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# The basics of electron transport in spintronics: textbook with lectures, exercises and solutions

Vincent Baltz

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# The basics of electron transport in spintronics : lecture notes and exercises

Vincent BALTZ

Physicist, PhD, HDR – CNRS, France



# Foreword

This manuscript is intended for readers who wish to acquire a solid basis for understanding electron transport in spintronics and the fundamental principle of some associated applications. It provides the reader with sufficient initial knowledge to be able to invest further in this field of research. To this end, some of the prominent key notions still widely used in the field were selected with care, with the aim of providing a simple, concise and efficient framework. These selected notions are explained based on simple examples and analytical calculations. The usual terms and specialist jargons are clearly explained, and how subtleties complicate the phenomenon, without altering the physical understanding, is addressed without diversions.

The text has grown out of lectures given to MSc level undergraduates and postgraduates. Therefore, it is designed for advanced undergraduates and postgraduates, as well as researchers and engineers, with a background in condensed matter physics and magnetism. Interested readers are encouraged to complement their prior knowledge by consulting for example the following books: [N. W. Ashcroft and N. D. Mermin, Solid state physics, Saunders College Philadelphia \(1976\)](#) ; [C. Kittel, Introduction to Solid State Physics, John Wiley & Sons \(2004\)](#) ; [J. M. D. Coey, Magnetism and Magnetic Materials, Cambridge University Press \(2010\)](#) ; [E. du Tremolet de Lacheisserie et al \(eds\) Magnetism I & II, Edp Sciences \(2002\)](#).

The text is structured in seven parts. Each part feeds on the previous one. A careful effort has been made to indicate clearly how the parts and the physical phenomena they describe are related to each other. While the text focuses on the fundamental aspects, the implementation of the fundamental physical effects in typical applications is described in details. Orders of magnitude of the various parameters and key phenomena are provided in each section, based on experimental data. The references are concise in order to provide a reasonable initial framework for the interested reader to go further. Physical effects that are more peripheral to current concerns are mentioned wherever relevant and references for understanding them are provided. An [Index of key concepts](#) allows the reader to navigate easily from one key notion to another.

[Chapter I](#) gives a concise overview of what spintronics is and what this discipline contributes to society. [Chapter II](#) lays the foundations of spin-dependent electron transport. Based on the two-current and  $sd$  scattering models, we first present why the electron scattering probability is spin-dependent, and how this effect impacts electron transport, with the example of the current-in-plane (CIP) giant magnetoresistance (GMR) effect. We then present how the addition of spin-orbit interactions mixes the spin states and reshuffles the spin-dependent scattering probabilities, with the example of the anisotropic magnetoresistance (AMR) effect. In [Chapter III](#), we describe in detail the effect of spin accumulation, encountered whenever electrons flow across an interface, due to the distinct current densities between materials of different types. In this chapter, we detail how the electron dynamics are described in this case, and what kind of spin-flip relaxation mechanisms prevail. We elaborate on the conditions on a key parameter called the spin-coupled resistance necessary to maintain the spin polarization across the interface, and present how spin accumulation is at the heart of the current-perpendicular-to-plane (CPP) GMR effect. How intrinsic interface effects such as the spin memory loss, as well as how non-collinearity and non-uniformity alters spin accumulation is presented. [Chapter IV](#) focuses on the process by which spin angular momentum can be

transferred from current (spin angular momentum accumulation) to magnetization, and the type of torque (STT) that this transfer generates. The key parameters involved, including the spin mixing conductance, will be discussed in detail. How STT alters electron transport by providing an additional relaxation channel for the spins will be tackled as well as how STT can trigger oscillations and magnetization reversal. The spin pumping (SP) reciprocal effect to STT will be introduced. Guidance on the morphology of STT in non-uniform magnetic textures, as well as hints on the magnetoelectronic circuit theory widely used in current spintronics will be given. This Chapter ends with a section about spin-orbit torques (SOT), which result from the transfer of angular momentum between lattice and electron orbital (crystal field potential), electron orbital and spin (spin-orbit coupling), and spin and magnetization (*sd*-exchange interactions). In [Chapter V](#) we describe how the intrinsic crystal and spin symmetries, represented by the different symmetry groups, are particularly important notions that have a significant impact on spintronics. With the example of the different Hall effects, whether they be unquantized or quantized, we present what the Berry formalism implies for intrinsic physical effects. The importance of breaking spatial or temporal inversion symmetries is detailed and illustrated. [Chapter VI](#) concludes the lectures with a non-exhaustive presentation of some of the current topics to which the interested reader may wish to turn, building on the achievements of the previous chapters, which now allow him/her to go further and understand other related or more elaborate phenomena of spintronics. Finally, [Chapter VII](#) provides a series of eight comprehensive exercises and corrections. They are specifically designed to illustrate the notions seen in the previous Chapters, and open up to other spintronic effects based on the acquired notions. These exercises deal with AMR (§II), domain wall AMR (§III), the drift-diffusion equation for spin accumulation (§III and IV); spin conductivity mismatch in CPP-GMR (§III), spin pumping and the inverse spin Hall effects (ISHE) (§IV), the extrinsic spin pumping contribution to the damping of magnetization dynamics (§IV), spin Hall magnetoresistance (SMR) (§II, III and IV), and the intrinsic anomalous Hall effect (AHE) (§V).

**V. Baltz**

Grenoble, June 2022

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