A Neural Tangent Kernel Perspective of GANs
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We consider the NNs in the NTK regime. This enables a theoretical study of their evolution w.r.t. training time $t$: $\partial f_t / \partial t = \mathcal{L}(f_t, \nabla^2 \mathcal{L}(f_t));$

**Discriminator Inner Loop**

Under mild assumptions, $f_t$ is uniquely defined and:

$$\begin{align*} & \forall \alpha \in \mathbb{R}, \ f_{t+\alpha} = \alpha \mathcal{T}_w + f_t \quad (\text{w.r.t. } \alpha) \text{ in Stein flow, as fast as possible (locally).} \\
& \mathcal{T}_w : \text{smoother over the whole input space by sending it to } H_{\mathcal{T}}. \\
& H_{\mathcal{T}} \text{ depends on discriminator architecture.} \\
& \text{Differentiability of the Discriminator:} \\
& \mathcal{D}_{\text{fully differentiable (almost) everywhere.}} \text{over } \mathcal{D} \text{ in NTK framework.} \\
& \mathcal{D}_{ \text{spatial gradient of the discriminator } \nabla \mathcal{L}(f_t) \text{ is well-defined.} } \\
& \mathcal{D}_{ \text{parameter gradient of the generator } \nabla^2 \mathcal{L}(f_t) \text{ is well-defined.} } \\
\end{align*}$$

**Resulting Convergence Results**

Our finer-grained framework allows us to derive novel convergence insights, with highlighted results below.

**Gradient Flow of Generated Distribution:**

$$\begin{align*} & \partial U_{\mathcal{L}(f_t)} / \partial t = \mathcal{L}(f_t, \nabla^2 \mathcal{L}(f_t)); \\
& \mathcal{L}(f_t, \nabla^2 \mathcal{L}(f_t)) = \mathcal{L}(f_t) \quad (\text{w.r.t. } f_t) \text{ in NTK flow, as fast as possible (locally).} \\
& \mathcal{L}(f_t) \text{ is the distribution of } f_t. \\
& \mathcal{L}_t \text{ approaches } \mathcal{L}_{\text{infinite}} \text{ as } t \to \text{infinite.} \\
\end{align*}$$

**IPM GANs $(a = b = s)$ and NTK MMD:**

We find $f_0 = f_t + (\mathcal{L}_{\text{infinite}} - \mathcal{L}(f_0))$, hence, if $f_0 = 0$:

$$\mathcal{L}(f_t) \propto MMD_{\text{NTK}}(a, b).$$

**Empirical Study**

We assess the adequacy of our framework by observing how close finite and infinite-width regimes are.

**Toolkit GAN(TK)$^2$**

- **GAN analysis toolkit based on our framework.**
- **Written with JAX with the Neural Tangents library.**
- **Link to paper and code.**