad (7)$$

Since the control matrix C of the state space equation obtained has full row rank 2, this decoupled dynamic system is controllable. The system input can therefore be defined as $u = -Kx$, where K is the state feedback gain matrix. The state space equation is obtained as follows:

$$\dot{x} = (A - BK)x \quad (8)$$

To ensure stability and speed of response, the desired closed-loop eigenvalues (poles) of $A - BK$ must be negative. Two eigenvalues, λ_1 and λ_2 , were set at -120 . The state feedback gain matrix K can then be determined by direct substitution. The characteristic polynomial for the desired system is:

$$|\lambda I - A + BK| = \left| \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/I \end{bmatrix} [k_1 \quad k_2] \right| = \lambda^2 + \frac{k_2}{I}\lambda + \frac{k_1}{I} \quad (9)$$

This characteristic polynomial must be equal to:

$$(\lambda + 120)^2 = \lambda^2 + 240\lambda + 14400 \quad (10)$$

By equating the coefficients of the terms of the like powers of λ , we obtain:

$$K = [k_1 \quad k_2] = [240I \quad 14400I] \quad (11)$$

The simulation was carried out in Matlab software with $I = 0.4\text{kgm}^2$ and the time-step set at 0.00001. The initial and final values of the rotating angles were as follows: $\theta_i = 0^\circ$ and $\theta_f = 30^\circ$. Tracking performance and input torque are obtained as follows:

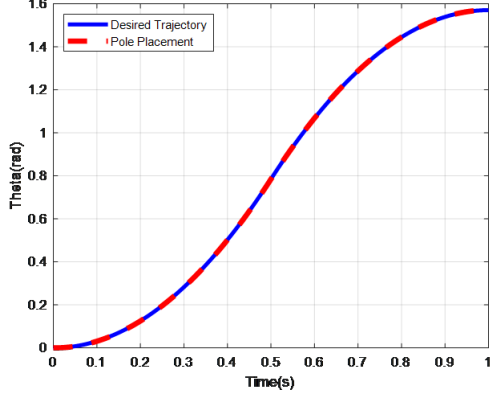


Figure 3. Trajectory Tracking of Pole Placement

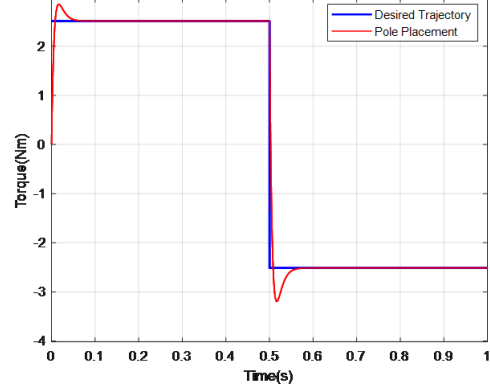


Figure 4. Torque Generation of Pole Placement

From the simulation results, it was observed that the decoupled dynamic system with the pole placement method results in excellent tracking performance by the bang-bang profile. The ISE is 0.0182 and the maximum torque 3.1933 Nm. However, this is a full state feedback control method; when the system is not measurable for velocity, other linear control techniques are considered.

3.3 Lead Compensation

The primary function of the lead compensator is to reshape the frequency-response curve to provide a phase lead angle sufficient to offset the excessive lag phase associated with the components of the fixed system. Since our new linear system, obtained by dynamic decoupling, was a double integrator model, a lead compensator could be used to stabilize the system by increasing the phase margin. A lead compensator in the following form will be used:

$$G_c(s) = K_c \frac{T_s + 1}{\alpha T_s + 1} \quad (12)$$

where K_c , T and α are the coefficients determined with the maximum phase lead angle. Note however that the system's dynamic characteristics need to be modified by increasing the cut-off frequency, which increases the dynamic response speed to track the bang-bang profile. The modified dynamic system expression is then:

$$G(s) = k \frac{I}{s^2} \quad (13)$$

where the value of k is determined to increase the system cut-off frequency w_c . The lead compensator is then applied to help the modified system track the desired trajectory by increasing a certain phase margin. In this case, the selected value of k is 20,000. The gain crossover frequency is therefore increased to 89.44rad/s. This implies an increase in the speed of response. We assumed that the necessary maximum phase lead angle ϕ_m is 70° , and therefore the coefficients in the lead compensator, could be determined. The Bode plot of the compensated system is as follows:

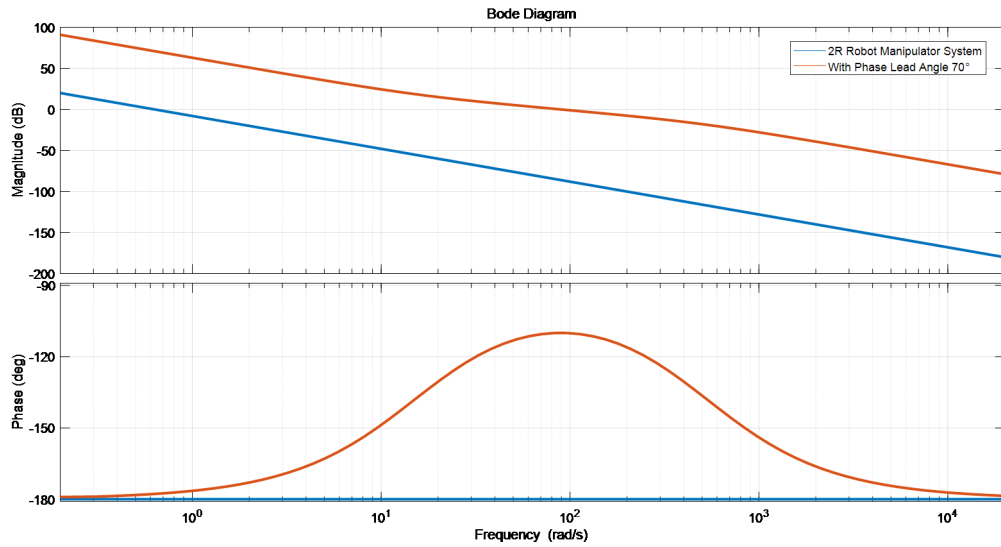


Figure 5. Frequency domain response with and without a lead compensator

As observed in the Bode plot, the maximum phase lead margin was increased to 70° at the gain crossover frequency, which improved the stability. To see the tracking performance and input torque, the simulation was carried out in Matlab/Simulink. The value of the time-step was set at 0.00001s.

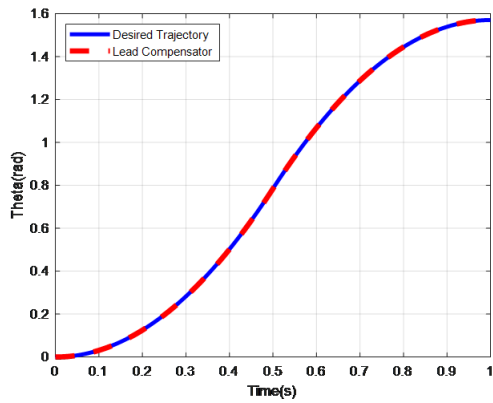


Figure 6. Trajectory Tracking of Lead Compensator

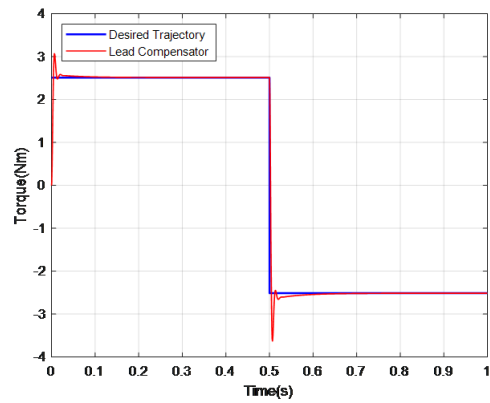


Figure 7. Torque Generation of Lead Compensator

It is evident that the system with the modified gain crossover frequency and lead compensator with 70° phase is capable of tracking the desired bang-bang profile trajectory. The integral square of errors is 0.0399 and the maximum torque 3.6289. In addition, a table with a variety of phase lead angles was created to investigate the influence of different phase lead angles on two criteria:

Tracking Performance	ISE	Maximum of Torque (Nm)
6° Phase Lead Angle	2.10 10 ⁻³	7.18
10° Phase Lead Angle	2.30 10 ⁻³	6.89
20° Phase Lead Angle	3.20 10 ⁻³	6.32
30° Phase Lead Angle	4.50 10 ⁻³	5.78
40° Phase Lead Angle	6.80 10 ⁻³	5.27
50° Phase Lead Angle	1.08 10 ⁻²	4.76
60° Phase Lead Angle	1.90 10 ⁻²	4.23
70° Phase Lead Angle	3.99 10 ⁻²	3.63
80° Phase Lead Angle	1.18 10 ⁻¹	2.87

Table 1. Results of Different Phase Lead Angles

The table shows that the ISE is diminished and the maximum torque increased when the phase lead angle is increased. In application, a large phase lead angle is required to lower the actuator capacity and energy consumption. However, for a system requiring high tracking accuracy, a small phase lead angle is needed. Obviously, there is a trade-off between the two criteria, which also means that the required performance can be achieved by adjusting the phase lead angle, which facilitates the design of the robot manipulator controller.

4 Conclusion

The control of robot manipulators for high-performance and high-speed tasks has always been a challenge for control engineers. Nonlinear control has been developed but it can encounter difficulties such as tracking inaccuracy at high speed and a heavy computational burden. It can be difficult to obtain satisfactory control performance. To resolve this problem, a new mechatronics approach is proposed to meet the demand for control performance. Rather than concentrating on the design of the control algorithm, this approach focuses on redesigning the mechanical structure to obtain a linear and decoupled dynamic system. This offers greater convenience for controller design. The arrangement of centers of mass and inertia redistribution for the links were described to obtain the decoupled and linear dynamic equations for the manipulator. It was demonstrated that the input torques in the dynamically decoupled manipulator we obtained are directly proportional to the input angular accelerations. The classical linear control techniques of pole placement and lead compensation were therefore adopted to track the desired bang-bang profile trajectory. With these classical linear control strategies, the decoupled system can be stabilized rapidly and the desired control performance obtained. The results of the simulations demonstrate the efficiency of the proposed method.

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